ELECTRICAL COMMUNICATION

Technical Journal of the
International Telephone and Telegraph Corporation
and Associate Companies

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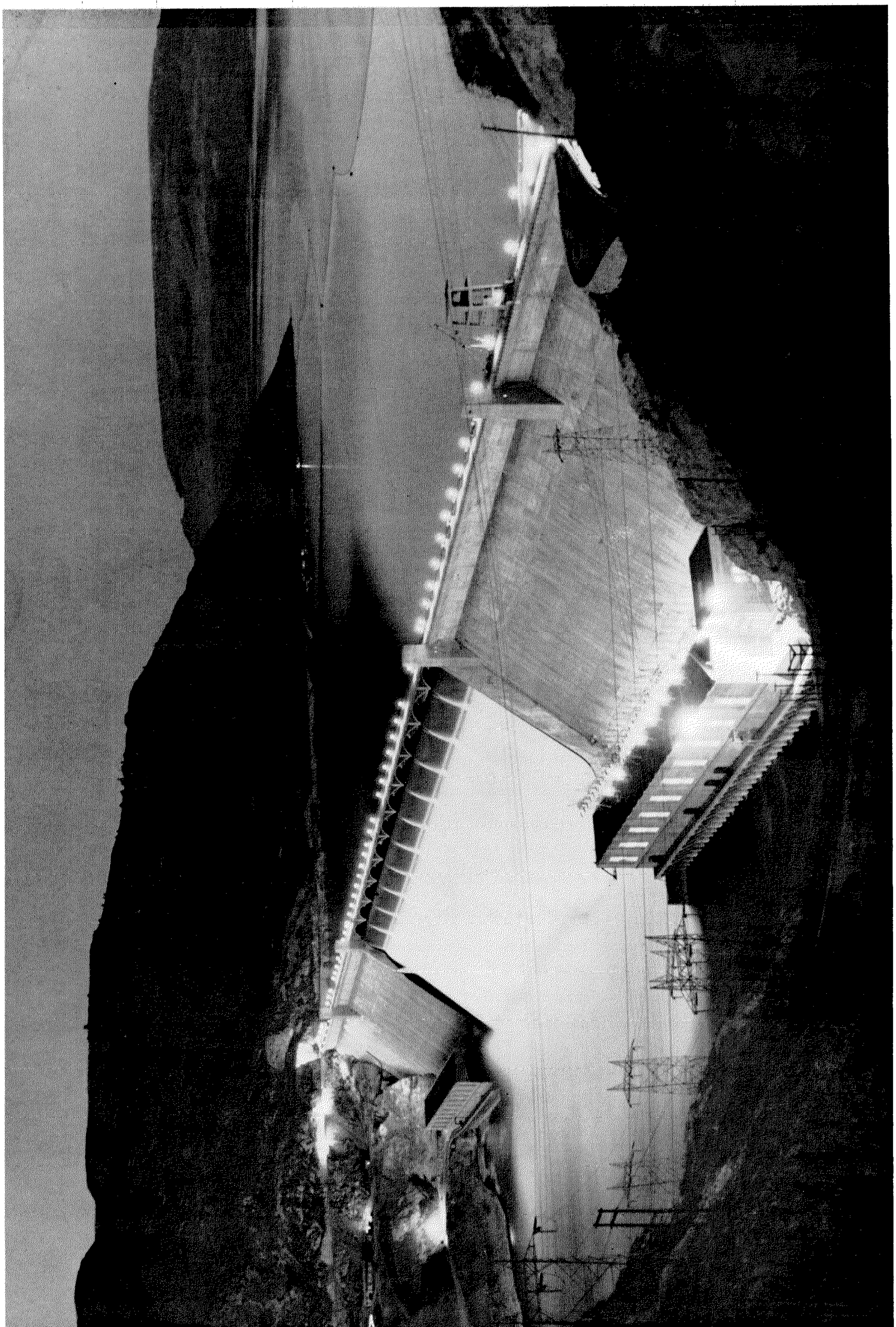
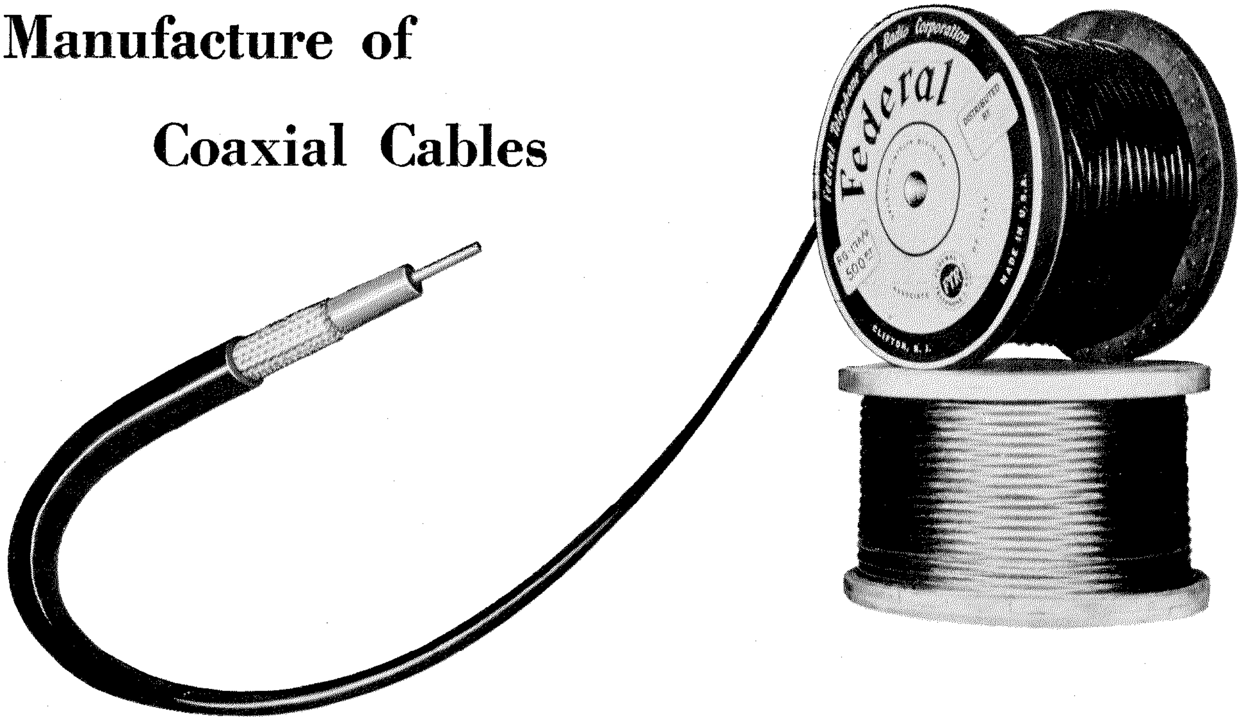


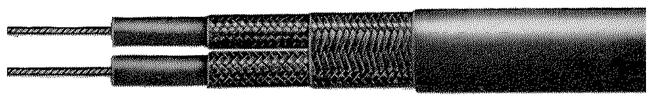
Manufacture of Coaxial Cables



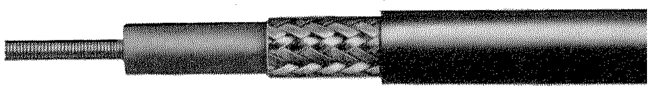
FEDERAL Telephone and Radio Corporation, at Clifton, New Jersey, is one of the largest manufacturers of coaxial-type cables in the world. These cables are extensively used for the transmission of radio-frequency power, particularly at the higher frequencies where low losses and good shielding from external interference are important.

The following few pages show pictorially the various stages in the manufacture of one of the simpler types of this cable. In the illustration above, the end of a reel of cable has been stripped to show the central conductor, a solid copper wire; next is a layer of polyethylene plastic insulation, over which is a braided-wire shield, and finally, a noncontaminating vinylite plastic sheath for over-all protection. If the cable is intended for rough use in an exposed location, a braided layer of aluminum or steel armor wires may be applied over the sheath—the bottom reel in the above picture contains such armored cable. Some of the other types of cable that are fabricated on the versatile machinery at the plant are shown at the right in approximately actual size.

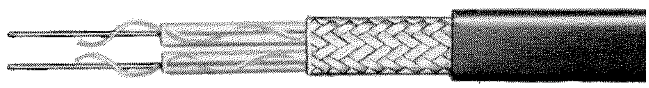
The extrusion machine shown in numbers 3 and 4 of the series may be considered the heart of the entire process. This machine must be kept running continuously by splicing the end of one reel of wire to the start of the next. The extruder is stopped only when a different type of cable is to be made. At one time, a continuous run of 400 miles of type *RG-8/U* cable was made on one of the four extruders at the plant. Wire passes through an extruder at roughly the speed of a fast walk, and very close coordination of speeds and temperatures must be maintained constantly.



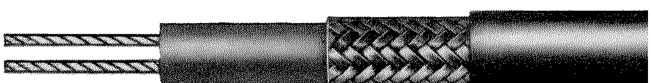
A balanced line with three separate braided shiel



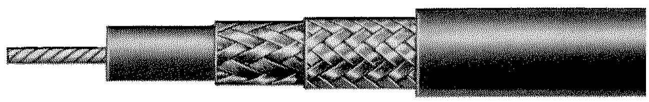
High-impedance delay line using a spiraled conduct



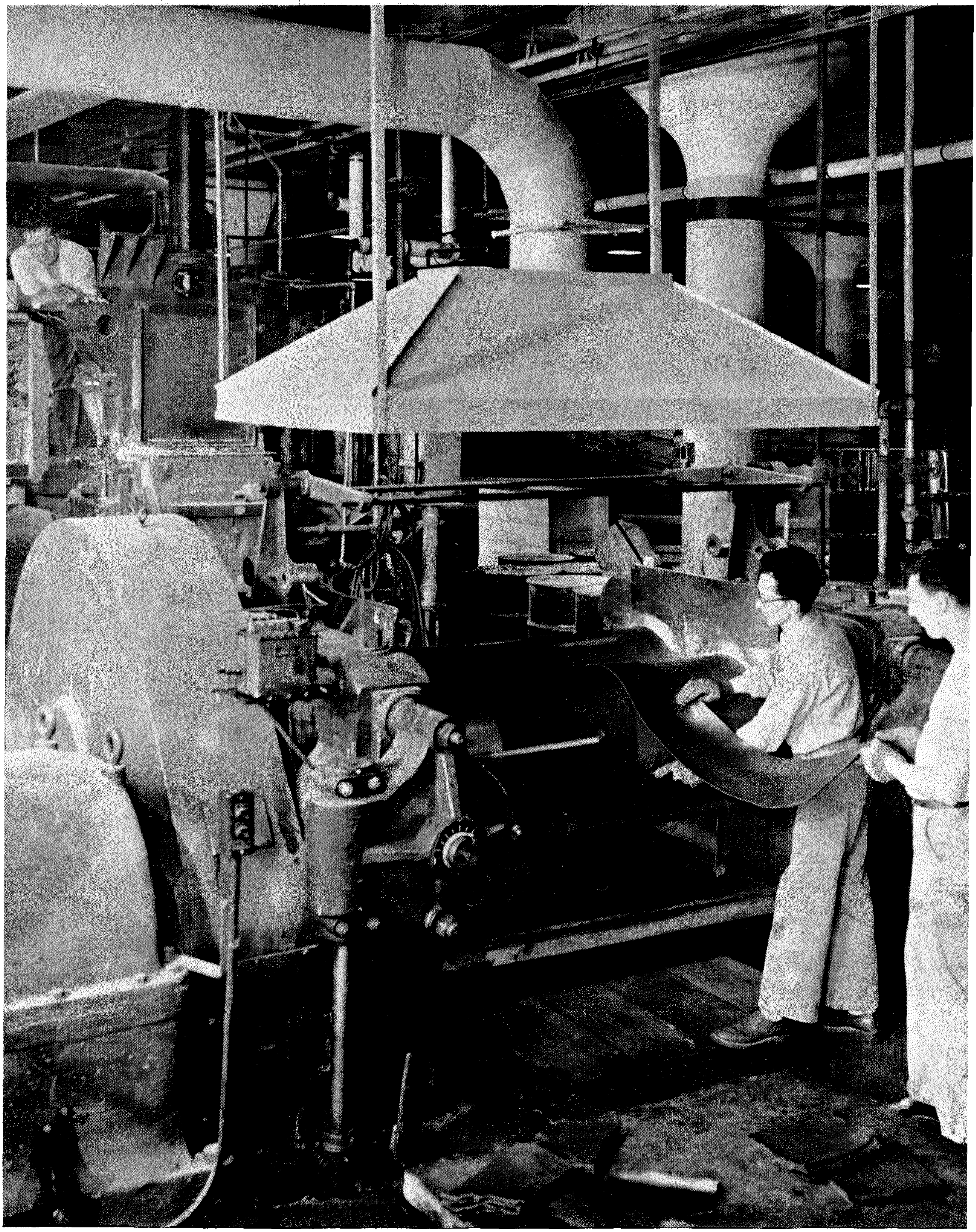
Balanced air-spaced line for high-impedance circu



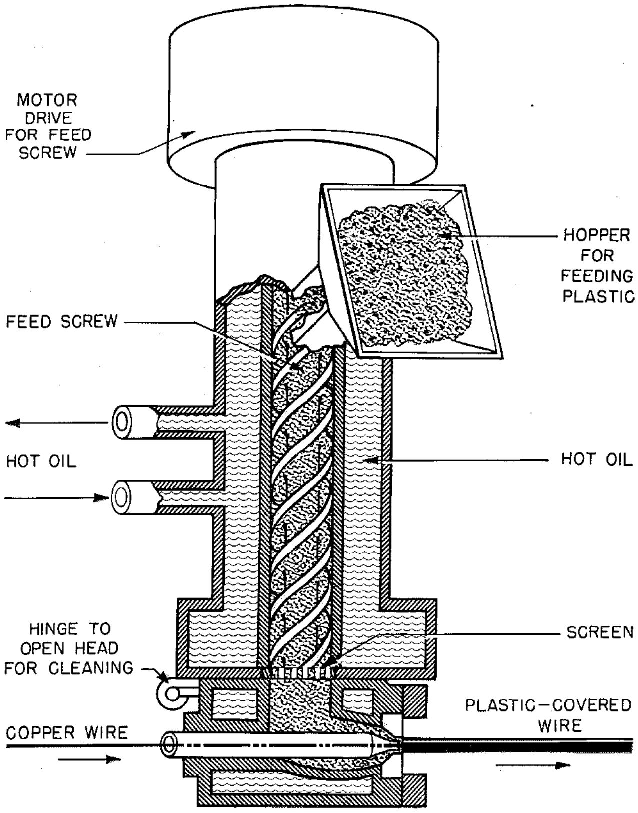
Balanced twinax line; two conductors with one shi

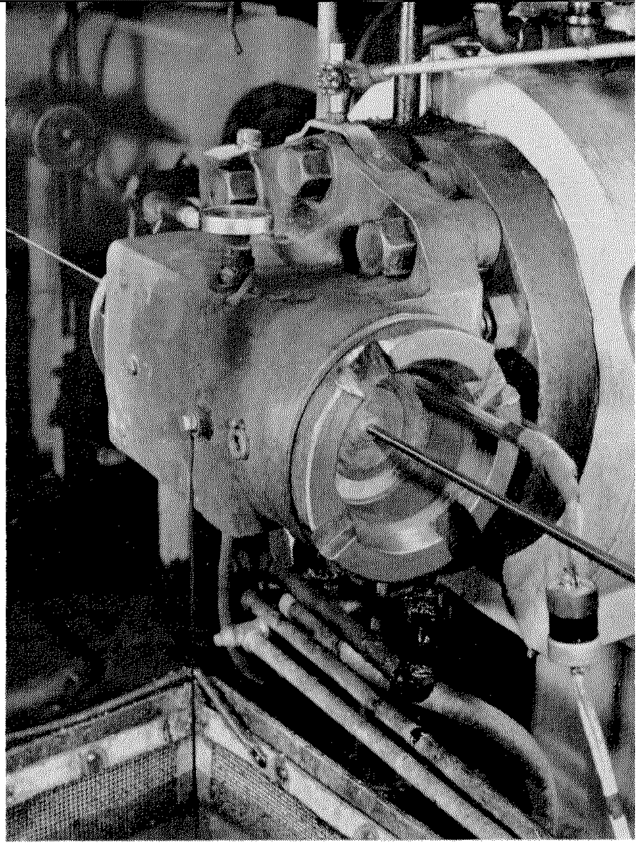


Double-shielded coaxial line with stranded conduc

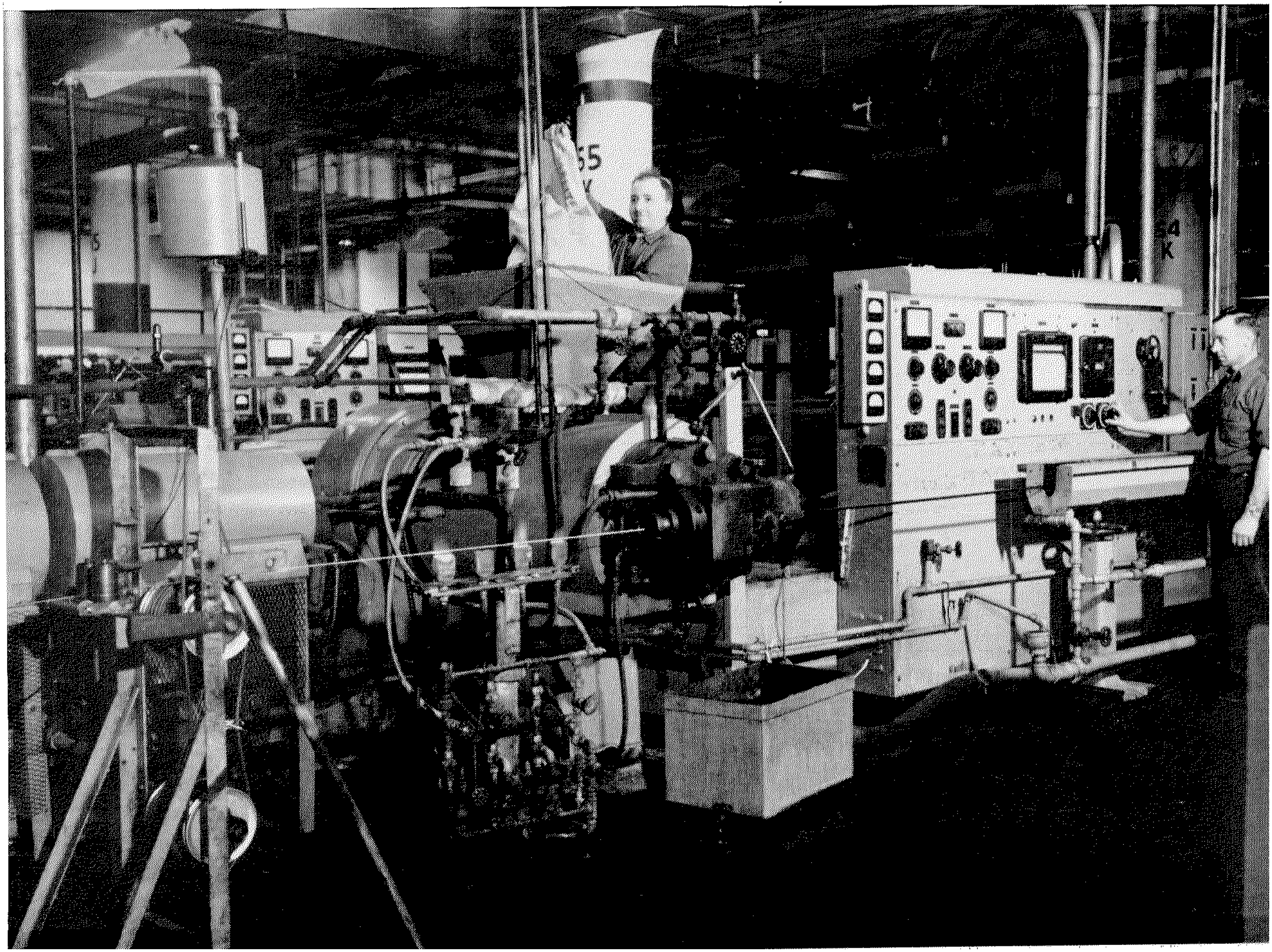


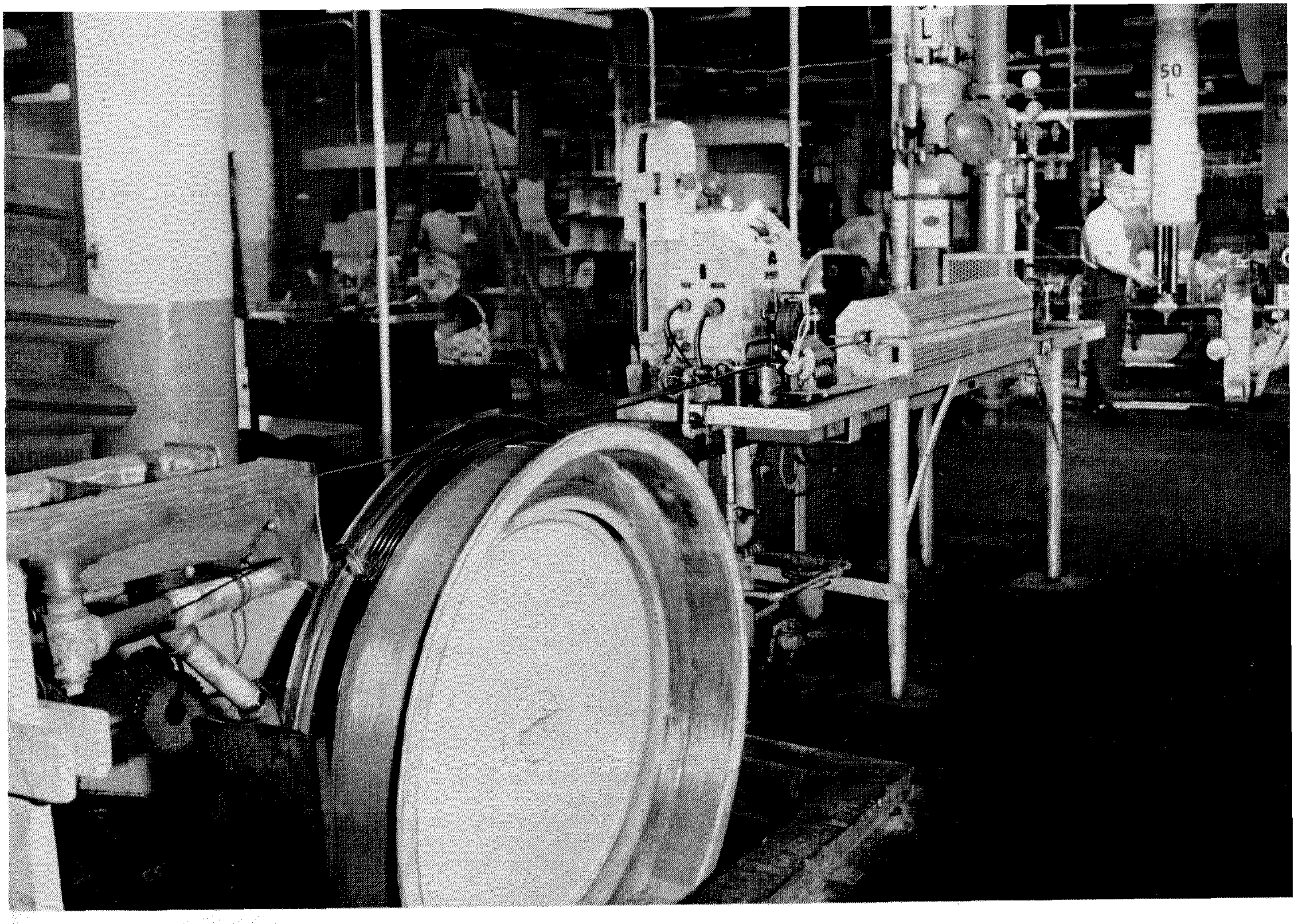
1. Both polyethylene plastic for insulation and vinylite for the sheaths are compounded in the Banbury mixer in the background and then milled on the heated rolls. Plastic in the form of a powder is mixed with a liquid plasticising agent and any necessary coloring matter in the steam-heated mixer, and the doughlike mass is rolled out in thin strips. These are cut to convenient sizes, and after being ground to a granular form in a chopper the plastic is ready to be placed in the hopper of the extrusion machine.

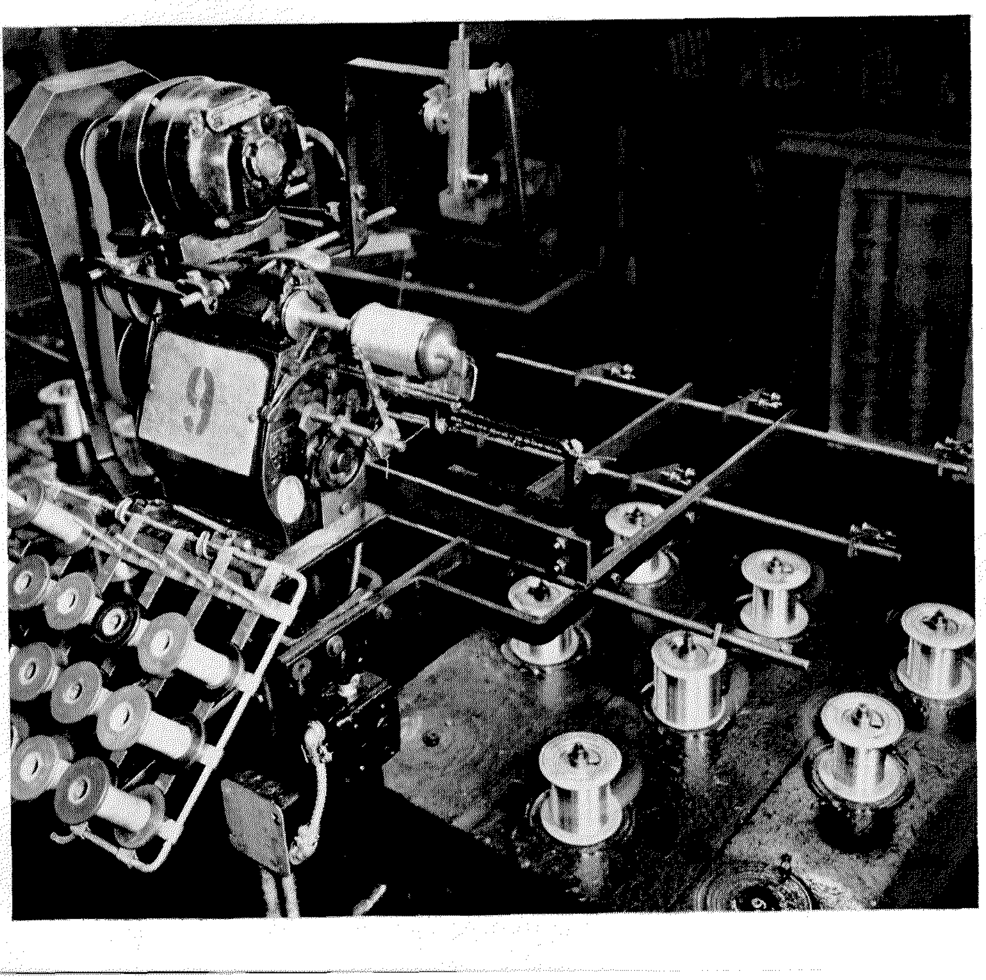




2. An over-all view of the front end of an extruder is shown below. The wire passes from a reel at the left, through a tension-maintaining device, and then through a series of gas flames that warm the wire and drive off any moisture. After the hot plastic is applied in the extruding head, the wire goes through a 300-foot trough of water. The temperature of the water in the various parts of the trough is adjusted so that the extruded plastic is cooled very slowly. Details of the extruding head are shown in the diagram and view above.



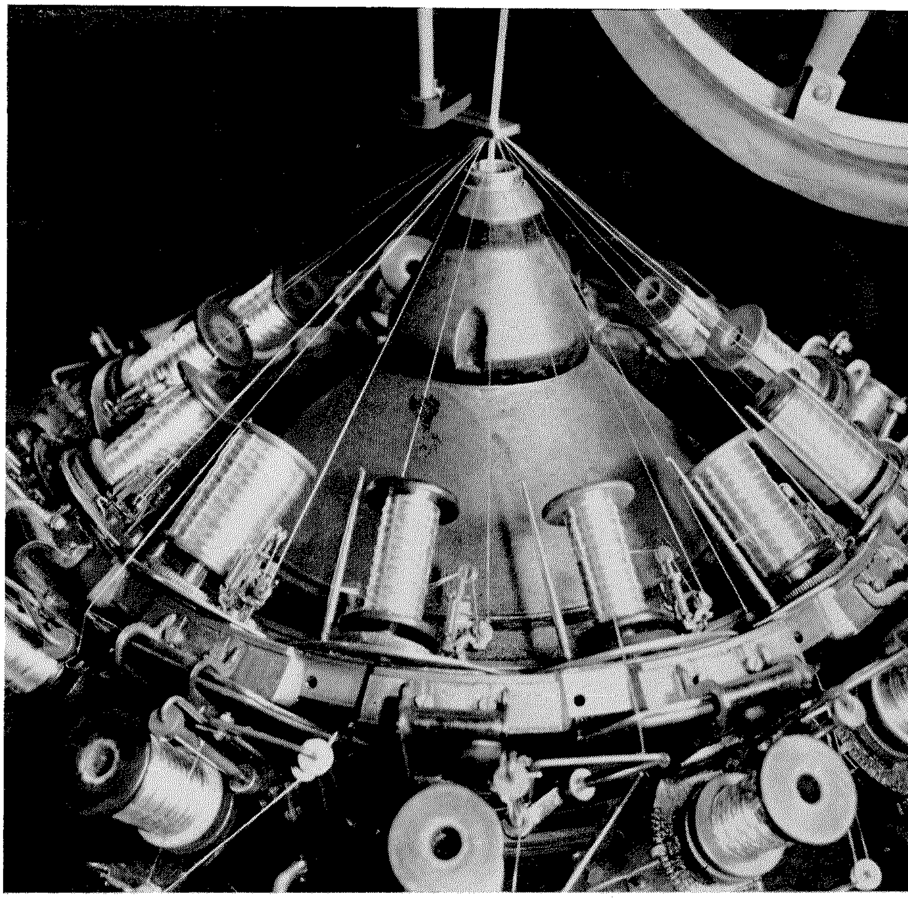




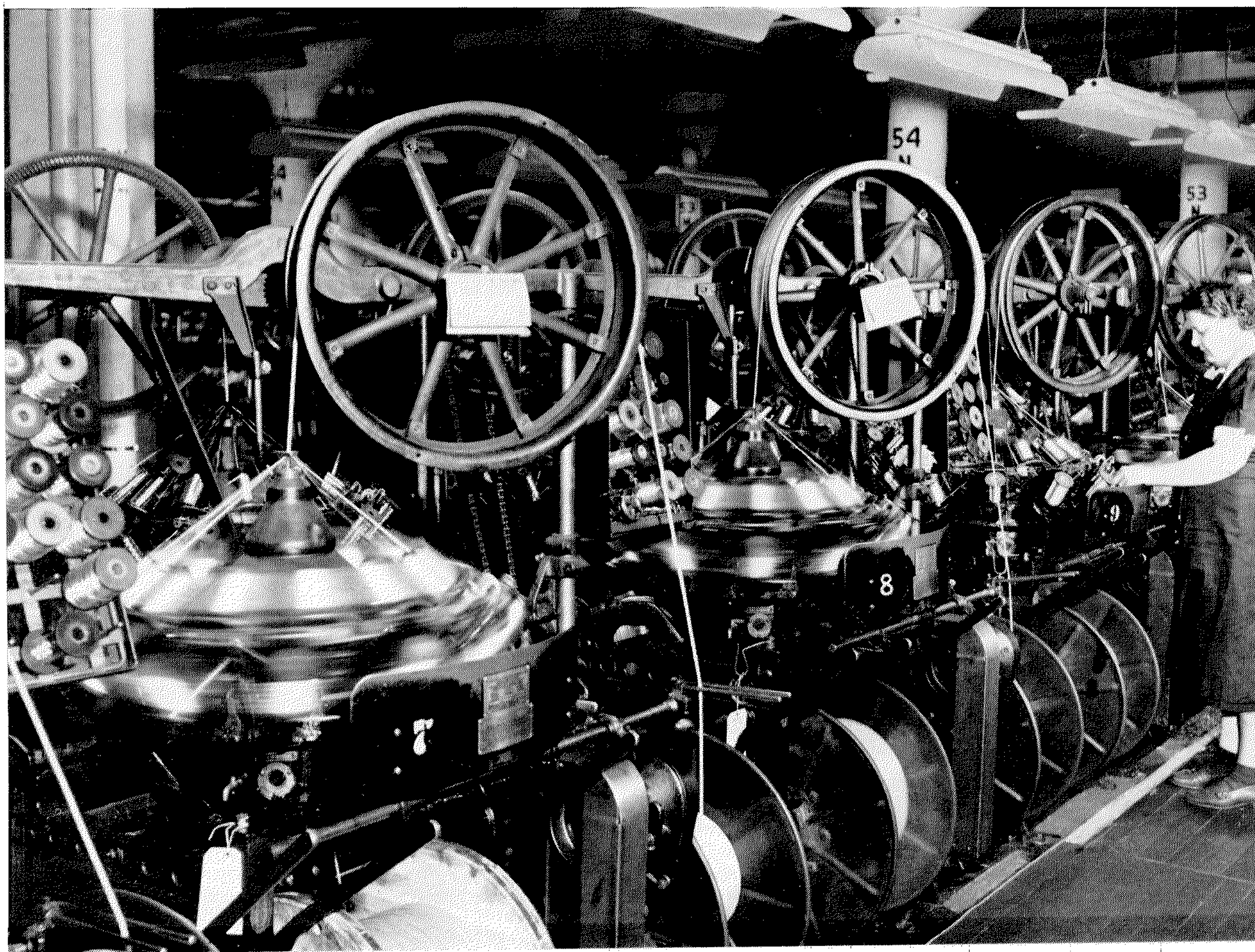
3. The view above shows the back end of the extruder, the large wheel in the foreground being the capstan that pulls the wire from the reel through the extrusion head and the long trough of water. On the table in the center of the picture are an air-blast drier, a footage-counting machine, and a high-voltage breakdown tester, in which 10 to 20 kilovolts are applied to check the homogeneity of the polyethylene insulation.

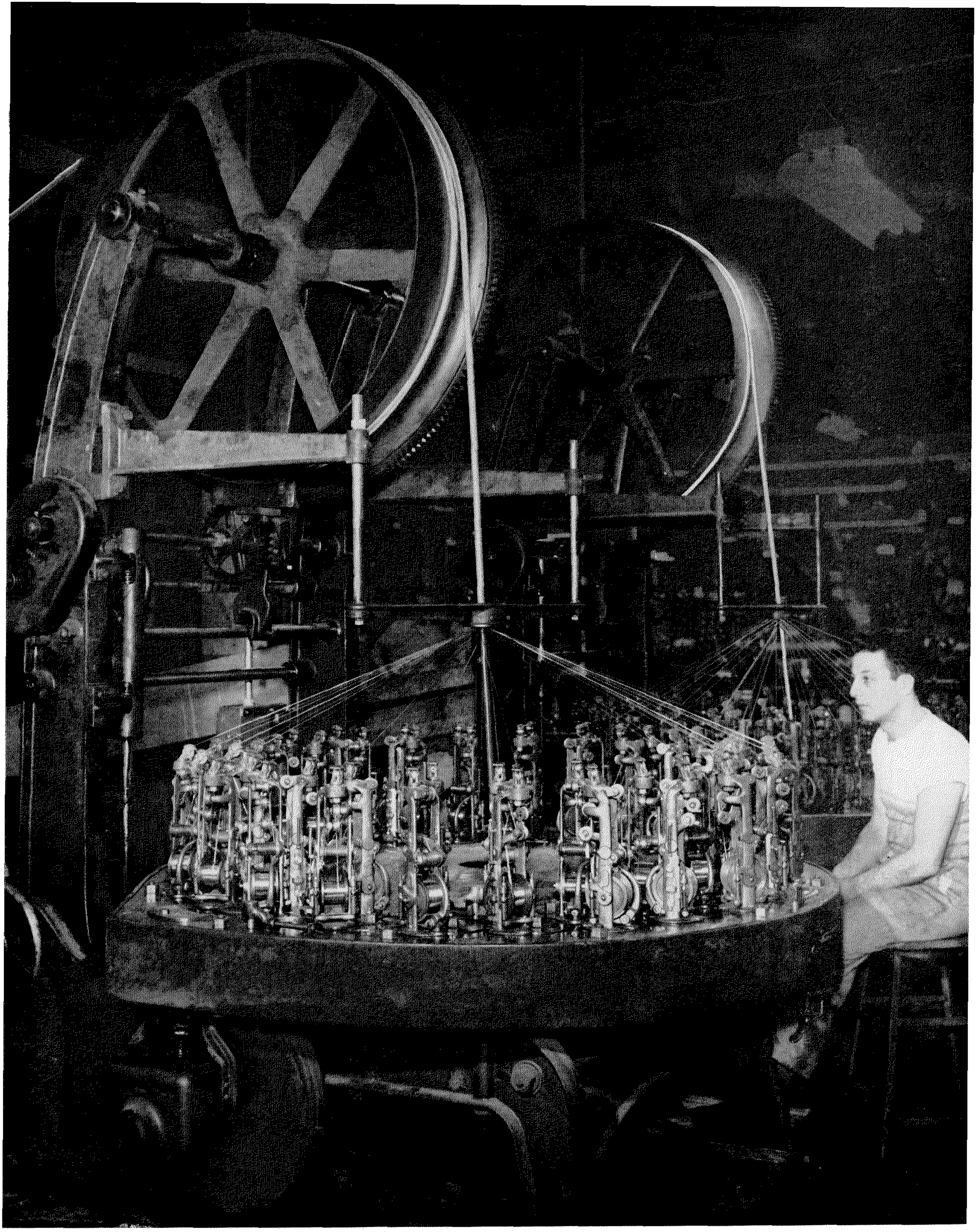
4. The next step is the application of the braided-wire shield to the cable. The shield consists of a number of ribbons of wire interwoven to form a complete covering over the polyethylene insulation. Each of these ribbons is formed of anywhere from 2 to 7 or 10 separate wires of very small diameter that must be laid exactly parallel to each other to form the ribbon. Perhaps the most critical operation in braiding is the filling of the bobbins with these thin wires. The operation is illustrated at the left, where 7 wires are being wound on a bobbin.

5. A close-up of a braider is shown at the right. The insulated wire is pulled upward through the center of the cone. Two circles of 12 bobbins each rotate in opposite directions around the wire. The strands from the lower circle of bobbins are alternately passed above and below the strands of the upper circle. Thus, 24 separate ribbons are tightly interwoven on the cable. This same machine may be used to apply a braid of steel wire over the outside of a small armored cable.



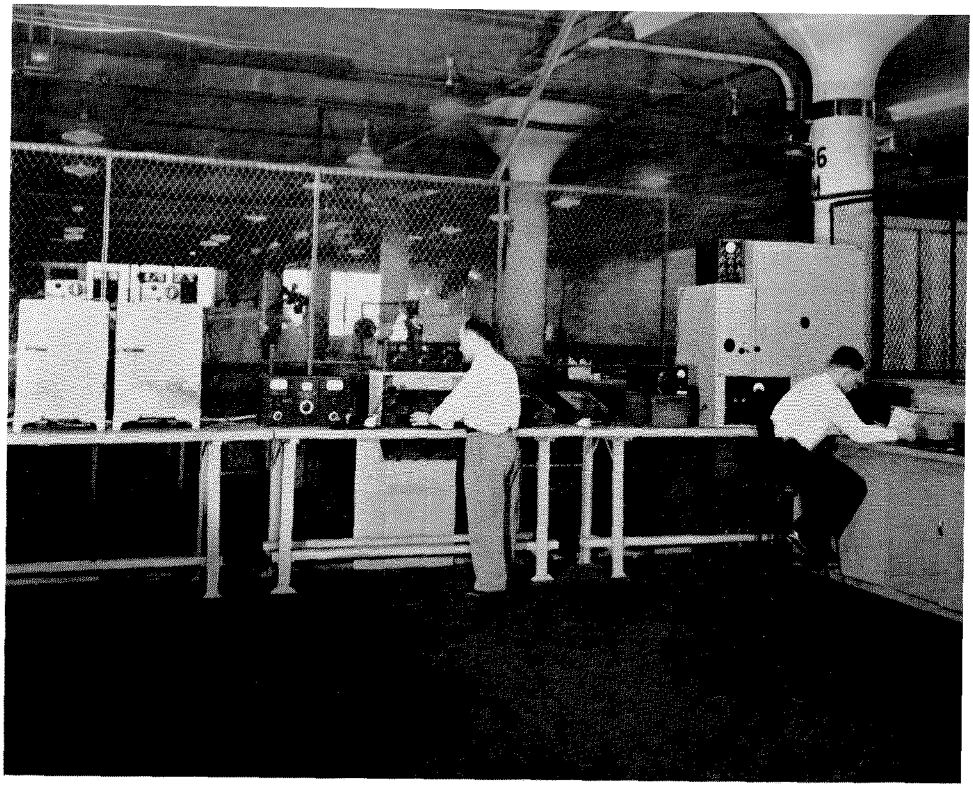
6. A single operator is capable of replacing depleted bobbins and performing other necessary duties on a series of 10 braiders. Such a row of machines is shown below. The insulated wire is pulled from the reel below the circles of bobbins by the large wheel at the top of each machine. The braided cable is rewound on the second reel under the braider. The machines operate very rapidly, approximately 6 to 10 feet of cable being covered with braid in a minute.



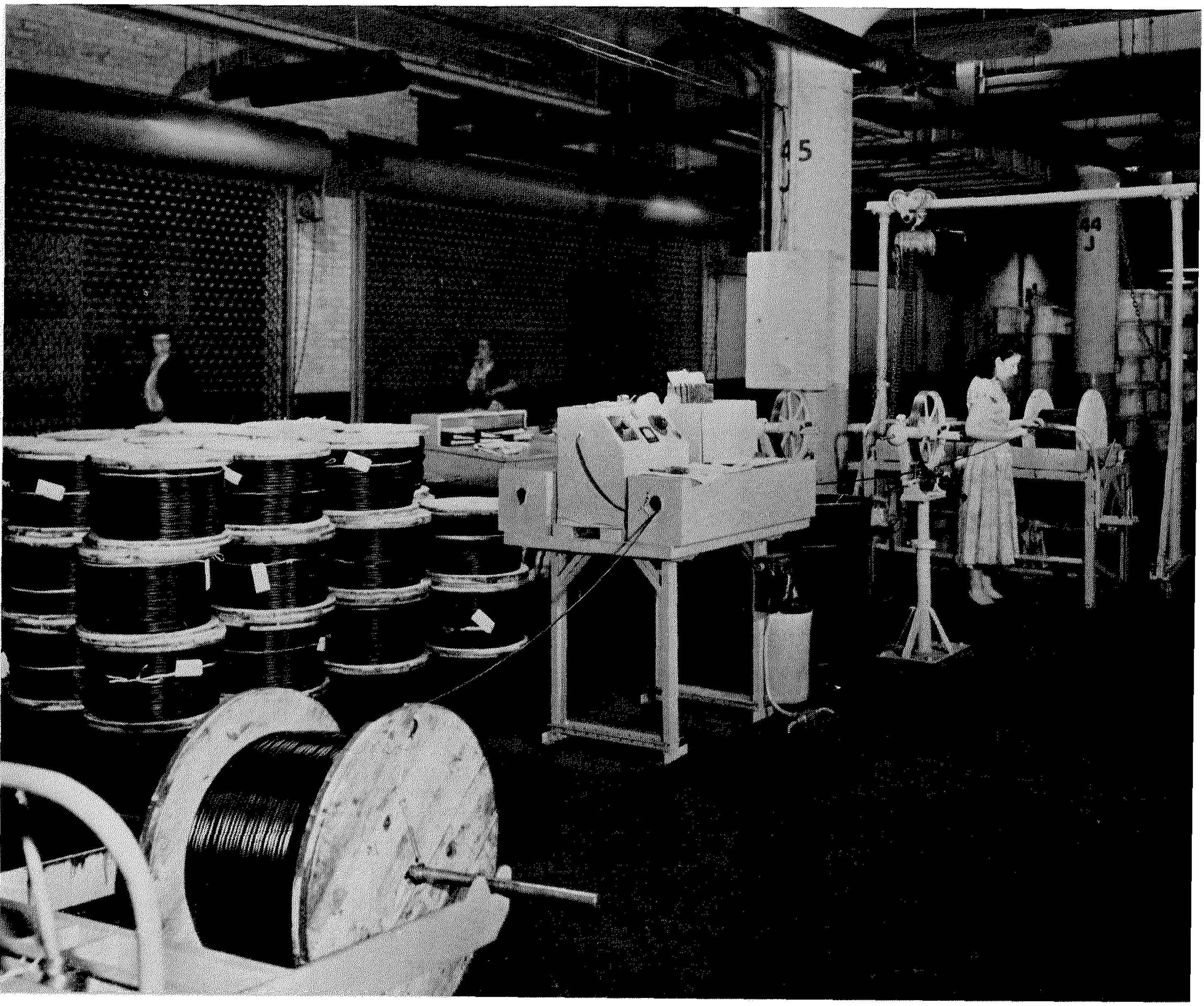


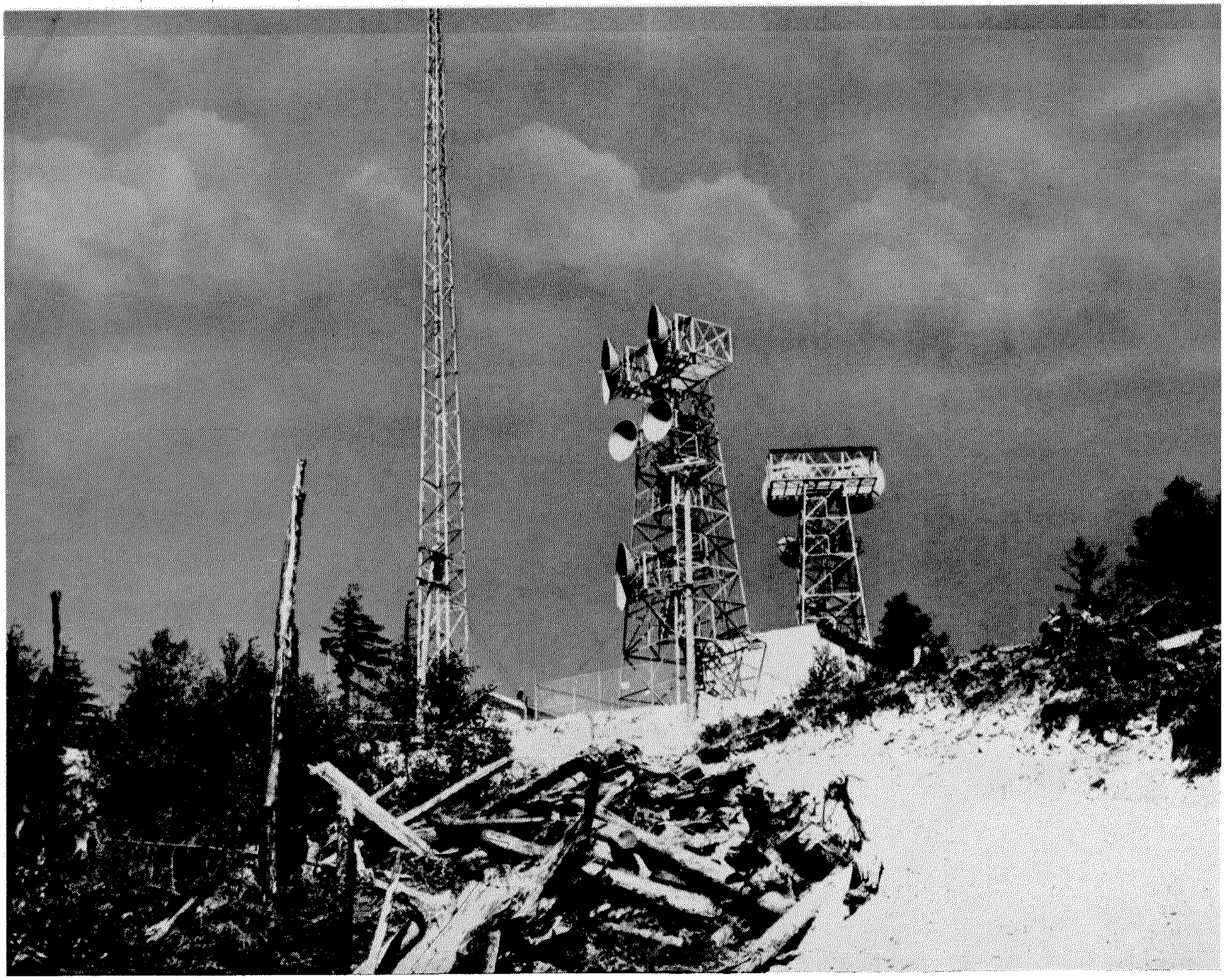
7. **The above photograph shows a braider that is used for cables of larger diameter than those illustrated in 6. On this machine, the ribbons of braiding wire are carried on the bobbins in the shuttles. The 48 shuttles on the table are divided into two series of 24 each that rotate in opposite directions around the central cable, alternately passing on opposite sides of each other. After braiding, the cable is returned to an extruder, where, with a slight modification of the dies in the head, the vinylite sheath is applied.**

8. The maintenance of a high standard of quality requires that a complete laboratory be available to check the electrical and mechanical properties of the product. In the area illustrated at the right, instruments are used to test samples for voltage breakdown strength, characteristic impedance, electrical losses, low- and high-temperature flexibility, and the effects of age.



9. The view below shows another part of the inspection and shipping department where high voltage is applied between the braid of the cable and the outside of the sheath to test the insulating properties of the sheath. While this test is being made in the box at the center of the picture, the operator is reeling up the cable and will cut it for shipment on reels in the lengths ordered.





Atop Squak Mountain

Fault Location by Pulse-Time Modulation*

By R. W. HUGHES and NELSON WEINTRAUB

Federal Telecommunication Laboratories, Incorporated; Nutley, New Jersey

THE PROBLEM of locating faults on a power line has long plagued the power industry. It is particularly troublesome since approximately 90 percent of all faults are not sustained and hence the location of the fault must be determined on an instantaneous basis. In order to keep a record of the time and location of all faults, a method based on the measurement of the time required for a wave to travel from the fault to a substation appears most practical.

* Reprinted with added photographs from *Electrical Engineering*, v. 69, pp. 1009–1011; November, 1950.

The system proposed by Stevens and Stringfield [1] is based on this method and is explained in detail as follows:

A. A fault occurs somewhere between *A* and *B*, the two ends of the power line. This starts a steep wave-front surge along the line in both directions at the rate of 0.186 mile per microsecond.

B. Assuming *A* to be the recording station, the arrival of the surge there causes an electronic counter to be triggered.

[1] R. F. Stevens and T. W. Stringfield, "Transmission Line Fault Locator Using Fault Generated Surges," *Transactions of the American Institute of Electrical Engineers*, v. 67, Part 2, pp. 1168–1178; 1948.

C. At station B, the surge is detected, converted to a convenient-sized pulse, and transmitted from B to A via a broad-band radio link.

D. The arrival of this second surge pulse stops the electronic counter, and from the elapsed time the location of the fault may be calculated.

E. By the addition of some simple supplementary equipment, the counter can be arranged to read distance to the fault directly, and this permits the recording of the information for a permanent record in a fashion similar to the following.

May 10, 1949 03:45:11 168.6 MI

It is of interest to note that the fault-locating system is based on the following facts.

A. Faults produce steep wave fronts, which may be detected from the normal power frequencies.

B. The speed of propagation of these wave fronts along a transmission line is constant within a very small percentage.

C. The attenuation of the wave fronts is small enough to allow propagation of the order of several hundred miles, including passage through substations.

D. Available electronic counters are capable of measuring time intervals as small as 0.625 microsecond. Since the speed of propagation is 5.37 microseconds per mile, it is then feasible to measure propagation times corresponding to an accuracy of almost 0.1 mile.

The proposed Bonneville Power Administration pulse-time-modulation system is shown in Figure 1. The first portion, now installed, includes the stations: J. D. Ross, Rainier, Chehalis, Olympia, Squak Mountain, Seattle, Covington, and Snohomish. As shown in Figure 2, it consists of a terminal, two video repeaters, an audio repeater, an audio branching repeater, and three terminals. Only eight of the 23 channels are being utilized at present. Over this

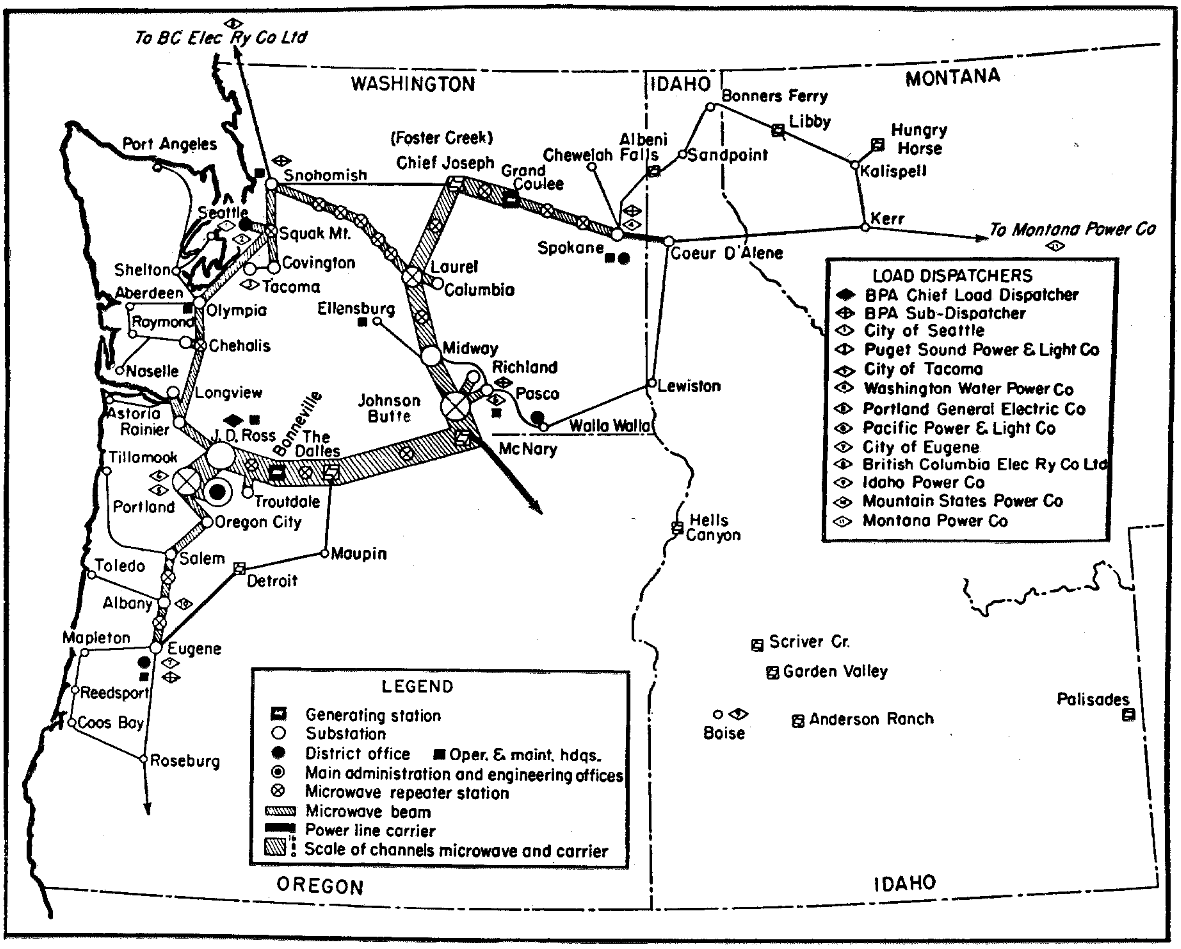
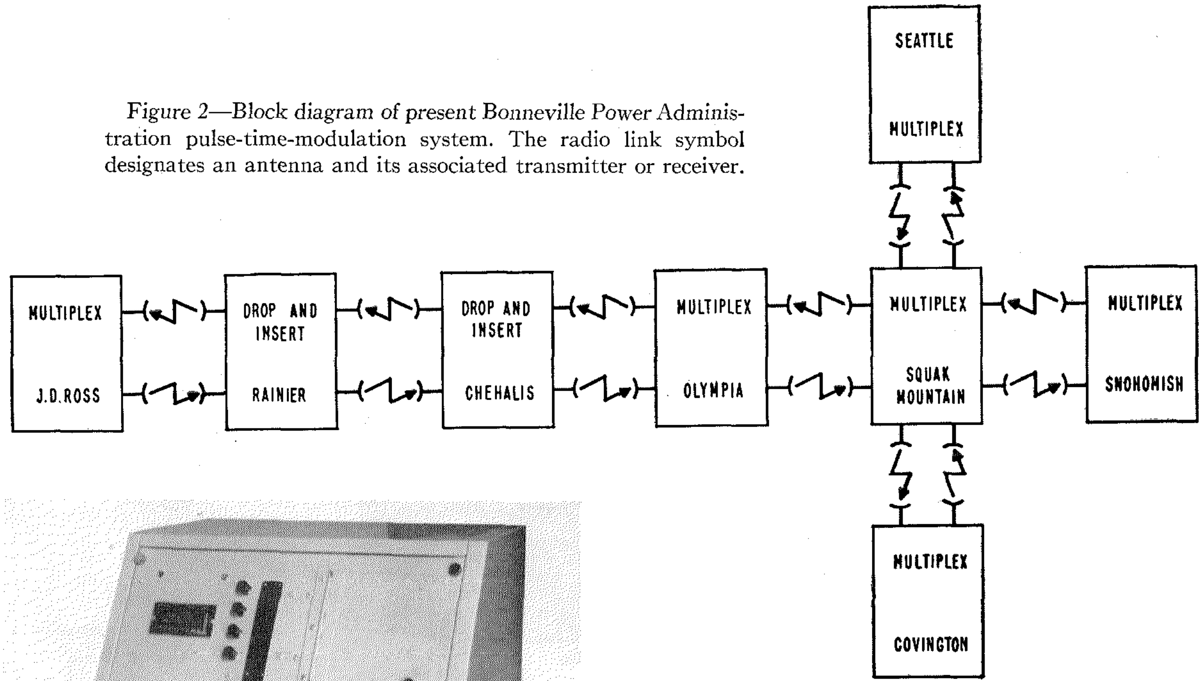


Figure 1—Proposed Bonneville Power Administration communication system as of 1954. Reprinted from R. F. Stevens, "Microwaves Supplement Present Channels," *Electrical World;* January 1, 1949.

microwave link, the fault pulse must be transmitted and hence provisions made for supplemental equipment. This equipment and its necessities are described successively in the following paragraphs.

The basic requirement that the pulse-time-modulation radio relay system pass a 10-microsecond pulse with rise and decay time of 0.2 microsecond maximum is permissible with no modification whatsoever, since the bandwidth of the system is greater than necessary to support the fault pulse. Also, the modulated pulse train

Figure 2—Block diagram of present Bonneville Power Administration pulse-time-modulation system. The radio link symbol designates an antenna and its associated transmitter or receiver.



Front view of power-line fault-locator printing equipment. Extending from the upper section may be seen the paper tape on which is printed the date, time to within a second, and distance to within a tenth mile from the end of the power line to each fault.

could be interrupted for several hundred microseconds with a barely perceptible effect on voice communication, so no modifications are necessary to avoid interruption of normal traffic. Paradoxically, it is the normal traffic that will affect the fault pulse. To be explicit, the leading or trailing edge of the fault pulse might be displaced in the event a channel pulse of the communication pulse train occurs at the same time. To avoid this, it is necessary to blank out the pulse train prior to the insertion of the fault pulse. For reasons involving repeater points, the trailing edge of the fault pulse was chosen as the time-defining edge. Hence, at the insertion point of the fault pulse, the pulse train is blanked out for approximately 15 microseconds, starting simultaneously with the fault pulse, but lasting five microseconds longer to protect the trailing edge. This is accomplished in an 8-tube unit called the "fault-pulse modulator," normally rack-mounted adjacent to the modulator multiplex output. It requires 5 inches of rack space and is provided with

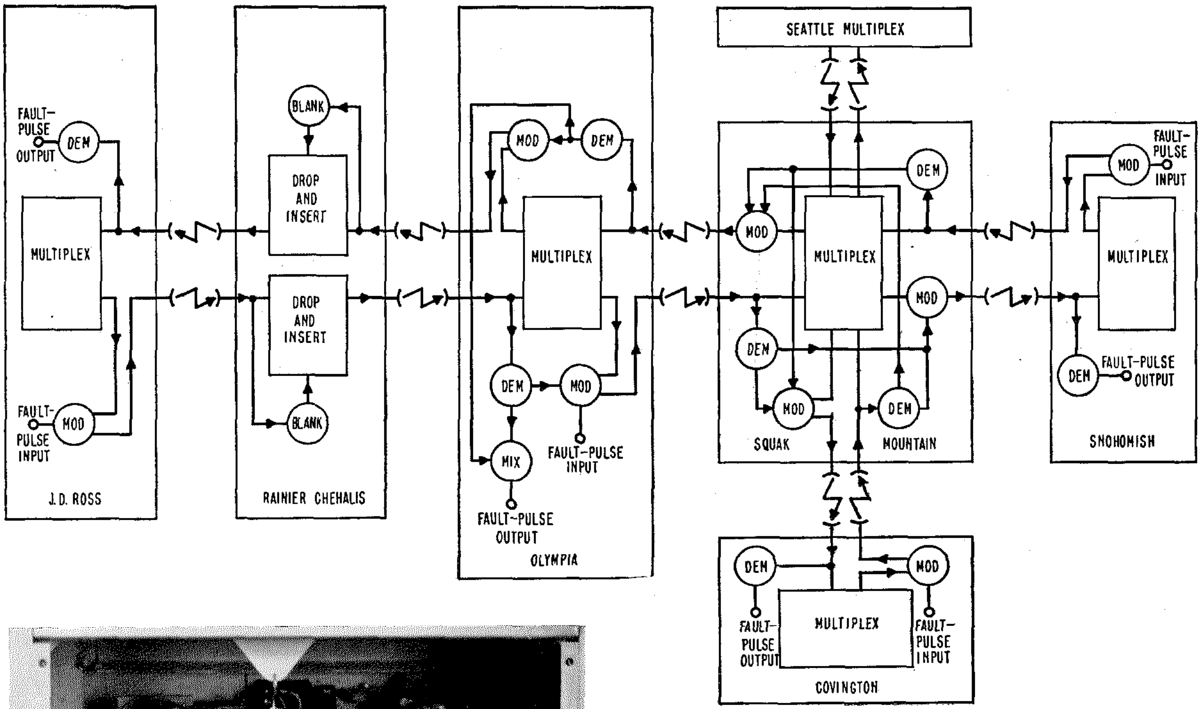
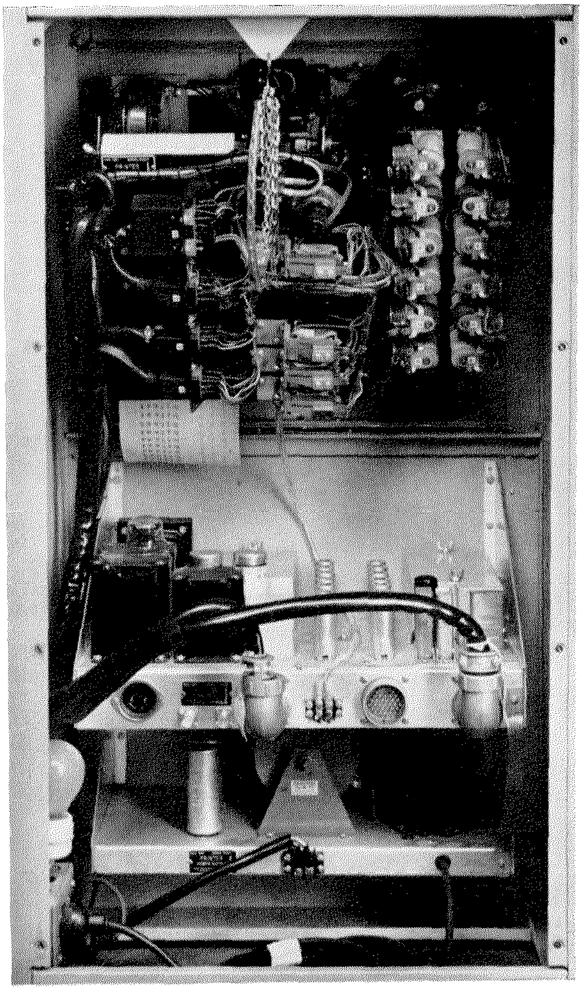


Figure 3—Present Bonneville Power Administration system block diagram showing fault-location equipment. The Rainier and Chehalis stations being identical and adjacent, only one is shown. Circles identify equipment for handling fault pulses: Blank = blanking-signal generator, DEM = demodulator, MOD = modulator, and MIX = mixer.



Rear view of printer cabinet.

two inputs for fault pulses to allow for dual inputs from branching power lines.

By video repeater is meant a repeater wherein the microwave signal is detected as a video-pulse train, but no demodulation to voice is provided. The video-pulse train simply remodulates the transmitter. This would permit the passage of the fault pulse without any modification. When one or more channels are demodulated to voice, however, the old channel pulse is dropped out and a new one inserted (hence, the name drop and insert unit). If either the blanking gate or the new channel pulse should coincide with the fault-pulse trailing edge, a timing error would be introduced.

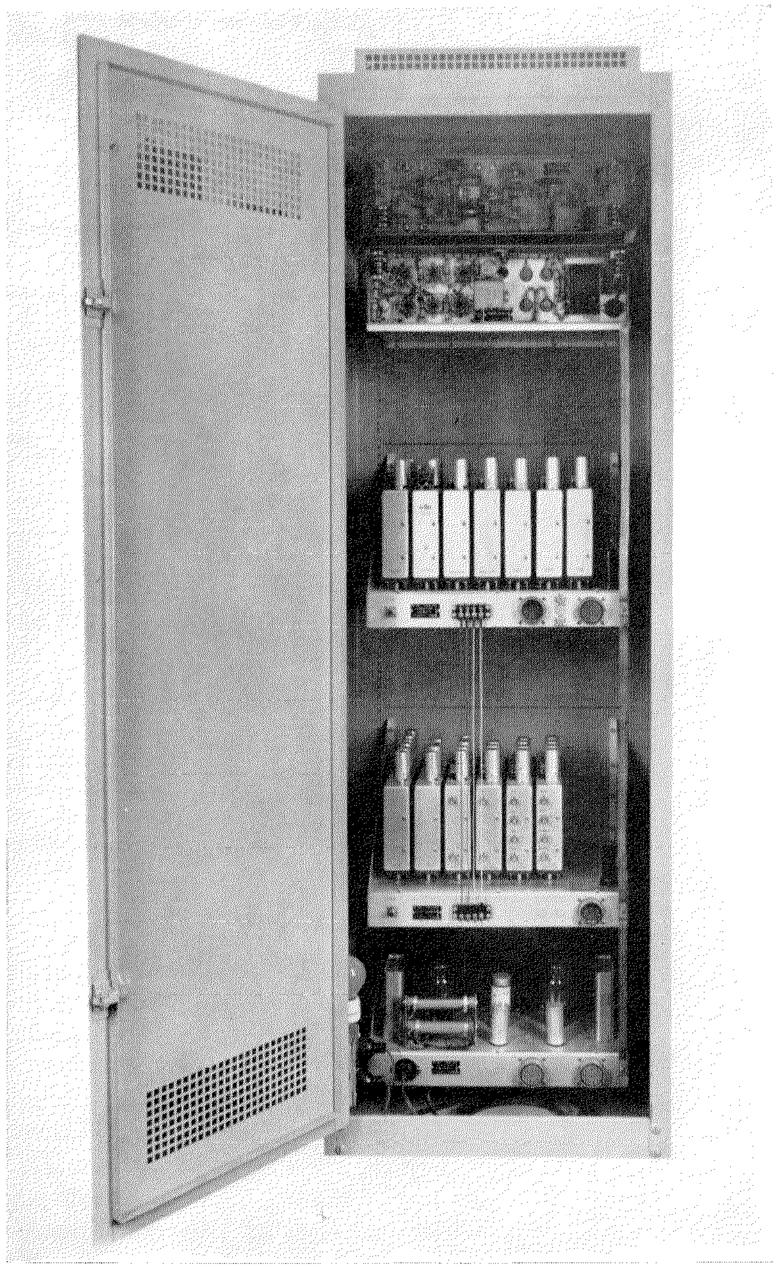
To avoid this, a unit must be provided at repeaters having drop and insert units to blank out the drop and insert signals during the fault pulse. Since it requires about one microsecond to detect the fault pulse from the channel pulses, the blanking signal to the drop and insert unit is delayed by that time and hence it is possible for one microsecond to be lost off the leading edge of the fault pulse. This is why the trailing edge of the fault pulse was used for time definition. It is also why the long 10-microsecond pulse was specified, since as short a fault pulse as three microseconds can be detected at the terminal. This allows several drop and insert losses without affecting final detection.

Detection of the fault pulse at the receiving station is accomplished by an integrating circuit to select the longer fault pulse, followed by a differentiating circuit to define sharply the trailing edge. The result is reshaped for transmission to the fault locator. This is provided by the fault-pulse demodulator normally located close to the demodulator multiplex since it also receives the received video-pulse train. The 4-tube unit requires 3 inches of rack space.

At an audio repeater such as Olympia and Squak Mountain, both the fault-pulse demodulator and modulator are required, since the fault pulse must be detected and reinserted in a new pulse train.

The Bonneville pulse-time-modulation system is shown again in Figure 3, but this time it includes all the fault-pulse equipment necessary for the system.

Note that the use of time-division multiplexing permits a high degree of flexibility for the fault-locating plan. As an example in the Bonneville system, provision is made at Olympia for an additional fault-pulse insertion to allow for future expansion of the microwave radio relay system. Also, at the Squak Mountain repeater, provision was made to allow fault-pulse insertion from either Snohomish or Covington. In the case of more than one power line supplied by the same radio relay system, ambiguity may be avoided by the use of coded fault pulses, thereby allowing one recording station to provide facilities for two power lines.



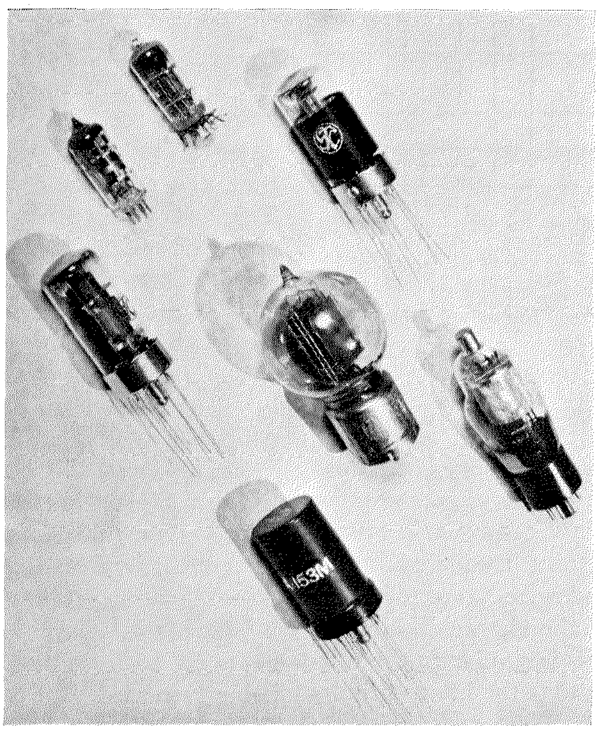
A fault causes a pulse to propagate to each end of the power line. At one end, the pulse starts the electronic counters shown above. The counters are stopped by the second pulse, which is transmitted by radio from the other end of the line. From this time interval, the location of the fault is computed automatically and a printed record is produced immediately.

Life of Valves with Oxide-Coated Cathodes

By C. C. EAGLESFIELD

Standard Telephones and Cables, Limited; Ilminster, England

THE LIFE of valves with oxide-coated cathodes has long been a cause of speculation, and it may therefore be of value to put on record the results of investigations of the life-test records of telephone repeater valves of the valve division of Standard Telephones and Cables. All evidence given here is limited to repeater valves manufactured by that company because the details of their manufacture are known. However, since the making of cathodes is now fairly standardised, the conclusions are likely to be general.



There are very many diseases that can assail valves and cause their failure, among which may be mentioned mechanical breakages such as cracked envelopes, disconnected lead wires, or fractured heaters; or electrical faults such as unwanted emission from electrodes or leakage across insulators. These and many other troubles can end the life of a valve, but the present discussion is confined to the emissive properties of the cathode.

It is generally supposed that the lives of cathodes are only moderate; it may therefore come as a surprise that no evidence could be found to put a definite term to the cathode lives. The conclusion seems to be that the emission continues indefinitely.

However, an effect was found that could easily be, and in fact was, mistaken for a drop in the emissivity. This is the formation of a resistive barrier between the cathode core and the coating, which causes a feedback and thereby a change in the measured characteristics. This effect seems to occur so universally that the study of cathode life almost becomes a study of this resistance.

There is a difficulty with some of the evidence, particularly regarding extremely long lives. When we consider a life of twenty years, we are faced with the fact that the valves must have been made over twenty years ago, when the technique was different from that of today.

Since there is not sufficient information to trace the effect of the many variables, we can proceed only by ignoring them. By correlating the evidence in this way, we reach conclusions, but their validity is more probable than certain.

In the next section are described typical life histories and the three groups into which they fall. In the second section, it is shown how all these typical lives are accounted for by the growth of cathode resistance; also it is shown how this resistance can be identified and measured.

The third section discusses possible causes of the resistance and whether a cure is feasible. As it is reluctantly concluded that a cure is dubious and will take a long time to establish, a fourth section is devoted to ways of designing apparatus so that the effect of the resistance is made small.

• • •

1. *Typical Life Histories*

Before proceeding to consider the known histories of valves that have been subjected to controlled life-tests, it is well to give some thought to why such tests are conducted and what information is likely to be obtained from them.

The main purpose of the test is to establish that each valve type is likely to give satisfactory service and to keep a continual check on the manufactured product. To do this, a certain routine has to be followed.

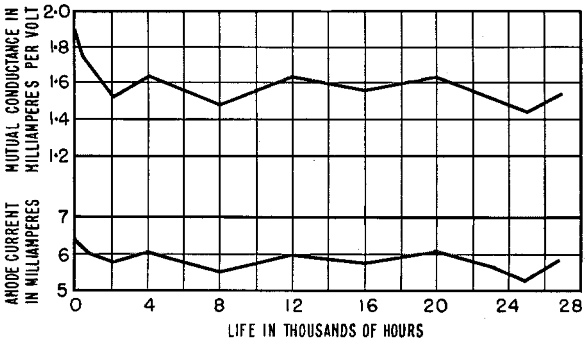


Figure 1—Life history of a batch of 12 small radio-frequency pentodes tested with 250 volts on the anode, 180 on the second grid, and −5·5 volts on the control grid.

Valves are run under specified conditions, and their characteristics are measured at gradually increasing intervals. These characteristics, such as currents, impedances, mutual conductances, etc., are usually those measured in the testing of a new valve, and rejection of the valve during life-test follows if the characteristics fall outside certain limits. This is similar to the testing of a new valve except that the life-test limits are wider.

Valves are also checked for their mechanical characteristics, which are just as important for satisfactory service. For instance, a loose base or top cap, a sagging filament, or a bowed cathode would cause rejection of the valve and, unless a repair were possible, removal from test.

There is another cause that makes for the removal of good valves from test—the question of space. As new batches are periodically put up, old batches must come down to make room. Also, for any particular type, there is usually a certain length of life that is regarded as satisfactory. Beyond that life, it is very likely that valves will be taken down, unless by some chance the space is not wanted. All these circumstances combine to make it difficult to find life histories of great length, and it is now realized that valves have often been taken down just when, from a general point of view, interesting information was to be expected.

Incidently, while the considerations listed would probably apply to any life-test department, it is relevant to mention that a serious fire took place in 1946, which cancelled many tests and caused some confusion in the records.

With these limitations in mind, we can now proceed to consider the evidence. First of all, it has been noticed that valve types may be divided into three classes as follows.

1.1 First Class

The first class has a rather short life, of the order of one or two thousand hours, after which time the emission has seriously deteriorated. This class of valve usually suffers from some characteristic fault, such as high gas pressure, which can be associated with the design of the valve. This is not to say that the valve is badly designed, but that the performance requirements have forced the designer's hand. Thus the short life is understood to be the price that has to be paid to get the required performance.

Examples in this class are specialized valves for use at centimetric wavelengths—in many such valves, full degassing on the pump is very difficult—or valves that, to save space or weight of equipment, are grossly overloaded.

Of course, there is continual development going on, and the lives of such types are always being increased. But the point here is that these very short lives are caused by poor vacuum and are therefore not relevant to the general argument. As has been pointed out by Metson,[1] normal receiving-type valves very quickly reach an extremely high degree of vacuum. We shall not discuss this first class any further.

1.2 Second Class

The second class of valves has an extremely long life, so long apparently that it is almost impossible to put a term to it. In this class, we can find many types for which life histories are available up to about 30 000 hours. As an example,

[1] G. H. Metson, "Vacuum Factor of the Oxide-Cathode Valve," *British Journal of Applied Physics*, v. 1, pp. 73–77; March, 1950.

consider Figure 1, which shows the average history of a batch of typical valves. These are small pentodes for radio-frequency amplification and have 3-watt cathodes; the conditions of testing and running on life are approximately the same. It will be seen that apart from a certain settling down in the first thousand hours and some small fluctuations thereafter, there is no significant change in the characteristics. This batch is still on test at the time of writing.

Many examples can be found up to the same length of life, but for a longer test consider Figure 2, which shows the average history of a batch of special triodes made for an emission study. These valves had 6-watt cathodes and again the test and running conditions were approximately the same. It can be seen that the change in characteristics up to 50 000 hours (6 years) is negligible. This test was stopped by the fire already referred to.

For a longer life history, we have to leave organized tests and consult actual service. Recently a telephone repeater station was dismantled and it was noticed that a number of valves had been in service since the repeater was installed, a matter of twenty years. These valves,

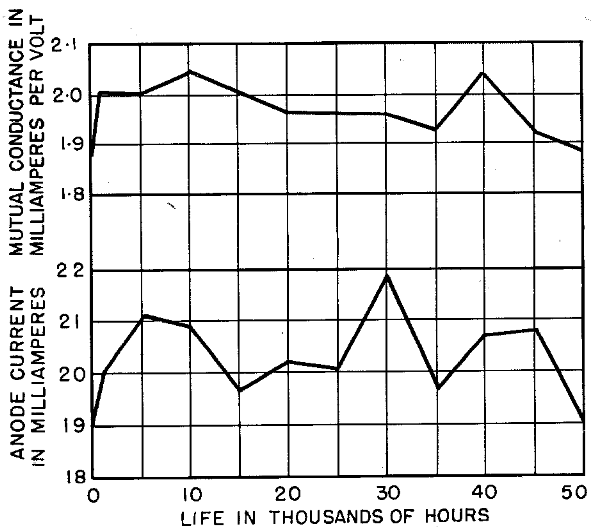


Figure 2—Life history of a batch of 5 triodes tested with 250 volts on the anode and −10 volts on the grid.

type *4101*, were returned by the courtesy of Dr. Ryall of the General Post Office and their history is exhibited in Figure 3. It can be seen that there can have been little change in their characteristics during the twenty years. This

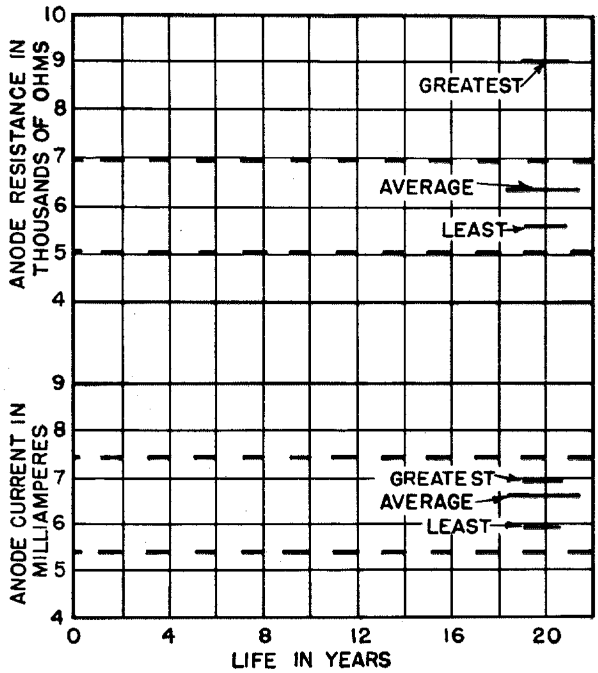


Figure 3—Characteristics of a batch of 18 type *4101* valves after 20 years of service tested with 130 volts on the anode and −9 volts on the grid. The normal test limits are indicated by dashed lines.

type had a 4-watt filament and, remarkably, no getter.

Of the three types quoted for this long-life class, it may be mentioned here that the cathodes differed in detail. The pentodes are coated with triple-carbonate on cores of *O* nickel, a grade of nickel very commonly used for cathodes and containing small additions of reducing impurities. The cathodes of the triodes were triple-carbonate on *S* nickel, a grade of nickel with rather smaller proportions of impurities. The filaments of the *4101* were mixed carbonates on a platinum-nickel alloy and the method of coating was very different from the method used today; so was the method of pumping. More recent samples of this type have had triple-carbonate coatings on filaments of *O* nickel, but unfortunately the longest organized life-test seems to be for 30 000 hours only, during which time the changes of characteristics were negligible.

1.3 Third Class

The third class of valve has an apparent life of several thousand hours; that is, a life intermediate to the very short ones of the first class and the very long ones of the second class. A typical example is shown in Figure 4, in which both the working bias and the mutual conductance show substantial changes in 4000 hours.

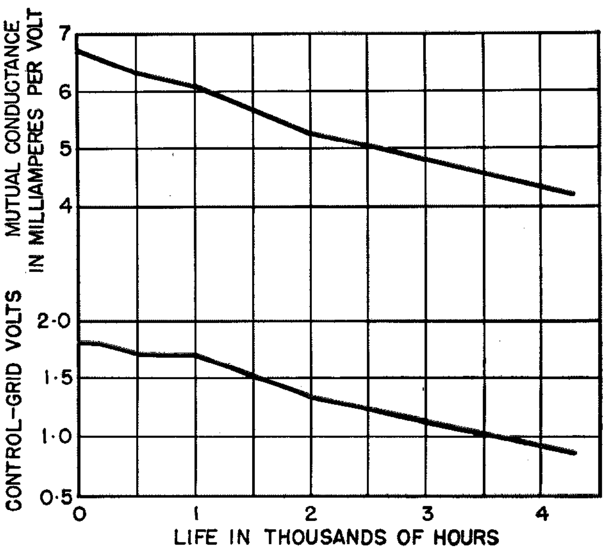


Figure 4—Life history of a batch of 6 miniature radio-frequency pentodes tested with 250 volts on the anode and second grid and with 10 milliamperes anode current.

Another example is shown in Figure 5, in which there is appreciable change of mutual conductance in 13 000 hours. Both these types are small radio-frequency amplifiers; the first has a 2-watt and the second a 3-watt cathode, in both cases of *O* nickel coated with triple-carbonate.

The deterioration of characteristics shown by this class of valve is usually sufficient to cause its rejection and removal from test, but special tests have shown that the deterioration reaches a certain point and then stops. The length of life is therefore very much a matter of definition.

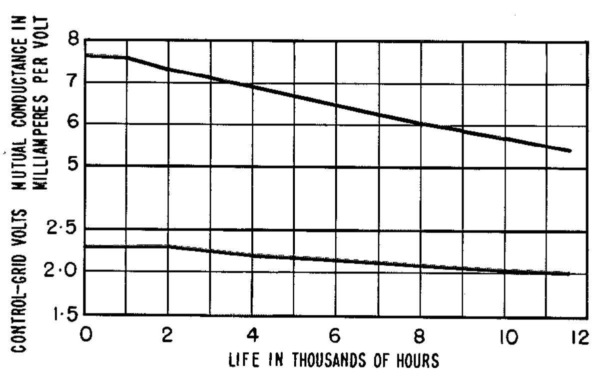


Figure 5—Life history of a batch of 8 radio-frequency pentodes tested with 250 volts on the anode, 150 volts on the second grid, and 10 milliamperes anode current.

1.4 Conclusions

We have now divided valves into three classes, according to whether their lives are short, long, or medium. The first class has been disposed of as having faults of design, but is there any way of forecasting whether a valve will fall into the second or the third class? At first, it was supposed that the answer might be connected with the cathode loading, that is, the ratio of cathode current to cathode area. In fact, this loading is small in many of the long-life valves, but it was not found to separate the classes. It was then noticed that the ratio of mutual conductance to cathode area did separate the classes: if the ratio were low, the valve fell in the long-life class and if it were high, the valve had a short life.

This suggested that the cause of the short lives might be a feedback due to a growth of cathode resistance. Such a resistance might be expected to be inversely proportional to the cathode area. The feedback would depend on the product of mutual conductance and resistance and therefore on the ratio of mutual conductance to cathode area.

It will be shown in the next section that a growth of cathode resistance is indeed the cause of the effect.

2. *Cathode Resistance*

When the life history of a valve shows a deterioration in working current, bias, or mutual conductance, it is perfectly feasible to postulate a cathode resistance of sufficient magnitude to explain the change. If this be done, one would expect the required resistance to be inversely proportional to the cathode area.

In Figure 6 is shown the life history of three valve types in terms of such a cathode resistance. The resistance is normalized for a cathode area of 1 square centimetre and is derived from the observed change of mutual conductance during life. Two of the types are those of Figures 4 and 5; the third is a radio-frequency pentode of somewhat greater rating: it has a 5-watt cathode (triple-carbonate on *O* nickel) and a mutual conductance of 6·5 milliamperes per volt at its usual working anode current of 38 milliamperes.

It will be seen that the normalized resistance builds up to a saturation value of about 40 ohms for all three types, with a surprisingly sharp angle where the rise meets the saturation level.

The resistance grows in a similar way for many other types; the three valves shown have been chosen because it happens that the tests have been continued long enough to show the saturation level clearly. All the life histories that have been examined can be explained by the hypothesis that a cathode resistance builds up to 40 ohm-square-centimetres and then stays constant. Such a resistance explains the change in characteristics of the medium-life valves; it is too small to affect the characteristics of the long-life class.

It is important to realize that the suggestion is that no change takes place in the emissivity of the cathodes, but that in all valves the resistance grows to a certain value and then stays constant. It may, or may not, change the measured characteristics appreciably, according to the design of the valve. But all valves grow the resistance and then stay without change indefinitely.

Are there any reasons for supposing that such a resistance exists physically?

The first way that was tried to detect it was to measure the shift in the grid-voltage–grid-current characteristic when cathode current is made to flow by applying an anode potential. The cathode current flowing through the supposed cathode resistance produces a bias, and from the known current and the shift in grid voltage the cathode resistance can be calculated. The method is not very accurate due to robbing of the grid current by the anode. Nevertheless, a resistance is readily detected in aged valves but not in new valves; moreover, the resistance measured in this way agrees fairly well with that needed to explain the change in the valve during life.

A second way of detecting the resistance is to measure the mutual conductance at a high frequency (greater than about 5 megacycles per second) as well as at a low frequency (less than about 50 kilocycles per second). With a new valve, there is no difference but with an aged valve the high-frequency mutual conductance is the greater. Such a difference is only to be explained by a resistance shunted by a capacitance having been formed at one of the electrodes. The cathode would seem the most likely electrode and in any case the previous measurements associates the resistance with the cathode.

The fact that the resistance has a substantial capacitance in shunt is remarkable and makes the measurement of the resistance a simple matter. Pulses and square waves can also be conveniently used.

The double-frequency method has been used to measure the cathode resistance of a number of valves that had deteriorated during life, supposedly for a drop in emission. In every case, a resistance was found and the resistance was approximately that required to explain the change in characteristics.

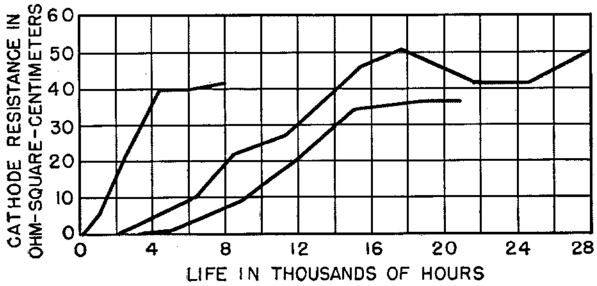


Figure 6—Growth of cathode resistance during life for three types of valves. The ordinate scale is the normalized resistance AR_c, A being the coated area of the cathode in square centimetres and R_c the measured resistance.

Thus, we may conclude fairly that the hypothesis outlined above is established, that the resistance is invariably there, and that it is the outstanding cause of changes in valve characteristics during life. This being so, it becomes of great interest to consider the possible causes of its growth.

3. *Cause of the Resistance*

Two possible causes have been suggested for the growth of the resistance: a mechanical theory by Raudorf and a chemical theory by Eisenstein.

Raudorf's theory[2,3] is that with age the coating shrinks away from the core, leaving contact between coating and core only at minute discrete spots. The reduction of contact area explains the resistance, which is localized round the contacts,

[2] W. Raudorf, "Change of Mutual Conductance with Frequency," *Wireless Engineer*, v. 26, pp. 331–337; October, 1949.

[3] W. Raudorf, "Letter to Editor," *Wireless Engineer*, v. 27, p. 164; May, 1950.

and the high capacitance is explained by the close spacing between the core and the body of the coating. Raudorf associated the shrinking of the coating with a network of fine cracks that he observed on the outer surface of the coating of aged cathodes, and stated that he found flat cathodes much superior to round cathodes.

Eisenstein's[4] theory is that a resistive film is formed at the interface of core and coating by the formation of compounds of barium and core impurities. These impurities are deliberately included in the core metal as reducing agents to promote activation: the most usual are silicon and magnesium and their effect is to release free barium. Eisenstein regards barium-orthosilicate as the most important cause of the resistance. This and other compounds of barium and core impurities have also been observed by Wright.[5]

Now, it is a difficult matter to decide with certainty which theory correctly explains the resistance that has been observed on our valves. The surfaces of the cathodes do not show the network of cracks observed by Raudorf and the coatings seemed to adhere closely to the cores. There is also the question of round and flat cathodes: Raudorf quotes a typical figure of 40 ohm-square-centimetres for round cathodes, which agrees closely with the results shown in Figure 6; but all the valves for Figure 6 have flat cathodes and we have found no systematic difference between round and flat cathodes.

As for the resistive interface, the cathodes certainly contained the impurities and doubtless the interfaces were formed; however, the difficulty is to be sure that all the resistance observed was due to this and not, in part, to a bad contact.

It is only when a cure for the effect is being considered that it is necessary to decide on the cause. It seems to the writer that Eisenstein's theory is more probable and contains fewer inconsistencies. It is interesting to use it as a basis for a possible cure.

The obvious course is to use cores that are free from impurities. However apart from the high cost of very pure nickel, there is the difficulty of activation. Reducing gases such as methane or ethane, can be introduced on the pump in small-scale manufacture, but this expedient is hardly practicable for mass production. In any case, even if an initial activation be completed, it seems likely that it would not be stable. For example, see Wright,[6] who reports a dropping activation when a pure core is used.

Another way might be to use cores that are much more active to get the resistance formed quickly, as part of the production processing.

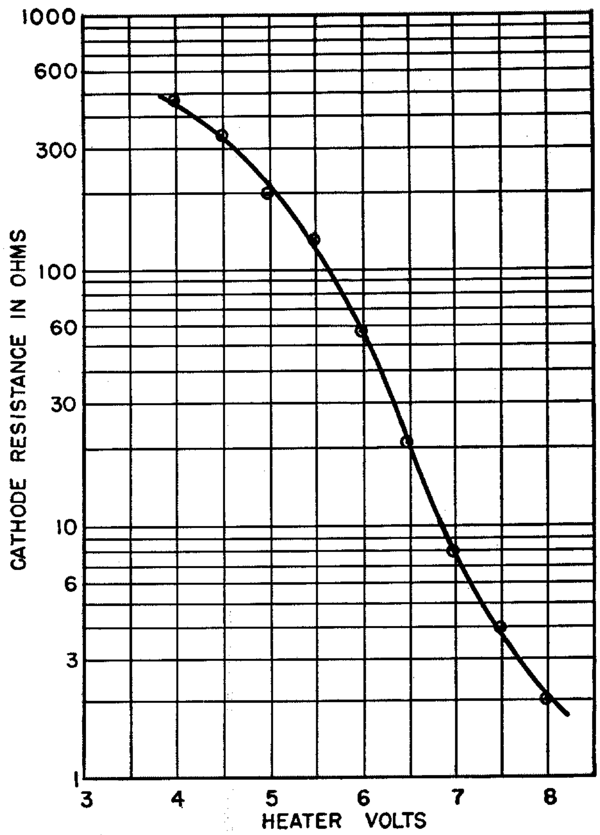


Figure 7—Variation of cathode resistance with heater voltage in a typical aged valve. The nominal heater voltage was 6·3 volts, corresponding to a cathode temperature of approximately 1000 degrees Kelvin. The coated area of the cathode was 1 square centimetre.

Even if this proved feasible, and the speeding-up required is very great, it might be a thankless task to persuade users that they were getting an improved article.

A third method might be to use cores that are more passive to delay the formation of the interface as long as possible. This would make activa-

[4] A. Eisenstein, "Letter to Editor," *Wireless Engineer*, v. 27, pp. 100–101; March, 1950.

[5] D. A. Wright, "Thermonic Emission from Oxide Coated Cathodes," *Proceedings of the Physical Society*, v. 62, Part 3, n. 351B, pp. 188–203; March 1, 1949.

[6] D. A. Wright, "Electrical Conductivity of Oxide Cathode Coatings," *British Journal of Applied Physics*, v. 1, p. 150; June, 1950.

tion more difficult and would only have a delaying action; the resistance would build up eventually.

A fourth possibility is to find a reducing impurity that would not be so highly resistive.

There is also the question of the influence of the temperature of the cathode. It seems very likely that a temperature lower than normal would delay the formation of the interface, but on the other hand its resistance would be higher when it was formed. This is illustrated by Figure 7, which shows the effect of the cathode temperature on the resistance. The lower the temperature, the higher the resistance.

An interesting deduction from Figure 6 is that the chemical process appears to come to an abrupt end when the interface reaches a certain thickness. There seems no difficulty in assuming the process to be of this type, in which the interface acts as a barrier between the impurity on the one side and the barium on the other. It would thus seem that after a certain time the formation of free barium is stopped, or at any rate greatly reduced. It appears to be sufficient to replace what may be lost by evaporation during normal running, but it seems possible that an aged valve could not recover so well from an accidental overload.

In considering any possible cure, it is well to remember the time scale of Figure 6, which shows the order of time required for an experiment. One is reluctantly driven to conclude that not only is the possibility of a cure dubious, but it is going to take a long time to establish its success.

This being so, it is very well worth while to consider what can be done in the design of apparatus to reduce the effect of the resistance, on the assumption that all present valves have it and all future valves will have it, for an indefinite time ahead.

4. *Design of Apparatus*

The known facts can be summed up by saying that within six months to two years running, valves will change from their initial state to a final state in which they have a cathode resistance shunted by a capacitance.

Measurements on valves in their final state suggest average figures of 40 ohm-square-centimetres for the resistance and 0·005 microfarad per square centimetre for the capacitance. It has been verified that the resistance is linear up to a loading of 40 milliamperes per square centimetre. The resistance is temperature dependent, but as there is little difference between the cathode temperatures of one valve type and another, this need not concern the user.

It would be useful to be more specific on the time of growth: this time probably depends on a number of factors, such as the cathode temperature and the quantity and kind of core impurities. But it also seems to depend on details of the pumping and processing. It is difficult at present to be more specific.

The user will probably not know the coated area of the cathode of any particular type of valve, but he may estimate it from the rated heater power, on the basis of 3 watts per square centimeter.

The easiest way to deal with this problem is to concentrate on the two states of the valves: if the apparatus is satisfactory for both states, it seems a fair deduction that it would be satisfactory during the period of growth. The designer may proceed with a trial design, based on the valves in their new state. He then estimates the resistance and capacitance that will grow at each cathode and, by experiment or calculation, tests whether the design is still satisfactory.

It is customary to allow a factor of safety to cover the deterioration of valves during life; the advantage of the systematic process outlined above is that, after it has been carried out, the apparatus should function almost for ever.

It is hardly possible to give very general instructions on how to choose circuits that will prove satisfactory: each case must be considered on its merits. However, consideration suggests that a liberal use of feedback gives the best chance of success. The reason is that the feedback produced by the life-impedance depends on the valve's effective mutual conductance and a permanent feedback effectively reduces the mutual conductance. A rough rule is that the permanent feedback should swamp the life-impedance feedback.

Take, first of all, a rather simple case, a single valve used for class-A amplification at high radio frequencies. Since the life-resistance is adequately by-passed by the life-capacitance, it will cause no feedback, but will only alter the bias conditions. This is easily allowed for by using a cathode resistor large compared to the life-resistance, to provide a bias. This usually gives excess grid bias, so the grid is returned, not to earth, but to a positive point. By this very simple device the valve's running conditions can easily be kept almost the same for its two states. This is an excellent illustration that the designer, when forewarned, can often take effective steps to meet future conditions. Without the warning, he would probably have provided a normal bias resistor.

Now consider an amplifier for audio frequencies. It is not likely that the valves for this service will show particularly large changes during life, as there is little need for a high ratio of mutual conductance to heater power. The feedback due to the life-impedance will be constant over the audio-frequency band. To reduce distortion, it is customary to provide a strong feedback from the output of the amplifier to an early stage and this would probably swamp the life-feedback. It seems likely that little modification would be needed to most audio-frequency amplifiers to make them satisfactory.

It thus seems that where the application involves frequencies either very high or very low, there should not be any great trouble. A more difficult case is the video-frequency amplifier, partly because the life-feedback then varies over the band and partly because the valves likely to be used are just those most susceptible to the effect.

Such amplifiers are normally required to have a flat frequency characteristic, and it is therefore necessary to compensate for their natural tendency to fall off at the higher frequencies. This is often done by increasing the effectiveness of the inter-stage coupling at the higher frequencies, but another way is to provide a frequency-dependent feedback, and this seems a better way for our present purpose.

Considering a single stage, the compensating feedback may be a resistance and capacitance in shunt in the cathode lead. Now there is usually a range of feedback for which the overall result is much the same, i.e., the same stage gain and frequency characteristic can be got by using high forward gain and high feedback or low forward gain and low feedback. It may thus be possible to swamp the life-feedback.

Where there are a number of stages, it must be considered whether to use feedback over several stages, at each stage, or a combination of both.

If great linearity is required in the input–output voltage characteristic, then feedback will be needed for this purpose. Such a requirement arises in multi-channel carrier amplifiers. For this case, it may prove best to use frequency-dependent feedback at each stage and frequency-constant feedback over the whole chain.

There are, of course, very many other valve applications in addition to those touched on above. These must be dealt with by the designer in a similar way. It is hoped that enough has been said to show that there is a hope of success if the problem is tackled in the design stage and to direct attention to the matter.

5. *Conclusion*

It has been shown by a study of the life histories of valves that it is hardly possible to assign any finite life for the emission from oxide-coated cathodes, but that there is an effect that could be mistaken for a finite life. This is that a resistance grows between the cathode core and the coating, which changes the performance of the valve. The life of the valve is divided into two parts, before and after the formation of the resistance. Although there seems little immediate prospect of making valves free of this resistance, it may often be possible to design equipment in such a way that the effect of the resistance is not important.

6. *Acknowledgment*

The writer is grateful to a number of his colleagues for assistance in preparing this paper and in particular to Mr. W. T. Gibson O.B.E., chief valve engineer, for helpful advice.

Secondary-Emitting Surfaces in the Presence of Oxide-Coated Cathodes*

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THE EXPERIMENTS described here show that the deleterious effect of oxide cathodes on secondary-emitting surfaces of silver–magnesium can be overcome by using tantalum instead of nickel as the base metal for the oxide coating.

• • •

It is well known that secondary-emitting materials are subject to contamination from heated cathodes. The deterioration in secondary emission caused by this contamination is so serious that tubes using a hot-cathode source and electron multiplication have usually[1–3] been specially designed so as to make the target (dynode) less accessible to material coming from the cathode. According to a recent paper by Mueller,[4] this difficulty has been overcome for filamentary cathodes and also, to some extent, for indirectly heated cathodes.

The experiments described here were entirely confined to indirectly heated cathodes, and as they resulted in a novel means to avoid the contamination, a short report on them may be of interest.

1. *Experimental Technique*

1.1 Tubes

While an oxide cathode is being formed, its temperature is raised for a short time to a value that may be about 200 degrees above its operating temperature. It is thus possible that the target contamination occurs, at least in part, during the forming process. Most of the experiments were therefore performed with tubes as shown in Figure 1, with a sliding target that did not face the cathode during formation. The arrangement is cylindrical. The straight cathode K (diameter $\frac{1}{16}$ inch) occupies the center; it is surrounded by a collector, which is shaped as a squirrel cage (24 tantalum wires, 3 of them 0.015-inch, the others 0.005-inch thick, held together by top and bottom tantalum ribbons; outside diameter $\frac{3}{8}$ inch). The target is coaxial with the cathode and collector and can slide on three

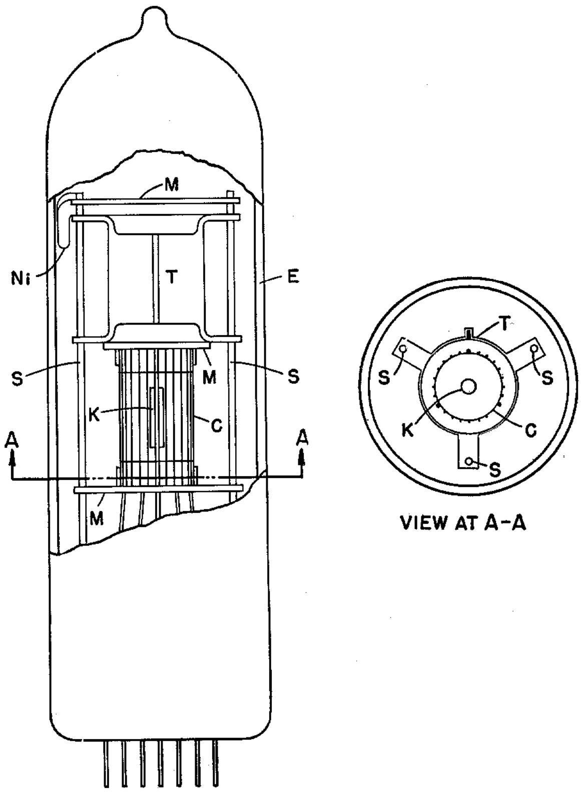


Figure 1—Sliding-target tube in which K is for cathode; C, collector; T, target; M, mica disks; Ni, nickel ribbon; S, support rods; and E, glass envelope.

* Reprinted from *Proceedings of the I.R.E.*, v. 39, pp. 191–193; February, 1951. This work was done in 1948 under contract W36-039-sc-33864 with the Evans Signal Laboratory.

[1] M. Chauvierre, "Secondary Emission Amplifier Tube," *Tele-Tech.*, v. 6, p. 69; July, 1947.

[2] H. M. Wagner and W. R. Ferris, "The Orbital-Beam Secondary-Emission Multiplier for UHF Amplification," *Proceedings of the I.R.E.*, v. 29, pp. 598–602; November, 1941.

[3] C. S. Bull and A. H. Atherton, "A New Secondary Cathode," *Proceedings of the Institution of Electrical Engineers*, v. 97, pp. 65–71; March, 1950.

[4] C. W. Mueller, "Receiving Tubes Employing Secondary Electron Emitting Surfaces Exposed to Evaporation from Oxide Cathodes," *Proceedings of the I.R.E.*, v. 38, pp. 159–164; February, 1950.

support rods S. On the pump, the tube is mounted upside down, so that the target slides by gravity into the position shown in the drawing. In operation, it slides down to surround the collector.

A similar construction, without the movable features, was used on fixed-target tubes, in which the target is permanently exposed to the cathode.

Eight-percent, 4-percent, and 1.7-percent magnesium–silver alloy was used as target material, but the final experiments all used the 1.7-percent alloy.

1.2 Forming Schedules

The procedure that worked best was as follows. The tube, after assembly of parts, is given a vacuum bake-out at 460 degrees centigrade, and then the cathode temperature is raised in steps, so as to outgas it. The cathode is then flashed to about 1150 degrees centigrade (true temperature), and then the collector is brought up to about 200 volts, with the emission rising up to 80 milliamperes. Thereafter, the cathode is cooled, oxygen is admitted, and the target is heated for about 1 minute with radio-frequency current. This oxidizes the target, but damages the cathode so that it has to be re-formed. Then the getter (if a getter was used) is flashed and the target once more outgassed by radio-frequency current after which the tube is sealed off.

Numerous variations of this process (such as a bake in ozone) were used at one time or other.

1.3 Secondary Emission

The secondary emission was measured under standardized conditions: 100 volts on the target and 200 volts on the collector; the primary current density on the target was 2.6 milliamperes per square centimeter.

The secondary-emission coefficient K was defined as

$$K = \frac{I_c}{I_c - I_t}, \qquad (1)$$

where I_c and I_t are collector and target currents. This formula neglects the fraction α of the primary electrons that are intercepted by the collector without ever reaching the target. If this fraction is taken into account, it is found that the "true" secondary-emission coefficient K_t is related to K as defined above by the formula

$$K - 1 = (K_t - 1)(1 - \alpha). \qquad (2)$$

Attempts to determine α met with limited success only; the best estimate is that $\alpha = 20$ percent in our tubes. This would mean that $K = 3$ really corresponds to a "true" coefficient $K_t = 3.5$. The data recorded below refer to K rather than K_t, because K gives a more direct measure of the practical gain than can be realized in a secondary-emission tube.

The K measurements were performed with direct current, but to guard against the possible presence of time delays, as in the Malter effect,[5] it was ascertained that the collector current perfectly reproduces the cathode-emission variations, at least up to 1 megacycle per second.

For life tests, it was necessary to keep the primary-emission current of the cathode constant. This proved difficult, because any excess emission led to a noticeable increase in collector temperature, which by radiation caused the cathode temperature to rise, thus increasing the emission still further. For the life tests, therefore, a thyratron circuit was used in the filament supply, controlled by the emission current so as to keep it constant.[6]

2. *Results*

Early results indicated that the decay in secondary emission depended very much on the cathode temperature. It became increasingly clear that the contamination came from the nickel base rather than from the oxide (barium-strontium carbonate, sprayed to a thickness of 0.002 inch with an amyl acetate binder, was used throughout). The base was a nickel sleeve (*799 DH* in most cases). By a properly chosen formation schedule, we finally succeeded in achieving a constant secondary-emission ratio over several hundred hours on sliding targets. But even more constant results were obtained, on fixed as well as sliding targets, after the nickel sleeve had been replaced by tantalum. This was done on the assumption that a material that evaporated less easily than nickel should be used.

Figure 2 shows life data taken on tube *365* (sliding target) and *368* (fixed target), both of which had a tantalum sleeve as base for the oxide coating. As a further proof that tantalum sleeves, even when run at overtemperature, cause no

[5] L. Malter, "Thin Film Field Emission," *Physical Review*, v. 50, pp. 48–59; July 1, 1936.

[6] This circuit was designed and built by H. Beach.

contamination, the record of tube *358* is offered: This tube was operated at the usual emission current of 10 milliamperes for 240 hours, showing a secondary-emission coefficient of $K=3.2$. From 240 to 375 hours, the emission was raised to 15 milliamperes, giving $K=3.4$. Thereafter, the emission was raised to 40 milliamperes for 8 hours. During this time, values of K between 2.3 and 2.5 were measured, but it is almost certain that space charge formed between target and collector. When the emission was again lowered to 10 milliamperes, K was 2.9 and stayed at this value. Similar, though less exacting, experiments with nickel always showed a rapid and permanent decay of K to values below 2.

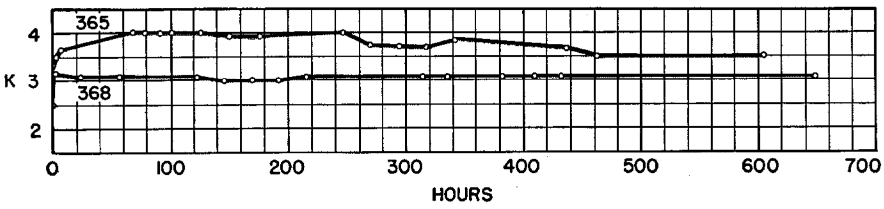


Figure 2—Effective secondary-emission coefficient K of two representative tubes with tantalum cathode bases as a function of hours of operation.

One other experiment may be quoted to support the idea that the nickel sleeve is the source of contamination. In a tube of somewhat different construction, it was possible to expose a target to a *799* nickel base without oxide coating. This base was heated to about 900 degrees centigrade, K dropped within 4 hours from its initial value of 3.2 to 1.9.

A similar experiment was performed in which barium was evaporated. This, too, caused a rapid decay in secondary emission. The decay caused by an oxide cathode has commonly been ascribed to barium[3,4,7,8] while there are also accounts[9] which seem to minimize the importance of barium. From our experiments, we are inclined to agree with this latter view.

3. Discussion

While these experiments prove that the main source of contamination comes from the nickel sleeve, there may be some doubt whether the nickel itself is to blame or some impurity in it. It may however be noted that nickel reaches a vapor pressure of 10^{-5} millimeters of mercury at 1160 degrees centigrade, i.e., slightly above the operating temperature, whereas tantalum would have to be heated to 2400 degrees centigrade to reach the same vapor pressure.[10] It is, therefore, quite possible that the contaminating substance is the nickel itself.

Using tantalum, or some other metal with a low vapor pressure, instead of nickel, is entirely practical. It has been argued that oxide cathodes on a nickel base have a higher efficiency than those on other base metals. Our experiments have not extended in that direction; however, even if it should be found that the cathode temperature has to be slightly higher on tantalum than on nickel, this would be a small price to pay for the resulting increase in tube life.

As good results were obtained in fixed-target tubes, it is seen that no special structure is required to prevent exposure of the target to the cathode during formation. This may be stated somewhat more generally as follows: Any contaminating agent that might come from the cathode during formation will be harmless if it is either not evaporated below the cathode-forming temperature (1100 degrees centigrade) or else if it does not react with the target and can be re-evaporated below the target outgassing temperature (about 600 degrees centigrade).

[7] J. B. Johnson, "Secondary Electron Emission from Targets of Ba-Sr Oxide," *Physical Review*, v. 73, pp. 1058–1073; May 1, 1948.

[8] J. L. H. Jonker and A. J. Overbeck, "Application of Secondary Emission in Amplifying Valves," *Wireless Engineer*, v. 15, pp. 150–156; March, 1938.

[9] G. E. Moore and H. W. Allison, "Thermionic Emission from Thin Films," *Physical Review*, v. 77, pp. 246–257; January 15, 1950.

[10] Data taken from S. Dushman, "Scientific Foundations of Vacuum Technique," John Wiley and Sons, Inc., New York, New York, and London, England; 1949: pp. 745–751.

Miniature Direct-Wire Remote Control at Loch Sloy Power Station

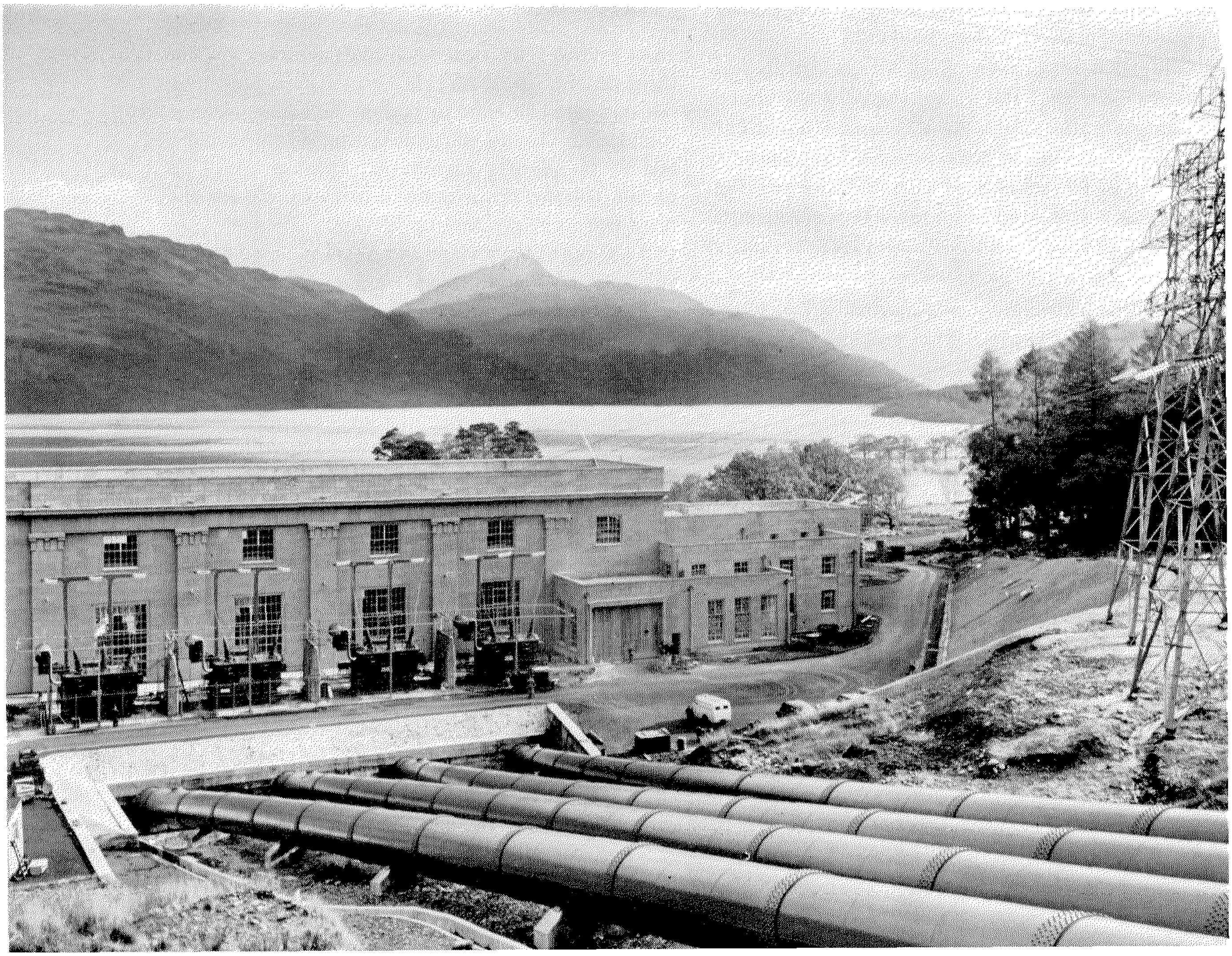
THE OPENING of the Loch Sloy, Scotland, power station put into service a new system for remotely supervising and controlling the operation of an entire hydroelectric installation.

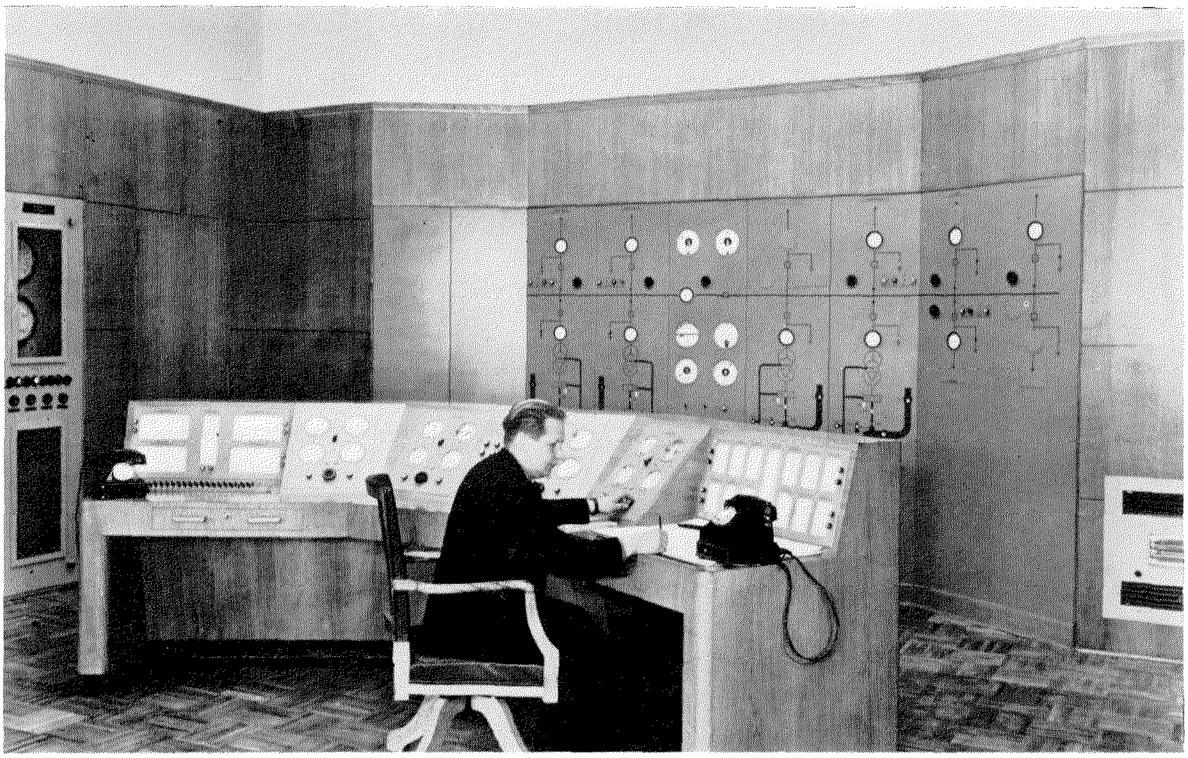
The miniature direct-wire control system used employs telephone-type apparatus and wiring to operate interposing relays on the turbine and switchgear panels. These relays are connected to the circuits of the normal "close" and "trip" coils usually associated with power-generating equipment. The control system, including the interposing relays, operates at 50 volts and the resulting small direct currents are easily carried by light 20-pound-per-mile conductors that are readily marshalled into cables and run to the operating room. The control keys and indicators are small and have been assembled in a compact arrangement that would be almost impossible with the heavy cables and large control switches of the old type.

The use of telephone techniques offers, in addition to economy of space and civil engineering, a considerable saving in cost in the manufacture and installation of telephone cable over that of vulcanized-india-rubber cable, for in this case the switchgear is located about three-quarters of a mile from the power house.

At Loch Sloy, a generator control desk and a mimic diagram afford full facilities for the supervision and control of four 32-megawatt turbo-alternators and six 132-kilovolt power lines. There is a control panel for each of the four

Power house and conduits carrying the water that drives the four generators at Loch Sloy.





Control room at Loch Sloy power plant. A single engineer has complete control of the entire installation including four 32-megawatt generators and six 132-kilovolt power lines.

generators, two alarm-display panels, and a 20-line cordless private branch telephone exchange in the control desk.

Each generator panel carries speed and voltage control keys, a circuit-breaker control key, and four indicating meters for output power, power factor, rotor voltage, and rotor current.

Two alarm-display panels are accommodated in the wings of the desk, one for the generators and the other for the power lines. There are 17 alarms for each generator and a fifth group contains miscellaneous alarms. There are 6 alarms for each of the 6 feeders and 12 additional alarms for the bus sections and line protection. Each alarm lamp is wired through a relay to the alarm-originating contacts on the associated protective device and, on the occurrence of an alarm, the lamp flashes and a bell rings. Actuation of the key for the group silences the bell and steadies the lamp. A total of 168 alarms is catered for on the desk, of which 121 are in use at present and the remainder are in reserve for future lines.

The wall-type indication and control diagram shows the electrical arrangement of the station and the water and oil systems of the turbines in mimic. The lines and symbols are in suitable colours on a neutral background and the circuit breakers and isolators are identified by three-colour single-aspect lamp indicators.

The center wall panel mounts six 6-inch meters, viz., megawatts, megavars, synchroscope, frequency, incoming volts, and busbar volts. Individual feeder currents are indicated on 3-inch instruments set in the feeder lines on the diagram.

The control engineer sitting at the desk has full command over all four generators and can synchronise, adjust loading, or switch them without leaving his chair. All turbine, generator, and switchgear protective devices are continuously monitored and any abnormality is instantly indicated. Thus, one engineer has at his finger tips full control of the entire electrical output of the station.

The miniature direct-wire remote control system and the supplementary direct-wire alarm equipment were developed, produced, and installed by Standard Telephones and Cables, Limited.

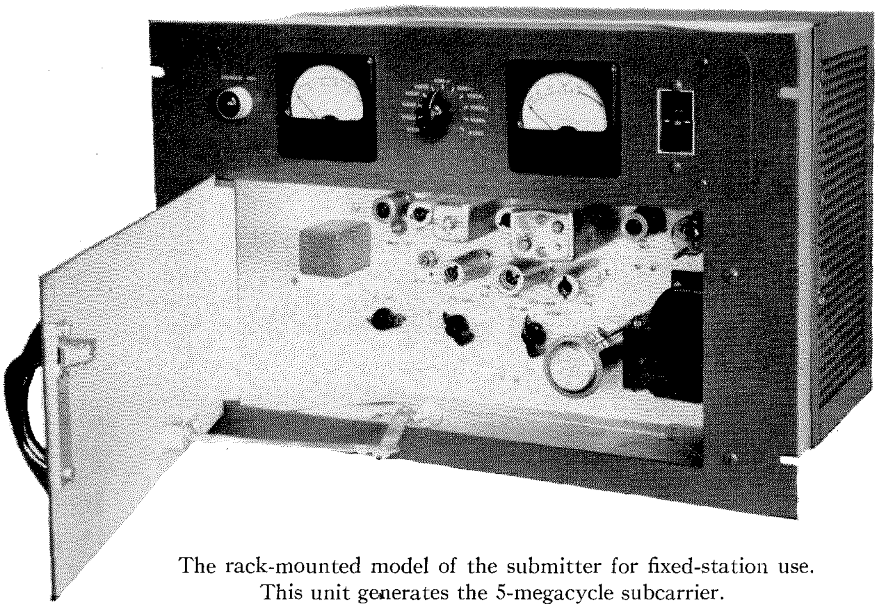
Television-Link Sound Diplexer

By L. STASCHOVER and H. G. MILLER

Federal Telecommunication Laboratories, Incorporated; Nutley, New Jersey

In transmitting television programs from a studio to a transmitter that may be remotely located in an inaccessible place, it is common practice to use a coaxial line or a radio link for the picture signal and wire lines for the sound program. The use of two separate media increases costs as well as the susceptibility to program interruption for a failure of either picture or sound is equally damaging. A method of transmitting the sound program over the standard studio-to-transmitter radio link has, therefore, been developed.

In the United States, the video-frequency band is 4.5 megacycles wide. This permits a sound subcarrier to be placed at 5 megacycles. The amplitude of this sound carrier, which is frequency modulated over a range of ± 50 kilocycles, does not exceed 10 percent of the equivalent picture-signal white level so there will be no crosstalk into the picture channel.



The rack-mounted model of the submitter for fixed-station use. This unit generates the 5-megacycle subcarrier.

There are two sources of crosstalk from the picture channel to the sound channel. One of these is through amplitude modulation of the sound channel from the picture channel, which may result from intentional nonlinear amplification in the link system. Such modulation may be removed by limiting action in the sound receiver.

The second cause of crosstalk is nonlinear time delay. This time or phase nonlinearity in frequency modulation is comparable to amplitude

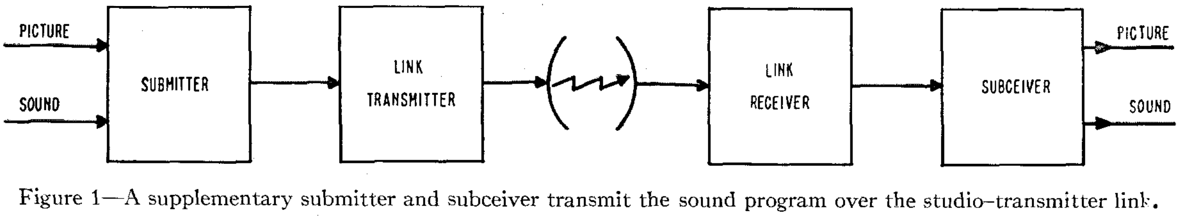
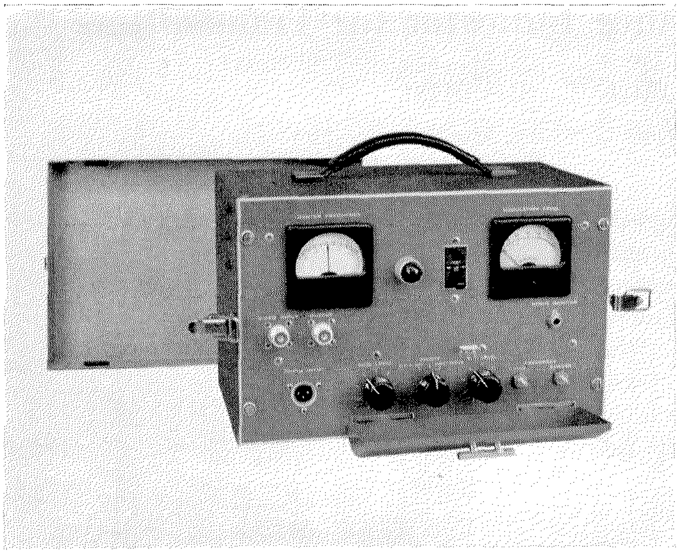


Figure 1—A supplementary submitter and subceiver transmit the sound program over the studio–transmitter link.



The portable model of the submitter is intended for mobile-station use. It measures 14 by 9 by 8 inches and weighs 16 pounds.

nonlinearity in amplitude modulation. Careful design of the radio- and intermediate-frequency sections of the entire system will keep this cause of crosstalk within satisfactory limits.

The subcarrier transmitter for the sound channel is called the "submitter." As may be seen in Figure 1, the picture and the sound electrical signals are supplied to it. It then generates the 5-megacycle subcarrier and frequency modulates it with the sound program. This sound channel is combined with the picture channel and supplied to the link transmitter.

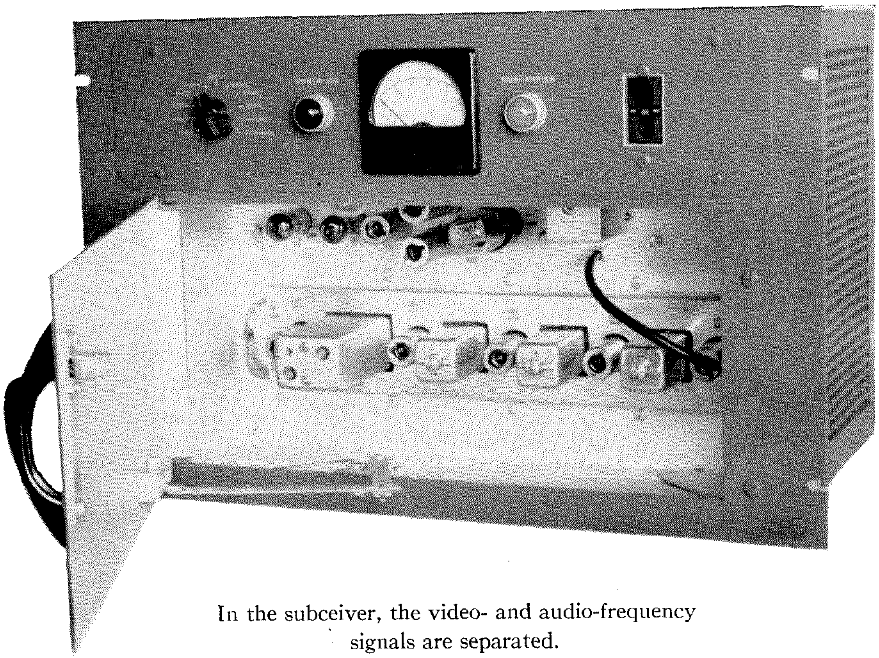
The demodulated output from the link receiver goes to the subcarrier receiver or "subceiver," which filters the sound channel from the rest of the band, amplifies and limits this 5-megacycle signal, and demodulates it in a balanced discriminator. A low-frequency filter and an audio-frequency amplifier deliver an output signal 18 decibels above 1 milliwatt into impedances of 600, 150, and 50 ohms, either balanced or unbalanced.

The low-pass filter in the subceiver is flat from below 20 cycles out to 12,500 cycles with a sharp cutoff above that frequency. Maximum attenuation occurs at the horizontal-line frequency of 15,750 cycles.

The submitter is designed for zero over-all gain, normally accepting and delivering 2-volt peak-to-peak signals on 75-ohm lines. However, the audio-frequency input may vary between +10 and −10 decibels referred to 1 milliwatt and normal output still be obtained.

A 5-megacycle discriminator samples the frequency-modulated output of the submitter and produces a direct-current error voltage that holds the generator to the required center frequency.

Residual crosstalk depends largely on the phase characteristics of the studio–transmitter relay link. The root-mean-square value of this crosstalk can usually be reduced readily to at least 50 decibels below rated output of the sound channel and compares favorably with intercity telephone circuits.



In the subceiver, the video- and audio-frequency signals are separated.

Transmission-Line Balance Indicator for Transmitters

By GEORGE T. ROYDEN

Mackay Radio and Telegraph Company; New York, New York

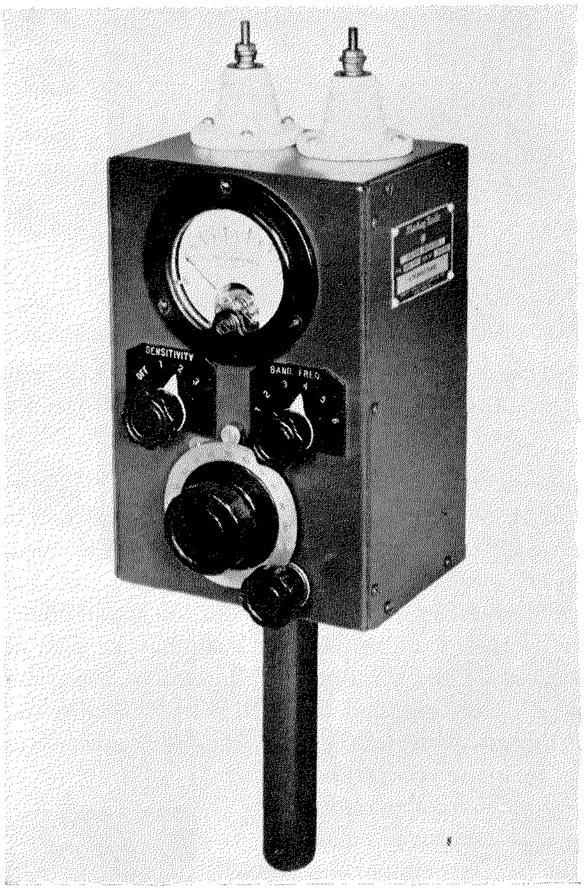
A BALANCE indicator has been designed to simplify adjustment of the output circuits of a radio transmitter to produce balanced currents in a two-wire transmission line. This indicator is particularly useful with transmitters having one tube in the output stage and feeding power through an Alford network to a two-wire transmission line as illustrated in Figure 1.

1. Description

The balance indicator is contained in a small aluminum box. There are a selective circuit, a germanium-crystal rectifier, a milliammeter, and switches for sensitivity and frequency ranges. Two bushing insulators are mounted on top and support the terminals through which the device is connected to the radio-frequency transmission line. Each bushing insulator also supports one plate of a small capacitor (approximately 1 micromicrofarad), the other plate being inside the box. Therefore, each line is connected through these small coupling capacitors to the selective circuit. A frequency calibration chart is mounted on the back. A wooden handle on the bottom facilitates use. On each terminal, there is mounted a wire with a hook on the end for hanging on the open-wire transmission line.

The selective circuit includes a variable capacitor and a coil having taps connected to the frequency range switch. A resistance voltage divider is connected through a small fixed capacitor in shunt with the selective circuit. The sensitivity switch connects the crystal rectifier to the desired position on the voltage divider. The rectified current is indicated by the meter.

The frequency range is from 4 to 30 megacycles per second. The box is $5\frac{1}{2}$ inches wide, $8\frac{1}{4}$ inches high, and $4\frac{1}{4}$ inches deep (14 by 21 by 11 centimeters). The handle extends 6 inches (15 centimeters) below the case and the terminals extend $2\frac{1}{4}$ inches (6 centimeters) above the case, making the over-all height $16\frac{1}{2}$ inches (42 centimeters). The weight is 4 pounds (1.8 kilograms).



Type *BNY-1197* balance indicator.

2. Theory of Operation

A two-wire transmission line is balanced when the current flowing in one wire is equal to that flowing in the other wire and there is a 180-degree phase displacement between these currents. To feed such a balanced line properly from a single-ended power amplifier stage requires accurate adjustment of the interconnecting circuit.

The Alford network, usually employed for this purpose, divides the output current precisely in half, part going directly to one wire of the transmission line and the other part being displaced by 180 degrees in phase before reaching the other wire. Two equal inductances in series and a variable capacitor in shunt to ground from the junction of the two coils are usually employed to accomplish this phase shift.

After connecting the balance indicator to the line, it is tuned to resonance at the transmitter frequency. Then the Alford network in the transmitter is adjusted so the rectified current as indicated by the balance indicator decreases to zero or a minimum near zero. While the transmitter output is unbalanced, current showing the unbalance flows through the circuit to the case and then through the capacitance between the case and ground. This capacitance is much larger than the coupling capacitances and therefore has little effect on the operation of the instrument.

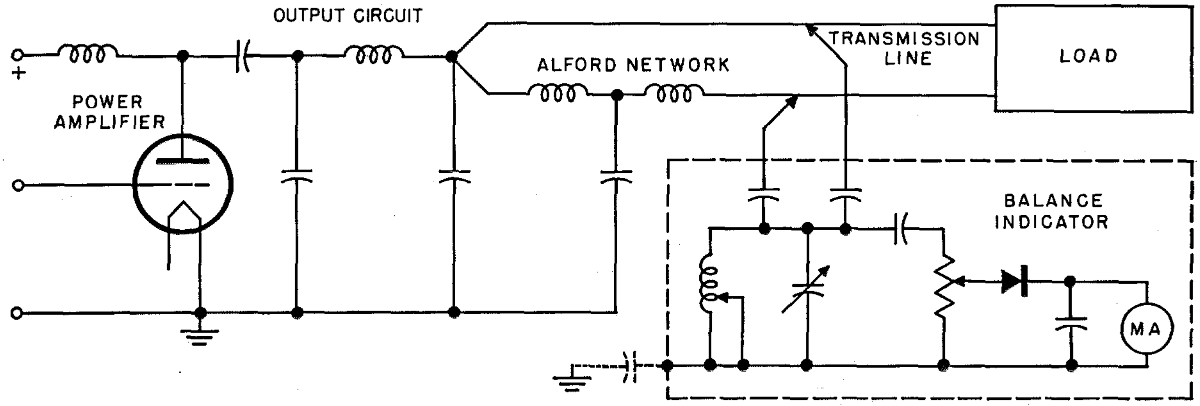


Figure 1—The balance indicator shows the voltage to ground from the midpoint of a capacitive voltage divider connected across the transmission line, thereby facilitating proper adjustment of the Alford network to produce line currents that are equal and 180 degree out of phase.

Assuming the load to be balanced with respect to ground, the voltage from each line wire to ground will be equal when the currents are balanced. The two small coupling capacitors are connected in series with each other across the line. Since they are adjusted to be equal, the voltage across them will be equal. Therefore, the junction should be at ground potential when the line voltages and currents are balanced.

Recent Telecommunication Development

Maximum-Amplitude Measuring Set

It is often desirable to measure the peak amplitude of transients such as occur in the firing of gas discharge tubes, electric welding, surges in capacitors, starting currents in motors, and surges in semiconductors, as well as abnormal phenomena that can be converted from mechanical or other forms into electric currents or voltages.

The maximum-amplitude measuring set will accept any voltage not exceeding 10 volts and will indicate simultaneously on two meters with an accuracy within 5 percent the maximum values of positive and negative peaks that persist for at least 2 milliseconds. The meters are reset to zero manually and the applied voltage may be either direct or alternating.

This portable instrument weighs only 25 pounds (11.7 kilograms) and is energized from the alternating-current mains. It was introduced recently at the Physical Society exhibition in London by Standard Telephones and Cables, the manufacturer. The set was designed by Standard Telecommunication Laboratories.

Suppression of Harmonics in Radio Transmitters

By GEORGE T. ROYDEN

Mackay Radio and Telegraph Company, Incorporated; New York, New York

LITERATURE on the subject of suppressing harmonic radiation from transmitters was reviewed and the most important items are summarized. The amount of harmonic suppression required is estimated for typical conditions. Several circuits that are favorable for harmonic suppression are described. It is shown that coupling two circuits by a common shunt capacitance is more effective in suppressing harmonics than to use inductive coupling. Formulas for estimating the harmonic output for several typical power-amplifier output circuits are included. Several applications of harmonic-suppression techniques are discussed and the results of some tests are given.

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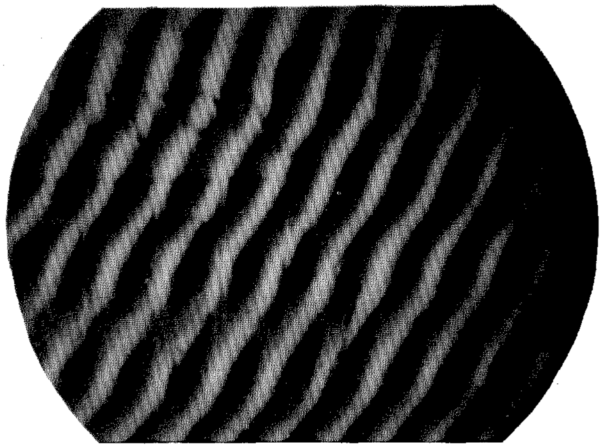

Radiotelegraph transmitters may interfere to various degrees with television reception. The above "venetian-blind" pattern results from strong interference, which if increased would prevent synchronization between transmitter and receiver and the picture would "tear" completely.

Radio transmitters intended for international telegraph communication usually operate with power outputs between 1 and 50 kilowatts at frequencies between 4 and 26 megacycles per second. For high efficiency, the final stage operates as a class-C amplifier. Conventional methods, including a high-Q plate tank circuit, have until recently been deemed adequate for the suppression of the undesired harmonics of the fundamental frequency. However, a considerable number of television receivers have recently been installed in the vicinity of some transmitting stations. This is serious, not only because of the many harmonics falling within the television band between 54 and 88 megacycles, but also because the radiotelegraph transmitters are located for economic reasons at a distance from large cities where they happen to be at the fringe of the service area for the television broadcast stations in those cities. Consequently, the radiated harmonic energy must be extremely small to avoid interfering with the weak television signals. With a large number of transmitters serviced by only a few operators and many transmitters being retuned to different frequencies each day, it is not feasible to resort to a trap circuit tuned to the unwanted harmonic and similar "tricks" employed by radio amateurs.

1. *Review of Literature*

The control of harmonics in radio transmitters has been a problem for many years.[1]* Kaar and Burnside[2] described transmitters feeding through transmission lines to a closed tuned circuit that is coupled to the antenna and very materially reduces harmonic radiation. Labus and Roder[3] studied various circuits and showed that a chain of resonant circuits coupled by shunt capacitances with the total volt-amperes being many times the watts output offers best harmonic suppression. Everitt[4] showed how the conventional π network between the plate and load circuits can be employed to reduce harmonic radiation. He also described how a series inductance and capacitance tuned to the harmonic frequency

* Numbered references appear in Section 11 on page 120.

and connected in shunt with the load greatly attenuates the selected harmonic. Kusonose[5] listed the general principles for attenuating harmonics and showed how a particular harmonic may be eliminated by tuning to that harmonic a trap circuit in series with the anode and adjusting the coupling between the inductances in the trap circuit and the main tank circuit to balance out the chosen harmonic.

Chambers[6] and associates described a 500-kilowatt broadcast transmitter in which harmonics are suppressed by using push-pull output stages, Faraday shields, a T-section harmonic filter (low pass), a concentric transmission line with grounded shell, and an impedance-matching section consisting of series inductance and shunt capacitance for coupling the line to the antenna.

A number of articles by Terman[7–11] and associates showed how the plate current of a class-C amplifier tube may be analyzed to determine the magnitude of each harmonic with respect to the direct current as a function of the angle of flow and peak amplitude.

Warner[12] observed a well-defined indication at the Federal Communications Commission that all services are going to be under the necessity of devoting a lot more attention to the elimination of spurious emissions, harmonics, hash, etc. He said that television is here to stay and all amateurs in the vicinity of a television receiver must emit only pure waves.

There have been many articles[13–29] published in *QST*, as well as other magazines intended for amateurs, showing how to reduce or eliminate harmonics. The following procedures have been advocated for amateur transmitters:

A. Do all frequency multiplying at a low power level.

B. Use class-B amplifiers up to the final stage.

C. Use a plate circuit for the final amplifier with a capacitor connected as directly as possible from plate to cathode.

D. Use a grid circuit for the final amplifier with a capacitor connected as directly as possible from grid to cathode.

E. By-pass all power and control leads with a full π network.

F. Shield all stages, making certain that interstage shielding is adequate.

G. The antenna coupling system should be separately shielded.

This mild form of interference may be caused by radiation from an inadequately shielded oscillator in a nearby television receiver. Interference from a radiotelegraph transmitter, while similar, will be influenced by keying, whereas that from another receiver changes only slowly.

H. Use narrow-band frequency modulation instead of amplitude modulation.

J. Adjust grid drive, grid bias, and output loading for minimum harmonic output.

K. Study layout of output circuit of final amplifier to ascertain the probable paths for harmonics. Determine the resonant frequency of these paths with a grid-dip oscillator and rearrange to move above 160 megacycles.

L. Explore radio-frequency fields near power and control leads with a sensitive indicator tuned to the harmonic frequencies. Continue corrective measures until harmonics are not detectable.

M. Explore radio-frequency fields near output and antenna transmission line for spurious outputs on harmonic frequencies, also harmonics of the oscillator frequency to determine if further reduction is necessary.

N. Insert a low-pass filter in the transmission line between the power-amplifier output stage and the antenna tuner.

P. Use a push-pull output stage.

Grammer[21] showed a filter consisting of a trap circuit resonant at a harmonic frequency in series with the inner conductor of a shielded concentric line linking the power amplifier output to the antenna coupling system and a shunt capacitor all within a shielded enclosure.

Reinartz[22] had applied the method proposed by Kusonose[5] of a trap circuit tuned to a harmonic and coupled to the tank circuit so as to cancel out the unwanted frequency in transmitters intended for amateur operation. He added an innovation by connecting a small capacitor between the hot side of the harmonic trap circuit and ground with the length of lead adjusted to provide minimum interference. Although Reinartz did not explain how it functions, it is probable that a series-tuned circuit to ground is provided for a higher harmonic.

Patterns[22] illustrating the distortions to the television picture caused by various types of interfering signals help to trace the source of interference.

Murdock[25] employed adequate shielding, a high-capacitance tank circuit in the power-amplifier stage, link-circuit coupling to antenna tuning circuit using coaxial cable with shielded coupling at each end, and a pair of coaxial transmission lines with shielded coupling to the antenna tuning circuit.

Mix[26] described a transmitter in which there are capacitors from plate to ground with short paths, adequate shielding, filtered power leads, and link coupling from the push-pull power amplifier stage to the separately shielded antenna tuning unit.

Seybold,[27] Pichitino,[28] and Grammer[29] described the design and application of m-derived low-pass filters for harmonic reduction.

The use of a push-pull amplifier with tubes operating in class B was suggested by Romander as an effective method to avoid the generation of harmonics. Such a proposal could be readily adapted in the design of a single-sideband transmitter and would be useful where both telephone and one or more telegraph channels were to be radiated simultaneously as advocated by Rabuteau.

2. *Degree of Harmonic Suppression*

A report issued by the Institute of Radio Engineers in 1930 recommended that any harmonic radiated from a broadcast transmitter should not exceed 0.02 percent of the field strength of the fundamental and never exceed 500 microvolts per meter at a distance of 1 mile. The meeting of the International Consultative Committee on Radio in Copenhagen in 1931 fixed a limit of 0.05 percent for broadcast frequencies and 0.1 percent for lower frequencies. Limits were not established for frequencies above 3 megacycles until the meeting of the International Consultative Committee on Radio at Cairo in 1938 agreed on a limit of −40 decibels for any harmonic but not in excess of 200 milliwatts. These figures were confirmed at the meeting at Atlantic City in 1947. However, the discussion in the next several paragraphs indicates that this limit is far above that necessary to avoid interference with reception near high-power transmitters.

For the purpose of illustrating how to determine the amount of suppression that is necessary, the conditions at Brentwood, New York, will be considered. This station is 40 miles from New York City. Measurements published by Goldsmith[30] indicate the field strength at this distance to be above 500 microvolts per meter from only the strongest television station.

Reinartz[22] showed that an interfering field strength below 5 microvolts per meter will not disturb a television signal of 500 microvolts per meter, which is considered by the Federal Communications Commission to be the limit of service area in rural sections. Residences on which television receivers may be installed are located only 2500 feet from the transmitters at Brentwood. The intervening path is over poor soil and through trees that introduce attenuation. Bullington[31] gives an approximate formula for the attenuation between low antennas far apart.

$$A=\frac{E}{E_0}=\frac{4\pi h_s' h_r'}{\lambda d}, \tag{1}$$

where

E = attenuated field in volts per meter
E_0 = radiated field in free space
$h_s' = h_s + jh_0$
h_s = height of sending antenna in meters
h_0 = minimum effective height
$h_r' = h_r + jh_0$
h_r = height of receiving antenna in meters
λ = wavelength in meters
d = distance in meters.

Substituting

$h_s = 3$ meters
$h_r = 10$ meters
$h_0 = 3$ meters (from Bullington,[31] Figure 3 for poor soil at 40 megacycles)
$\lambda = 4.3$ meters (at 69 megacycles)
$d = 760$ meters,

attenuation is found to be

$$A=\frac{4\pi 4.2\times 10.4}{4.3\times 760}=0.168.$$

The radiated power is related to field strength and distance by

$$P = E_0^2 d^2/30, \text{ watts.} \tag{2}$$

Substituting

$$E_0 = E/A = 5\times10^{-6}/0.168 = 30\times10^{-6}$$
$$d = 2500 \text{ feet} = 760 \text{ meters,}$$

the radiated harmonic power is calculated to be

$$P = 30^2\times10^{-12}\times760^2/30 = 17.4\times10^{-6} \text{ watts,}$$

which is 92 decibels below a power output of 30 kilowatts at the fundamental frequency. The intervening trees will introduce an attenuation of about 5 decibels. For television reception in that area, a high-gain antenna is recommended and, since most residences are situated at one side of the Brentwood transmitters, the directivity of the television antenna may be expected to provide 10 decibels or more discrimination. Therefore, the harmonics between 54 and 88 megacycles should be suppressed to at least 80 decibels below the fundamental to avoid interference with television reception.

Mobile services on frequencies between 30 and 50 megacycles are not active at present in the immediate vicinity of Mackay Radio's transmitter stations. Therefore, it would seem reasonable to suppress the radiated harmonic power at these frequencies enough to avoid interference within a radius of 5 miles. The sensitivity and characteristics of the frequency-modulation receivers used for mobile service are such that an interfering signal of 0.5 microvolt per meter may be tolerated. Assuming an antenna height of 3 meters at the receiver and substituting in (1), the attenuation is

$$A = \frac{4\pi 4.2\times4.2}{7.5\times8000} = 0.0037,$$

and the harmonic power is calculated from (2) to be

$$P = \frac{0.5^2\times10^{-12}\times8000^2}{0.0037^2\times30} = 0.039 \text{ watts,}$$

which is 59 decibels below 30 kilowatts. Therefore, harmonics between 30 and 50 megacycles should be reduced to a level at least 60 decibels below a fundamental power output of 30 kilowatts.

For mobile services on frequencies between 72 and 76 megacycles, the 80-decibel reduction previously indicated to avoid interference to television reception should be adequate.

3. *Measuring Apparatus*

Honnell and Ferrell[32] measured the harmonic-power output by coupling a balanced shielded loop to the transmission line from the transmitter to the antenna and connecting the loop through a second tuned circuit to a receiver having a calibrated attenuator in the intermediate-frequency amplifier circuit. The system was calibrated by measuring the power sent through the line to a dummy load from a separate transmitter operating at the frequency of the harmonic being measured. Both series and parallel currents could be measured in the open-wire transmission line.

Grammer[33] improved on the simple tuned circuit with rectifier and meter by using a tuned circuit coupled to a type-955 acorn triode with adjustable plate voltage so the tube detector is operated just below oscillation. The equivalent of a high Q is thereby obtained.

A high-Q circuit has been found convenient for measuring the voltage of the harmonic on the transmission line or at some accessible point in the transmitter. The inductor consists of a single turn of copper 1-inch wide and 0.03-inch thick (25 by 0.7 millimeters) connected at its center to the shield. This is tuned to the selected harmonics by a variable air capacitor connected across the ends of the inductor. One end of the inductor is connected through an air capacitor of about 1 micromicrofarad and a bushing insulator to the external circuit. A germanium diode is connected through a multiposition switch to taps on the opposite side of the inductor. The positions of these taps are chosen to give all, one-half, one-fourth, and one-eighth of the voltage across that half of the inductor thus providing several voltage ranges. The diode feeds a 50-microampere indicating instrument. All components are enclosed within a suitable aluminum box. The device is calibrated by means of a vacuum-tube voltmeter and a suitable source of radio-frequency power.

Another device employing three tuned circuits has been found more selective. For convenience the ganged variable inductors and the ganged variable capacitors are driven from a common shaft. The first circuit is coupled to the transmission line through a very small capacitor, about 1 micromicrofarad, and the succeeding circuits are coupled by small capacitors. The radio-frequency voltage in the final tuned circuit is rectified by a germanium diode and the rectified current passed through an adjustable resistance to a 50-microampere indicating instrument.

The light weight and small size of these devices permit them to be hung from the transmission line. The coupling capacitor is designed to withstand the peak line voltage with an ample factor of safety.

4. *Calculation of Harmonic Content*

To obtain high efficiency, the final amplifier is usually operated in class *C*, that is, the plate current flows for less than half of the cycle. The waveform of the plate current is a pulse which Terman[34] has shown to follow approximately a 3/2 power law. It is reasonable to assume the plate current to flow during 120 degrees for the purpose of estimating the harmonic content. For this angle, the ratio k of the harmonic current to the direct current is determined from Terman's Figure 86 to be 1.4, 0.85, and 0.27 for the second, third, and fourth harmonics, respectively.

For well-shielded circuits of good design, the harmonic voltage at the output terminals of the transmitter may be calculated approximately in terms of the above ratio, the direct current, and the circuit constants. For example, calculations will be shown for a transmitter with a π output network, as shown in Figure 1.

For the harmonic current, the impedance of A becomes A/n, that of B becomes Bn, and C becomes C/n, where n is the order of the harmonic. The impedance of S is negligible. The harmonic voltage appearing across the output capacitor can be shown to be

$$e=\frac{ACIk}{n^3(B-A/n^2-C/n^2)}. \qquad (3)$$

The radiated harmonic power depends on many factors such as the radiating properties of the transmission line and the antenna. It has been found that harmonic currents flow in parallel into an open-wire transmission line. A typical transmission line consisting of two size 6 (American Wire Gage) wires spaced 12 inches apart and

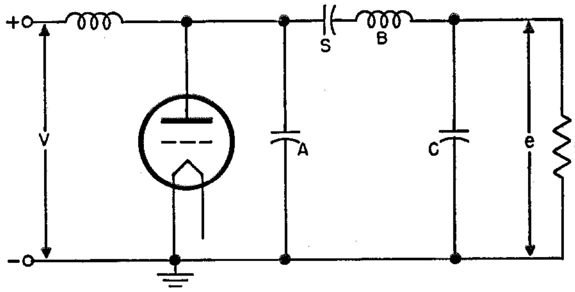

Figure 1—Transmitter output stage connected through a π network to the load.

10 feet above ground has an impedance H for parallel currents of 330 ohms. This value is taken for calculating harmonic power. Let

$$\begin{aligned}
W &= 30\,000 \text{ watts output}\\
V &= 9\,000 \text{ volts, direct voltage}\\
I &= 5 \text{ amperes, direct current}\\
f &= 20 \text{ megacycles}\\
A &= 80 \text{ ohms}\\
B &= 110 \text{ ohms}\\
C &= 30 \text{ ohms}\\
R &= 150 \text{ ohms.}
\end{aligned}$$

Then, for the second harmonic

$$\begin{aligned}
n &= 2\\
k &= 1.4\\
e &= 80\times30\times5\times1.4/2^3(110-20-7.5)\\
&= 25 \text{ volts,}
\end{aligned}$$

and the harmonic power leaving the transmitter is approximately

$$\begin{aligned}
w &= e^2/H\\
&= 25^2/330 = 1.9 \text{ watts,}
\end{aligned}$$

which is 42 decibels below the assumed power output. Similar calculations show the third harmonic power to be 54 decibels below the output power of 30 000 watts.

More harmonic suppression can be obtained by two circuits coupled together by a common shunt capacitance (sometimes called a double π network) as shown in Figure 2. For this circuit, it is necessary to choose C so that critical coupling results. This is approximately

$$C=\frac{A}{(r+rR/F^2)^{\frac{1}{2}}}, \tag{4}$$

where r = ratio of desired plate load to output resistance R.

The harmonic voltage appearing across the output capacitor can be shown to be approximately

$$e=\frac{ACFIk}{n^5(B-A/n^2-C/n^2)(D-C/n^2-F/n^2)}. \tag{5}$$

It is desirable to choose an output capacitor such that the Q of the output circuit is higher than 2 but preferably not over 3. For a value of 2.6,

$$F=150/2.6=58$$
$$r=7.5$$
$$C=10.3$$
$$B=80+14=94$$
$$D=26+50=76.$$

For the second harmonic

$$e=\frac{80\times 10.3\times 58\times 5\times 1.4}{2^5(94\text{–}20\text{–}3)(76\text{–}3\text{–}15)}$$
$$=2.6 \text{ volts,}$$

and the harmonic power leaving the transmitter is approximately

$$w=2.6^2/330=0.02 \text{ watts,}$$

which is 62 decibels below the assumed output. Similarly, the third harmonic power is found to be 82 decibels below 30 kilowatts.

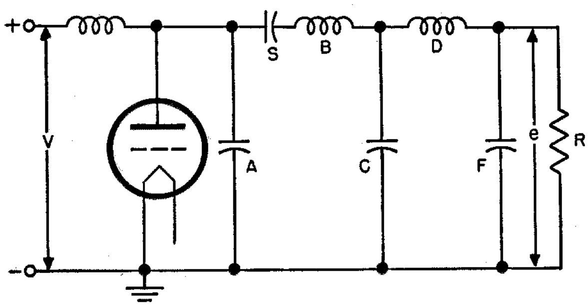

Figure 2—Transmitter output stage using two π networks having a common shunt capacitance.

Analysis of (3) and (5) indicate that there is little to be gained so far as harmonic suppression is concerned by designing for high-Q circuits in the output stage. This is contrary to the conclusions expressed by several authors. In (3), the expression $(B-A/n^2-C/n^2)$ in the denominator is approximately equal to A in the numerator, leaving the other factors C, I, k, n, and H to determine the harmonic-power output, so that

$$w=\frac{C^2I^2k^2}{n^6H}, \text{ approximately.} \tag{6}$$

Similarly in (5), the expression $D-C/n^2-F/n^2)$ in the denominator is nearly equal to F in the numerator, so that

$$w=\frac{C^2I^2k^2}{n^{10}H}, \text{ approximately.} \tag{7}$$

In a practical circuit, the impedances of the circuit components and their connecting leads are usually different at harmonic frequencies than the assumed impedances. Therefore, the approximations introduced in deriving (6) and (7) may be tolerated.

5. *Development*

Many of the transmitters presently in service use a secondary circuit that is inductively coupled to the plate tank circuit, the transmission line being fed from the secondary circuit. For such a circuit, the harmonic voltage induced into the secondary is

$$E_n=j2\pi nfMI_n, \tag{8}$$

where

$j=-1^{\frac{1}{2}}$
n = order of harmonic
f = frequency in cycles per second
M = mutual inductance in henries
I_n = component of current flowing in primary at the harmonic frequency in amperes.

For shunt capacitance coupling as shown in Figure 2, the harmonic voltage applied to the secondary circuit is

$$V_n=\frac{I_n}{j2\pi nfC}, \tag{9}$$

where C = coupling capacitance in farads, and the other terms are as before. The circuits are normally adjusted for critical coupling, which occurs when

$$(R_p R_s)^{\frac{1}{2}} = 2\pi f M = 1/2\pi f C, \tag{10}$$

where

R_p = equivalent series resistance of primary circuit in ohms

R_s = equivalent series resistance of secondary circuit in ohms.

Combining (8), (9), and (10) and solving for the ratio of harmonic voltage for inductive coupling with respect to that for shunt capacitive coupling

$$E_n / V_n = n^2. \tag{11}$$

This indicates an improvement of 12 decibels for the second harmonic, 19 decibels for the third, and 24 decibels for the fourth harmonic.

Measurements of the harmonic voltage at the output terminals of several transmitters having inductively coupled output circuits revealed the harmonics to be somewhat higher than calculated. Some harmonics were comparable with the calculated harmonic voltage at the anode, indicating no attenuation in the circuits following the anode. Further study indicated that these abnormally high harmonic outputs were due to stray capacitance beginning at the anode, or components connected to the anode, and terminating on components connected to the transmission line. The harmonic current flowing through this stray capacitance may pass over connecting leads having enough inductance to approach resonance at some harmonic.

It was found that this type of coupling via stray capacitances could be overcome by means of a Faraday screen between the plate tank-circuit coil and the coupling coil together with adequate shielding for the output circuit. The Faraday screen should be designed so the wires are grounded at but one point and the length of each wire should not exceed 0.1 times the wavelength of the highest harmonic frequency to be stopped, also the gap between wires should not exceed 0.03 times the wavelength.

6. π-Network Output Circuit

The π network, as shown in Figure 1, is the most economical way to obtain a reasonable suppression of harmonics with a minimum number of components and controls. When the components are properly located with respect to each other, to the anode, and to ground, it has been found that the harmonic voltage is somewhat lower than calculated as above for the low-order harmonics. The discrepancy is probably due to distributed capacitance of the series coil B making its reactance higher than Bn, also the inductance in the leads, which connect the shunt capacitors to ground and to the circuit, thereby making their reactances less than A/n and C/n. Measurements indicate an attenuation better than 50 decibels.

The π network is recommended for use where the station is isolated so that only a moderate suppression of harmonics is required.

7. Double π Output Circuit

The double π output circuit, as shown in Figure 2, is recommended where the station is so located that considerable suppression is required to avoid interference by harmonics. Several transmitters at Brentwood have been modified to use this output circuit. Measurements indicate the power at a harmonic frequency to be better than 70 decibels below the power at the fundamental frequency.

To obtain best performance, the coupling capacitor should be located so its grounded terminal is close to the point where the cathode circuit is grounded. The loop enclosed by the blocking capacitor S, the series coil B, the coupling capacitor C, and the plate tank capacitor A (usually the internal plate-grid tube capacitance, neutralizing capacitance, and stray capacitances suffice without added tank-circuit capacitance for high-power tubes at frequencies above approximately 8 megacycles) has an important effect on the highest frequency to which the tank circuit may be tuned.

The secondary circuit must be adequately shielded from the primary or plate tank circuit. The coupling capacitor should be placed so the lead from it to the secondary circuit has a minimum exposure in the tank-circuit compartment, preferably not more than 4 inches. The loop enclosed by the series coil D, the shunt capacitor F, and the coupling capacitor C, together with the interconnecting leads and ground return has an important effect on the maximum frequency to which the secondary circuit can be tuned. Therefore, the area of this loop must be kept as small as possible.

The capacitance F in the secondary circuit is arbitrarily chosen so that the loaded Q is higher than 2 but preferably not over 3. This is done to minimize circulating current and avoid unnecessary losses.

8. Low-Pass Filters

A number of low-pass filters consisting of a constant-k intermediate section and shunt m-derived end sections have been constructed. These are designed for operation over a narrow band of frequencies. The terminal half-sections are designed so the peaks of attenuation are near the second and third harmonic. The intermediate section provides ample attenuation for the higher-order harmonics. The elements are in duplicate for operation with a balanced 600-ohm line. Fixed vacuum capacitors were employed for the shunt elements, four being required for each filter. Short lengths of large coaxial cable were used as capacitors in shunt with the end coils. All coils were wound with size 6 (American Wire Gage) solid copper wire. The filter was enclosed in an aluminum case with shield partitions between sections.

Care must be taken with transmitters having exceptionally high stray capacitance between the plate circuit and the output circuit. If the circuit including this stray capacitance and the filter has series inductance approximating resonance, a high circulating current may cause failure.

9. Spot Filters and Stubs on Transmission Line

As an expedient, a simple filter may be applied to the transmitter to attenuate one particular harmonic. This consists of a capacitor and coil in series with each other from each line to ground. To be effective these must resonate for the harmonic frequency to be suppressed. Sometimes only one coil is used between ground and the junction of two capacitors that bridge the transmission line. An attenuation of 10 to 20 decibels has been obtained.

Open-ended stubs have been applied to the transmission line for the purpose of suppressing harmonics. The length is made one-quarter wavelength for the harmonic frequency to be suppressed less about 6 inches for correction of end effect. These are usually connected directly to the output terminals of the transmitter, sometimes two or three sets of stubs being employed for as many harmonics. As an alternative (or in addition to stubs at the transmitter when more attenuation is desired) two sets of stubs in conjugate relationship are applied to the transmission line. These should be located as close to the transmitter as feasible because standing waves for the harmonic frequency are set up on that portion of the line between the stubs and the transmitter. Radiation in certain directions will take place from this portion of the line. An attenuation of 10 to 20 decibels may be obtained by means of stubs.

10. Conclusions

The recent increased use of frequencies above those now used for fixed point-to-point service often requires modification of old transmitters to suppress harmonics and avoid interference with mobile and television services. A reasonable suppression may be obtained by employing a π network in the output circuit together with adequate shielding of the transmitter and its supply leads. If a particular harmonic still has sufficient strength to cause interference, a spot filter or stubs may be applied.

For general use in a locality where a high degree of suppression is necessary, the double π network is recommended for the output circuit. This has the advantage of providing adequate suppression but at the expense of several additional circuit components and their controls.

Before closing, it must be pointed out that not all interference is due to harmonics. Confirming our own experience, Kiser[35] has pointed out that most of the interference complaints investigated by the Federal Communications Commission in the New York area were found to be due to a frequency other than that of the television carrier, that is, the receiver is at fault. Some are due to poor image-frequency rejection and can be avoided by adding a tuned circuit (booster) between transmission line and receiver. Others are due to insufficient shielding to keep a high-power signal out of the intermediate-frequency amplifier and can be corrected by inserting a booster or a high-pass filter between the antenna and the receiver. Although a good technician can often correct these faults, a lot of trouble can be avoided by arranging for only well-designed television receivers to be sold for use in areas around high-power point-to-point radio transmitting stations.

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Quantization Distortion in Pulse-Count Modulation with Nonuniform Spacing of Levels*

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IT IS SHOWN that in a pulse-count-modulation system the distortion due to quantization can be minimized by nonuniform spacing of levels. Equations are derived for an arrangement of nonuniform level spacing that produces minimum distortion. It is also shown that minimum distortion is significantly less than distortion resulting from uniform quantization when the crest factor of the signal is greater than four.

• • •

1. Introduction

It has been shown in a previous paper[1] that the process of quantization in a pulse-count-modulation system introduces distortion. In that paper, equations were derived for the special case of equal level spacing. This paper studies the effect of nonuniform level spacing with the view of obtaining minimum distortion.

It is shown that by taking the statistical properties of the signal into consideration, the distortion can be minimized by a proper level distribution, which is a function of the probability density of the signal. In practice, nonuniform quantization is realized by compression followed by uniform quantization. The most common form of compression is the so-called logarithmic one, where the levels are crowded near the origin and spaced farther apart near the peaks. It is shown that with logarithmic compression the distortion is largely independent of the statistical properties of the signal.

2. Distortion for Nonuniform Spacing

Consider a quantized signal $y(t)$ as shown in Figure 1. Let the levels be symmetrically disposed about zero voltage level, but otherwise placed in an arbitrary manner in the interval $(-V, V)$. The problem of minimizing the distortion of the quantized signal by properly spacing the levels will be considered now. De-

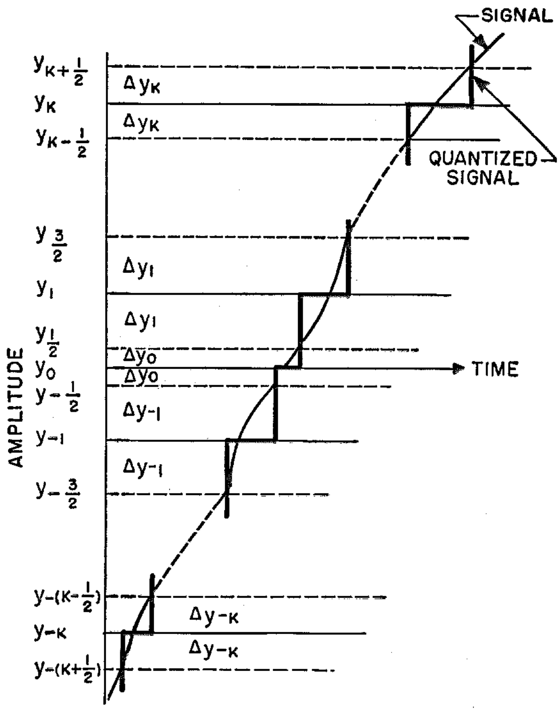

Figure 1—Nonuniform quantization.

note the levels by $y_{-n}, y_{-n+1}, \cdots, y_0, y_1, \cdots y_{n-1}, y_n$, where $y_k = -y_{-k}$ and $y_0 = 0$. Further, assume that a signal value of y which satisfies

$$y_{k-\frac{1}{2}} < y < y_{k+\frac{1}{2}} \tag{1}$$

is transmitted as level y_k. The y's with fractional subscripts are the crossover values.

The measure of distortion that is adopted is the mean-square-distortion voltage defined as

$$\sigma_k = \int_{y_{k-\frac{1}{2}}}^{y_{k+\frac{1}{2}}} (y - y_k)^2 P(y) dy, \tag{2}$$

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[1] A. G. Clavier, P. F. Panter, and D. D. Grieg, "PCM Distortion Analysis," *Electrical Engineering*, v. 66, pp. 1110–1122; November, 1947.

where σ_k is the distortion in the kth level. The error of the transmitted signal is $(y-y_k)$ and $P(y)$ the probability density of the signal y.

First, derive a relation between the various y's in (2) so that σ_k in the kth level is a minimum.

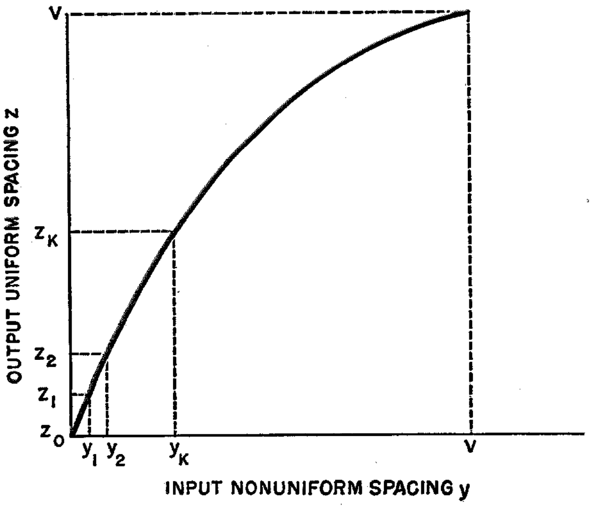

Figure 2—Compression curve for nonuniform level spacing.

$$y_k=\Delta y_0+2\Delta y_1+\cdots+2\Delta y_{k-1}+\Delta y_k$$
$$=\frac{K}{2V}\left[\frac{1}{P^{\frac{1}{3}}(y_0)}+\frac{2}{P^{\frac{1}{3}}(y_1)}+\cdots+\frac{2}{P^{\frac{1}{3}}(y_{k-1})}+\frac{1}{P(y_k)}\right]\frac{2V}{N},$$

$$\therefore\quad y \doteqdot A\int_0^z \frac{1}{P^{\frac{1}{3}}(z)}dz=\frac{\int_0^z \frac{1}{P^{\frac{1}{3}}(z)}dz}{\int_0^V \frac{1}{P^{\frac{1}{3}}(z)}dz},$$

where $z=V$ and $y=V$.

Suppose that the levels are so close that $P(y)$ may be considered as nearly constant over the region of integration and equal to $P(y_{av})$

where

$$y_{kv}=\frac{y_{k+\frac{1}{2}}+y_{k-\frac{1}{2}}}{2};$$

$$\sigma_k=\frac{P(y_{av})}{3}[(y_{k+\frac{1}{2}}-y_k)^3+(y_k-y_{k-\frac{1}{2}})^3]. \tag{3}$$

Differentiating σ_k with respect to y_k gives

$$\frac{d\sigma_k}{dy_k}=P(y_{av})[-(y_{k+\frac{1}{2}}-y_k)^2+(y_k-y_{k-\frac{1}{2}})^2]=0$$

or

$$y_k=\frac{y_{k+\frac{1}{2}}+y_{k-\frac{1}{2}}}{2}=y_{av}. \tag{4}$$

Thus, the condition for making σ_k a minimum is that y_k lie half-way between $y_{k+\frac{1}{2}}$ and $y_{k-\frac{1}{2}}$.

$$\left.\begin{aligned} y_{k+\frac{1}{2}}&=y_k+\Delta y_k,\\ y_{k-\frac{1}{2}}&=y_k-\Delta y_k.\end{aligned}\right\} \tag{5}$$

Substituting these values in (3), it follows

$$\sigma_k=\frac{2}{3}P(y_k)\Delta y_k^3. \tag{6}$$

The total mean-square distortion voltage is found by summing over all levels, giving

$$\sigma_d=\frac{2}{3}\sum_{-n}^{n}P(y_k)\Delta y_k^3. \tag{7}$$

Now, it will be proved that the mean-square distortion σ_d is a minimum when σ_k is constant, independent of the kth level.

By the definition of an integral, we may write

$$2\sum_{-n}^{n}P^{\frac{1}{3}}(y_k)\Delta y_k=\int_{-V}^{V}P^{\frac{1}{3}}(y)dy=2K, \tag{8}$$

where K is a constant, since the integral is a function of only its limits. Let $\mu_k=P^{\frac{1}{3}}(v_k)\Delta y_k$; then (7) and (8) become, respectively,

$$\sigma_d=\frac{2}{3}\sum_{-n}^{n}\mu_k^3, \tag{9}$$

and

$$K=\sum_{-n}^{n}\mu_k. \tag{10}$$

The problem is now reduced to minimizing the sum of cubes subject to the condition that the sum of the variables is a constant.

From Lagrange's method of undetermined multipliers, it follows that (9) is a minimum when

$$\mu_{-n}=\mu_{-n+1}=\cdots=\mu_{n-1}=\mu_n=\frac{K}{2\mu+1}. \tag{11}$$

From this, it follows

$$P^{\frac{1}{3}}(y_k)\Delta y_k=\frac{K}{2n+1} \tag{12}$$

or

$$\sigma_k=\frac{2}{3}\frac{K^3}{(2n+1)^3}.$$

The total minimum distortion power

$$\sigma_m = \frac{2}{3}\frac{K^3}{(2n+1)^2}$$
$$= \frac{1}{12(2n+1)^2}\left(\int_{-V}^{V} P^{\frac{1}{3}}(y)dy\right)^3. \qquad (13)$$

Since $P(y)$ is an even function and letting $N = 2n+1$ be the total number of steps,

$$\sigma_m = \frac{2}{3N^2}\left(\int_0^V P^{\frac{1}{3}}(y)dy\right)^3. \qquad (14)$$

The ratio of the mean-square distortion voltage to the mean-square signal voltage is

$$D_m^2 = \frac{\sigma_m}{\sigma} = \frac{2}{3N^2}\frac{\left[\int_0^V P^{\frac{1}{3}}(y)dy\right]^3}{\int_0^V y^2P(y)dy}, \qquad (15)$$

Equation (15) gives the minimum distortion resulting with optimum level spacing.[2]

An approximate method of obtaining levels may be obtained by writing (where k is positive)

$$y_k = \Delta y_0 + 2\Delta y_1 + \cdots + 2\Delta y_{k-1} + \Delta y_k$$
$$= \frac{K}{N}\left[\frac{1}{P^{\frac{1}{3}}(y_0)} + \frac{2}{P^{\frac{1}{3}}(y_1)} + \cdots + \frac{2}{P^{\frac{1}{3}}(y_{k-1})} + \frac{1}{P(y_k)}\right]$$
$$= \frac{K}{2V}\left[\frac{1}{P^{\frac{1}{3}}(y_0)} + \frac{2}{P^{\frac{1}{3}}(y_1)} + \cdots + \frac{2}{P^{\frac{1}{3}}(y_{k-1})} + \frac{1}{P(y_k)}\right]\frac{2V}{N}. \qquad (16)$$

The series may be approximated by an integral

$$y = A\int_0^z \frac{1}{P^{\frac{1}{3}}(z)}dz, \qquad (17)$$

where the variable on the right has been changed to z to avoid confusion. A is a constant of proportionality so chosen that when $z = V$; $y = V$. Hence,

$$y = V\frac{\int_0^z \frac{1}{P^{\frac{1}{3}}(z)}dz}{\int_0^V \frac{1}{P^{\frac{1}{3}}(z)}dz}. \qquad (18)$$

[2] This equation was first derived by P. R. Aigrain using a slightly different method.

By letting z vary from 0 to V, y will describe a curve as shown in Figure 2. As z takes on the values of $z_0 = 0$, $z_1 = 2V/N \cdots z_k = 2kV/N \cdots z_n = 2nV/N$, we get the point $y_0 = 0$, $y_1 \cdots y_k \cdots y_n$.

While approximate, this derivation has the advantage that the levels are obtained quickly. The relation (12) may be used to make spot checks since

$$y_{k+1} = y_k + \Delta y_k + \Delta y_{k+1}$$
$$= y_k + \frac{K}{N}\left(\frac{1}{P^{\frac{1}{3}}(y_k)} + \frac{1}{P^{\frac{1}{3}}(y_{k+1})}\right). \qquad (19)$$

From Figure 2, it may be seen that the uniform spacing along the z axis is transformed into a nonuniform spacing along the y axis. The compressed output z is then subjected to uniform quantization. To recapitulate, nonuniform spacing of levels may be realized by passing the signal through a compressor with a given characteristic and applying uniform quantization to its output. Obviously, this also implies that a corresponding expansion is incorporated in the receiver.

3. *Logarithmic Compression*

A compression curve that is relatively easy to obtain and has been used in practice is the so-called logarithmic compression curve. The positive half of the compression characteristic is given by

$$v_2 = k\log\left(1 + \frac{\mu v_1}{V}\right), \qquad (20)$$

where

μ = compression parameter
v_1 = input voltage
v_2 = output voltage
V = maximum input voltage
k = an undetermined constant.

To find k, let $v_2 = V$ when $v_1 = V$, so that the maximum values of the input and compressed waves are equal. This gives

$$v_2 = \frac{V}{\log(1+\mu)}\log\left(1 + \frac{\mu v_1}{V}\right). \qquad (21)$$

The parameter μ controls the degree of compression and may be chosen so that large changes in the input produce relatively small changes in the output. As shown in Figure 3 for $\mu = 1000$, a

60-decibel change in the input will cause only a 20-decibel change in the output. When μ is large, the levels are crowded about zero; $\mu=0$ corresponds to uniform spacing.

Differentiating, (21) yields

$$dv_2=\frac{\mu}{\log(1+\mu)}\frac{V}{V+\mu v_1}dv_1. \tag{22}$$

As the compressed signal is divided into N uniformly spaced levels, $\Delta v_2=2V/N$. This gives

$$\Delta v_1=\frac{2\log(1+\mu)}{\mu N}(V+\mu v_1). \tag{23}$$

In the notation of the previous section, $\Delta y_k \doteq \Delta v_1/2$ so that

$$\Delta y_k=\frac{\log(1+\mu)}{\mu N}(V+\mu y_k)=\alpha(V+\mu y_k), \tag{24}$$

where

$$\alpha=\frac{\log(1+\mu)}{\mu N}. \tag{25}$$

From (23), it follows that the ratio of the largest to the smallest level is given approximately by

$$\frac{\Delta y_n}{\Delta y_0}\doteq 1+\mu. \tag{26}$$

It is interesting to note that

$$\frac{(dv_2/dv_1)_{v1=0}}{(dv_2/dv_1)_{v1=v}} \text{ is also equal to } 1+\mu.$$

This ratio, when expressed in decibels, is often referred to as the compression of the system.

The distortion power is given approximately by (7) when the number of levels is large

$$\begin{aligned}\sigma_d&=\frac{2}{3}\sum_{-n}^{n}P(y_k)\Delta y_k^3\\&=\frac{2\alpha^2}{3}\sum_{-n}^{n}(V+\mu y_k)^2P(y_k)\Delta y_k\\&=\frac{2\alpha^2}{3}\sum_{-n}^{n}[V^2P(y_k)+2V\mu y_kP(y_k)\\&\qquad+\mu^2y_k^2P(y_k)]\Delta y_k.\end{aligned} \tag{27}$$

Since $P(y_k)$ is an even function and $y_k=-y_k$, the second term in the summation vanishes. Thus,

$$\sigma_d=\frac{2\alpha^2}{3}\sum_{-n}^{n}[V^2P(y_k)+\mu^2y_k^2P(y_k)]\Delta y_k.$$

Converting this into an integral with $2\Delta y_k=dy$ and using (41) and (46) of the appendix yields

$$\begin{aligned}\sigma_d&=\frac{\alpha^2V^2}{3}\int_{-V}^{V}P(y)dy+\frac{\alpha^2\mu^2}{3}\int_{-V}^{V}y^2P(y)dy\\&=\frac{\alpha^2V^2}{3}+\frac{\alpha^2\mu^2\sigma}{3},\end{aligned} \tag{28}$$

where σ is the mean-square voltage and V is the peak value of the signal. Hence, the distortion is given by

$$D=\left(\frac{\sigma_d}{\sigma}\right)^{\frac{1}{2}}=\frac{\log(1+\mu)}{3^{\frac{1}{2}}\mu N}(C^2+\mu^2)^{\frac{1}{2}}, \tag{29}$$

when C is the ratio of peak to root-mean-square value of signal. For a given C, the distortion is a function of μ only and will be a minimum for optimum μ as shown in Figure 4.

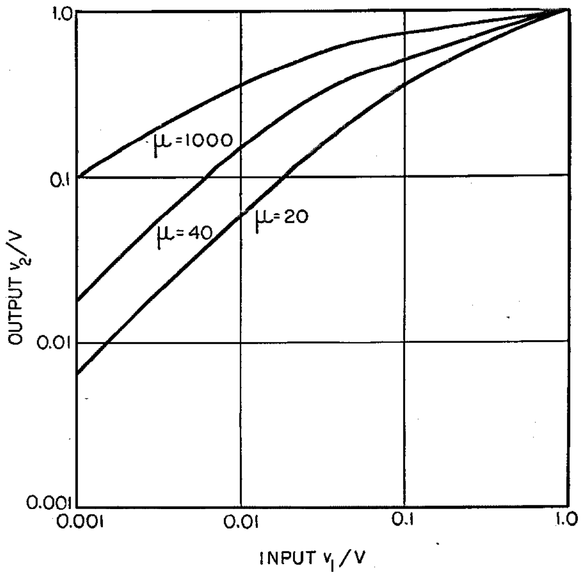

Figure 3—Logarithmic compression.

$$v_2=\frac{V}{\log(1+\mu)}\log\left(1+\frac{\mu v_1}{V}\right).$$

When μ is large compared to C, this becomes

$$D=\frac{\log(1+\mu)}{3^{\frac{1}{2}}N}. \tag{30}$$

Hence, when μ is large, the distortion is largely independent of the signal.

The distortion for uniform quantization may be obtained by letting μ become zero in (29).

$$D=C/3^{\frac{1}{2}}N. \tag{31}$$

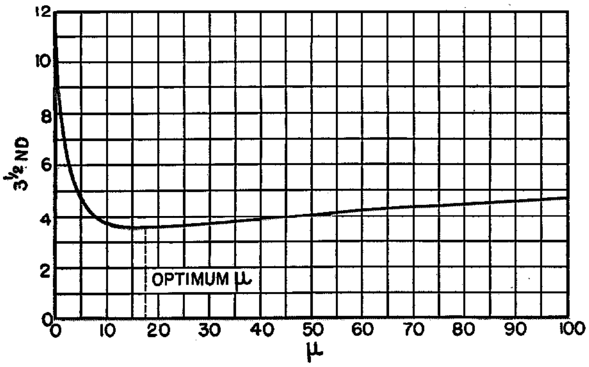

Figure 4—Distortion plotted against the parameter μ.

$$C=12 \text{ and } 3^{\frac{1}{2}}ND=\log(1+\mu)\left(1+\frac{C^2}{\mu^2}\right)^{\frac{1}{2}}.$$

For a sine wave,

$$C=2^{\frac{1}{2}}$$

and

$$D=\frac{2^{\frac{1}{2}}}{3^{\frac{1}{2}}N}=\frac{2}{6^{\frac{1}{2}}N}.$$

When a signal is uniformly quantized, the distortion depends on the ratio of the peak to the root-mean-square value. This is illustrated in curve 1 of Figure 5; curve 2 gives the minimum distortion for optimum logarithmic compression when the compression parameter μ is selected by curve 3.

4. *Application to Specific Signals*

To illustrate the various points involved, let the preceding results be applied to the class of signals specified by the probability density given by

$$P(y)=\frac{1}{(3+2\lambda)^{\frac{1}{2}}\sigma^{\frac{1}{2}}\beta(1/2,\lambda+1)}\left(1-\frac{y^3}{(3+2\lambda)\sigma}\right)^{\lambda}, \tag{32}$$

which is discussed in the appendix. Here λ is a parameter that determines the ratio of the peak value to the root-mean-square value. For $-1<\lambda<0$, the signal is relatively flat, while for $0<\lambda<\infty$, the signal has sharp peaks. When $\lambda=0$, all values are equally probable. If we let

$$A=\frac{1}{\sigma^{\frac{1}{2}}(3+2\lambda)^{\frac{1}{2}}\beta(1/2,\lambda+1)},$$
$$a=\sigma^{\frac{1}{2}}(\beta+2\lambda)^{\frac{1}{2}},$$

we find that the minimum distortion is given by

$$\begin{aligned} D_m{}^2(\lambda)&=\frac{2A}{3N^2\sigma}\left[\int_0^a\left(\frac{1-y^2}{a^2}\right)^{\lambda/3}dy\right]^3\\ &=\frac{Aa^3}{12N^2\sigma}\left[B\left(\frac{1}{2},1+\frac{\lambda}{3}\right)\right]^3\\ &=\frac{2\lambda+3}{12N^2}\,\frac{B^3(1/2,1+\lambda/3)}{B(1/2,1+\lambda)}. \end{aligned} \tag{33}$$

In this case

$$C=(2\lambda+3)^{\frac{1}{2}}. \tag{34}$$

The distortion for uniform-level spacing is from (31)

$$D=\frac{1}{3^{\frac{1}{2}}N}(3+2\lambda)^{\frac{1}{2}}. \tag{35}$$

The quantity ND is plotted versus C in Figure 6. It is seen that for small values of C, there is little advantage in using the minimum spacing. It is not until the crest factor is above 6 that the difference between the two distortion figures is 6 decibels. The distortion for optimum spacing attains a final value of about $1.65/N$. Thus, the chief advantage of the optimum spacing occurs when the crest factor of the signal is high.

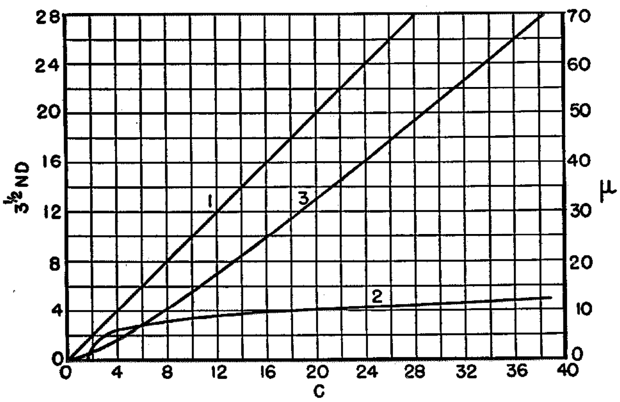

Figure 5—Distortion characteristics. Curve 1 is for uniform quantization $3^{\frac{1}{2}}ND=C$. Curve 2 is for optimum logarithmic compression.

$$3^{\frac{1}{2}}ND=\log(1+\mu)\left(1+\frac{C^2}{\mu^2}\right)^{\frac{1}{2}}.$$

Curve 3 is for the case where

$$C=\left[\frac{\mu^2}{\frac{1+\mu}{\mu}\log(1+\mu)-1}\right]^{\frac{1}{2}}.$$

5. *Appendix—Statistical Properties of the Signal; Pearson Distribution*

The probability density given by

$$P(y)=\frac{1}{(3+2\lambda)^{\frac{1}{2}}\sigma^{\frac{1}{2}}B(1/2,\,\lambda+1)}\left(1-\frac{y^2}{(3+2\lambda)\sigma}\right)^{\lambda} \tag{36}$$

is known as the Pearson distribution. In (36), σ is power and λ is a parameter satisfying $-1<\lambda<\infty$. Further, the variable y is restricted to the interval

$$-[(3+2\lambda)\sigma]^{\frac{1}{2}}\leqslant y\leqslant+[(3+2\lambda)\sigma]^{\frac{1}{2}}.$$

Finally, $B(m,n)$ is the beta function.

For special values of λ, $P(y)$ reduces to simple forms. We have for $\lambda=0$

$$P(y)=\frac{1}{2(3\sigma)^{\frac{1}{2}}},\quad -(3\sigma)^{\frac{1}{2}}<y<(3\sigma)^{\frac{1}{2}},$$

which is a rectangular distribution with a peak-to-peak amplitude of $2(3\sigma)^{\frac{1}{2}}$. The case $\lambda=-1/2$ reduces to the distribution function of a sine wave whose maximum value is $A=(2\sigma)^{\frac{1}{2}}$. If $\lambda=-1$, we find that $P(y)=\infty$ when $y=\pm\sigma^{\frac{1}{2}}$; $P(y)=0$ elsewhere. The distribution consists of impulses at $y=\pm\sigma^{\frac{1}{2}}$ of strength 1/2; the signal is a square wave. When $\lambda\rightarrow\infty$, the probability density reduces to the normal distribution.

The probability density (36) is plotted in Figure 7 for several values of λ with constant power. Actually $\sigma^{\frac{1}{2}}P(y)$ is plotted against $y/\sigma^{\frac{1}{2}}$ to get a set of curves applicable to all values of σ. From these curves, it is seen that $\lambda=0$ is the dividing line between two classes of signal. If λ is negative, the signals tend to concentrate near the maximum value. For λ positive, values near zero are emphasized.

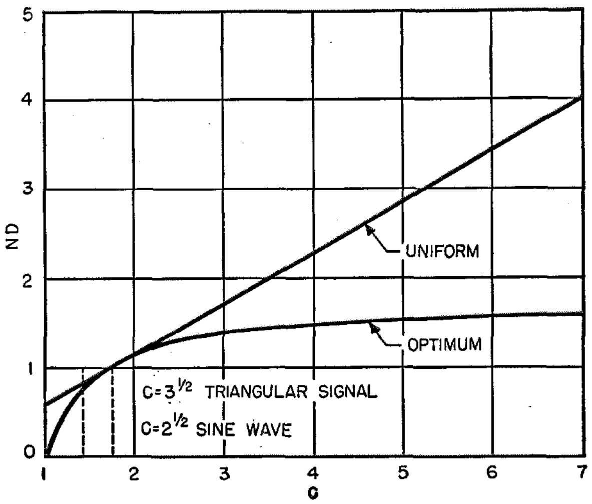

Figure 6—Distortion plotted against crest factor for the Pearson distribution.

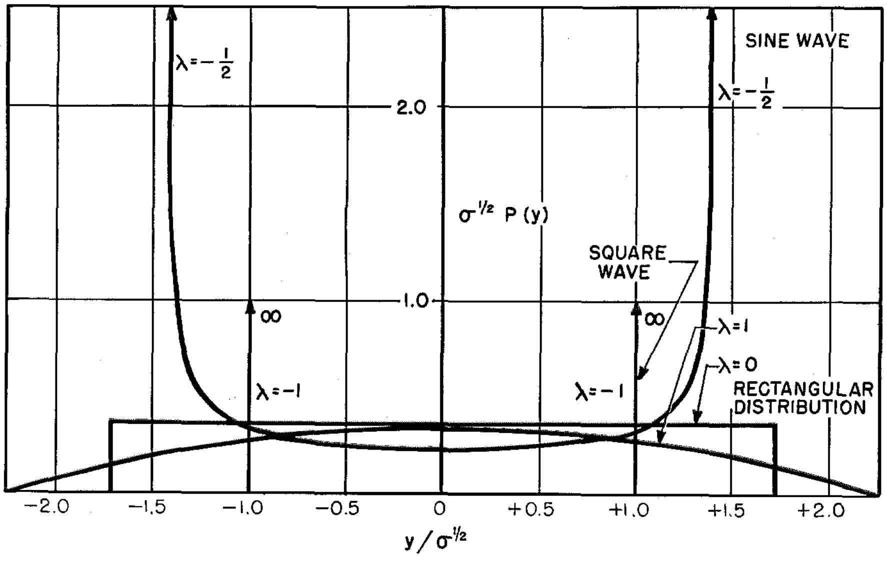

Figure 7—Pearson distribution curves of $\sigma^{\frac{1}{2}}P(y)$ plotted against $y/\sigma^{\frac{1}{2}}$.

Four/Two-Channel Time-Division-Multiplex Telegraph System for Long-Distance Radio Circuits*

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INCREASED RELIABILITY of frequency-shift transmission has made the use of time-division-multiplex systems feasible on long radio circuits. Time-division multiplex provides a means, economical of frequency spectrum, to handle traffic that is too great for one printer channel or that originates from or is destined to different places. A four-channel multiplex system at a speed of 50 words per minute per channel, using the five-unit Baudot code, has a unit pulse length of 0.010 second, which normally provides sufficient margin against signal shift due to radio multipath effects or other causes.

A combination electronic and electromechanical system was chosen for reliability and simplicity of maintenance. At the sending end, the electromechanical distributor provides a simple means for interleaving the signals for the various channels. It also provides a means for extending a channel on a start–stop-printer basis without the use of individual electromechanical translating devices, to a branch office so located that clutch pulses can be sent to it economically. Alternatively, a multiplex extensor may be used for more-distant branch offices.

Any inaccuracies of signals from the sending distributor are eliminated by passing them through a simple electronic regenerator. Where the multiplex signals are passed over a keyed tone channel between the multiplex terminal and the radio transmitter, the sending-end regenerator makes possible the synchronization of the tone-channel carrier to the multiplex baud speed, which practically eliminates carrier distortion. The sending-end regenerator also makes it possible to send manual morse signals without disturbing the synchronism of the multiplex receiving regenerator, as all morse signals begin and end at the same times that multiplex signals would begin and end. The resultant distortion is too small to be noticeable to the ear.

At the receiving end, the signals are regenerated electronically with greater precision and reliability than by prior methods using electromechanical relays in conjunction with a rotary distributor. An automatic margin indicator provides one kind of indication if the margin falls below 40 and another kind if below 20 percent.

After regeneration, the signals are distributed very simply by an electromechanical distributor whose brushes do not require great contacting precision for this function. The use of an electromechanical distributor also provides a simple means of applying a modified Verdan scheme for error detection and permits monitoring of the channel signals just as they are leaving the distributor face plate.

The Verdan scheme of error detection is based on sending each character twice and comparing the received combinations. If they are the same, it is assumed that no error has occurred as the probability of an identical error occuring in two receptions of the same combination is very remote. If they differ, an error has occurred in one of the transmissions and an indication is made on the copy by printing a question mark. This scheme cuts the traffic capacity in half, which is equivalent to using a 10-unit code. However, as the error-indicating equipment can be cut in or out quickly, it need be used only when the automatic margin indicator shows that the margin of regeneration is low. Thus, if 40 percent of the traffic is handled with error indication, the average length of the code would be 7 units. If only 30 percent is handled with error indication, the average length of code would be 6.5 units.

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1. *Apparatus*

1.1 Sending End

The four channels are normally operated from standard sending operating tables equipped with multiplex perforators and multiplex transmitters. The distributing, translating, monitoring, regenerating, and general operating equipment is located on the sending distributor rack shown in Figure 1. The distributor has six pairs of rings and is driven by a phonic motor at a speed of 300 revolutions per minute from an electronically driven 200-cycle-per-second fork.

1.2 Receiving End

Reception is normally on four standard receiving operating tables equipped with start–stop printers and facilities for extending their respective channels through them to branch offices.

Regenerating and synchronizing equipment is mounted on the regenerator rack of Figure 2. It includes a 200-cycle-per-second fork, similar to the one in the sending distributor rack.

Distributing and monitoring equipment is located on the receiving distributor rack, which may be seen in Figure 3. The distributor is the same as the one used on the sending distributor rack and is driven from the fork on the regenerator rack.

Error-indicating equipment and channel-extension apparatus are mounted on the error-indicator rack displayed in Figure 4.

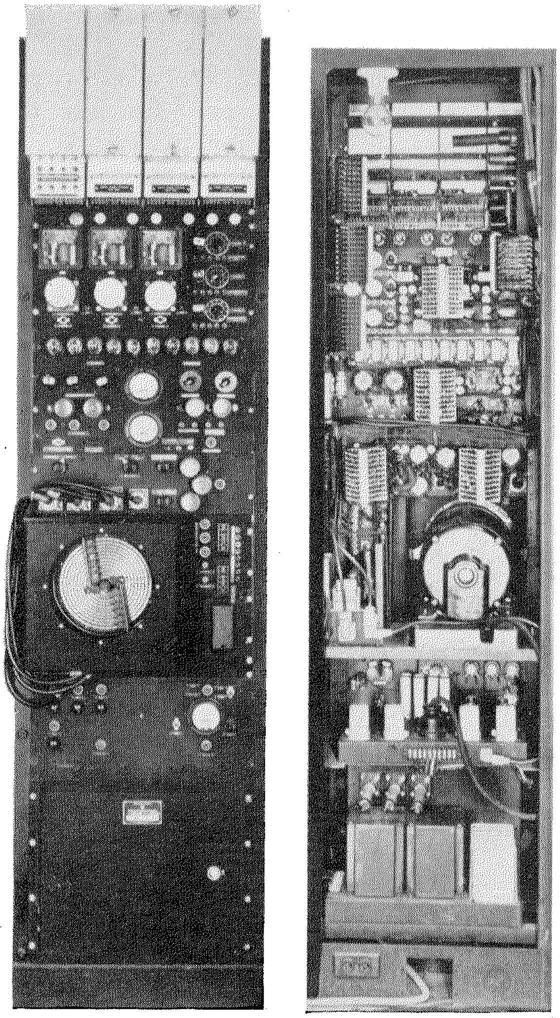

Figure 1—Front and rear views of the sending distributor rack. The distributor is driven by a phonic motor.

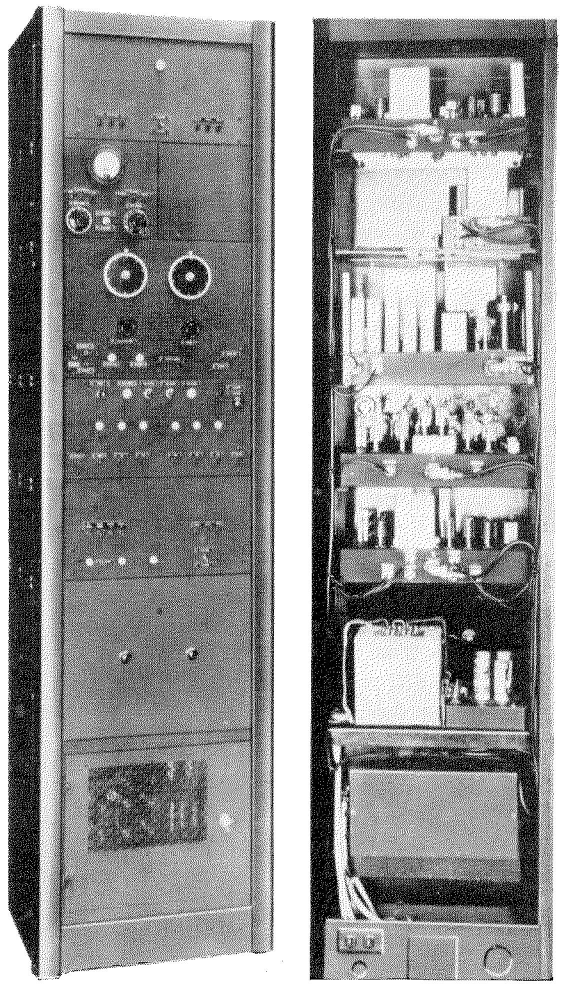

Figure 2—Views of the electronic regenerator rack. The receiving distributor is driven by a fork in this cabinet.

2. *Sending-End Circuit*

2.1 General

The sending-end circuits are shown in Figure 5. The various channel segments on the sending ring show that channels A and C are interleaved as are channels B and D, which follow. This was done for ease in changing from four-channel operation to two-channel operation, which can be accomplished by throwing switches (not shown). The marking polarity is positive on two of the channels, A and D in this case, and is negative on the other two channels (B and C), in accordance with the usual practice, to provide signal reversals for synchronizing even when all four channels are idle.

Channel A is always operated from a multiplex transmitter in the main office but channels B, C, and D can be extended to branch offices on a start–stop-printer basis.

2.2 Extension Circuits

Circuits are shown for extending only the C channel by closing the seizure switch in the branch office. The closing of this switch causes the clutch pulse to operate the seizure relay and the C-channel switch relay. The latter switches the C-channel sending segments from the multiplex transmitter to the extension storage capacitors through the C-channel idle-signal switch. Each clutch pulse releases the start–stop transmitter distributor clutch so that the signal

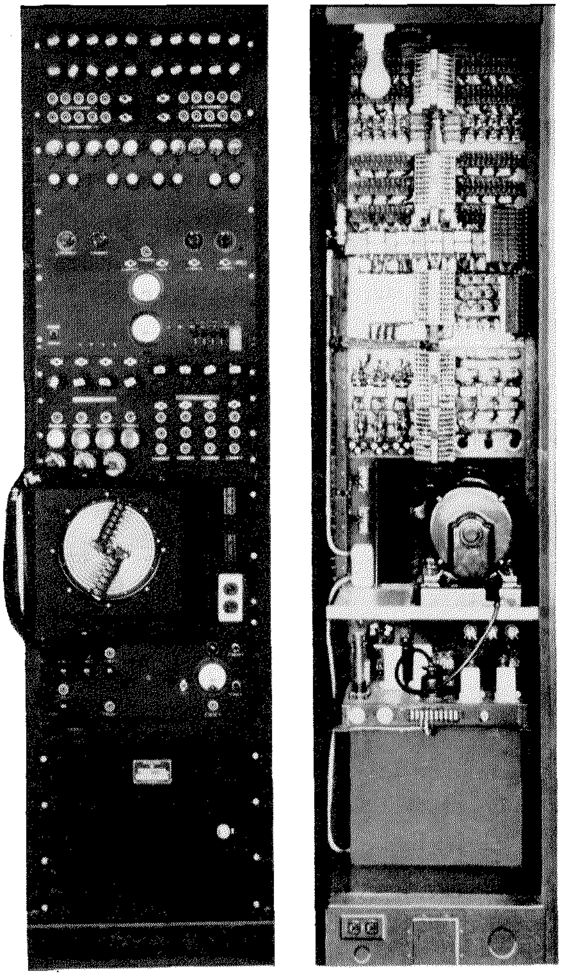

Figure 3—Receiving distributor rack mounting the distributing and monitoring equipment at the receiving end.

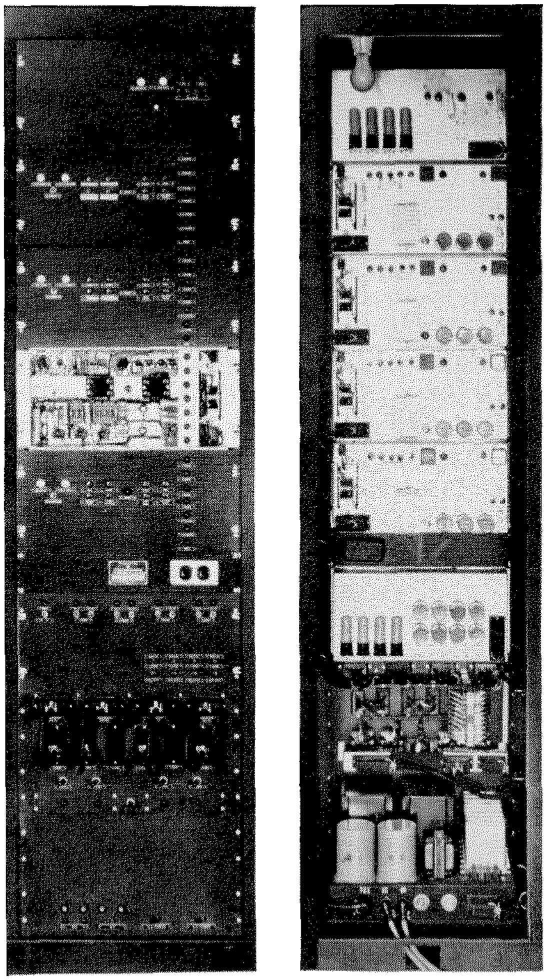

Figure 4—Error-indicator rack. In the front view, the panel has been removed from one error-indicator chassis.

combination for one character will be sent to the main office and the extension-line relay operated. From the tongue of this relay, polar signals are sent through the extension translator rings into the channel-extension storage capacitors from which they are sent through the proper segments of the sending ring. The clutch pulse, originating from a segment of the local ring, is phased by means of the phasing control rheostat so that the brush on the extension translator rings will be passing over each pulse segment when the signal for that pulse is arriving from the tongue of the extension-line relay. The segments on these rings are spaced so that the brush will pass from the beginning of one segment to the beginning of the next segment in 0.022 second, which is the duration of each selection pulse of the start–stop combination when operated at a 61-word-per-minute speed.

When the auto-stop contacts of the branch-office transmitter open due to a taut tape, the transmitter distributor clutch is not released by the clutch pulses and a solid marking current is sent to line which would cause the extension storage capacitors to be charged with marking potential and the "letter shift" combination would be set up. This would be a false combination and would result in errors if upper-case characters were being sent when the tape became taut. To prevent this, the idle-signal relay tongues transfer the sending segments from the storage capacitors to spacing potential, so that the "idle" signal is sent over the multiplex channel when a false "letter-shift" combination is received. The false "letter-shift" combination has no start pulse and this distinguishes it from the true one. This condition is detected by means of idle-signal tubes, which are controlled from a segment on the translator ring over which the brush passes when the start pulse is received. These tubes control the idle-signal relay.

2.3 Error-Indicator Circuits

For error indication, each character combination is sent twice. On a four-channel basis, the same combination is sent on two successive revolutions of the distributor brushes and the traffic capacity is 25 words per minute per channel. When sending is from a multiplex transmitter, it is only necessary to open the transmitter step-pulse circuit on alternate revolutions of the brushes. This is done by flip-flop relays, operated at the proper time from a segment on one of the distributor rings. When sending is from a branch office, the clutch pulse is interrupted on alternate revolutions of the brushes and the extension relay is cut off from the translator rings on the same revolution by means of a flip-flop relay so that the same signal combination remains on the storage capacitors as on the previous revolution.

By using two channels for original transmission and the other two channels to repeat the combinations for comparison, a traffic-carrying capacity of 50 words per minute on each of two channels can be obtained. If two channels are used this way, the other two can be used with error indication at 25 words per minute per channel or either or both may be used at 50 words per minute per channel without error indication.

Error indication, on a two-channel basis, is put into effect at the sending end by throwing a switch. The circuits for operation on *C* channel with repetition on *B* channel, by throwing *C*- and *B*-channel error-indicator switch, are shown in Figure 5. The first three pulses are repeated by a direct connection of the sending segments but the other two are repeated from overlap storage capacitors, as the tape on the *C*-channel transmitter is being stepped while they are being sent.

2.4 Monitor Circuits

Two multiplex five-magnet tape printers are available for connection to any channel for monitoring the signals as they leave the sending ring. Since the signals from all channels are already aggregated on this ring, they must be separated again by channels on the monitor translator rings. The printer selections are controlled by monitor-control tubes, which reinvert the signals of the two inverted channels.

2.5 Sending Regenerator Circuits

Signals from the solid sending ring are regenerated before they are used to modulate the tone generator of the keyed tone channel between the telegraph office and the radio station. As may be seen in Figure 6, they are passed by means of an isolation cathode-follower tube into the secondary of the input transformer of the positive-to-polar-pulse converter. Differentiated square waves, at baud speed, from a fork-driven counter

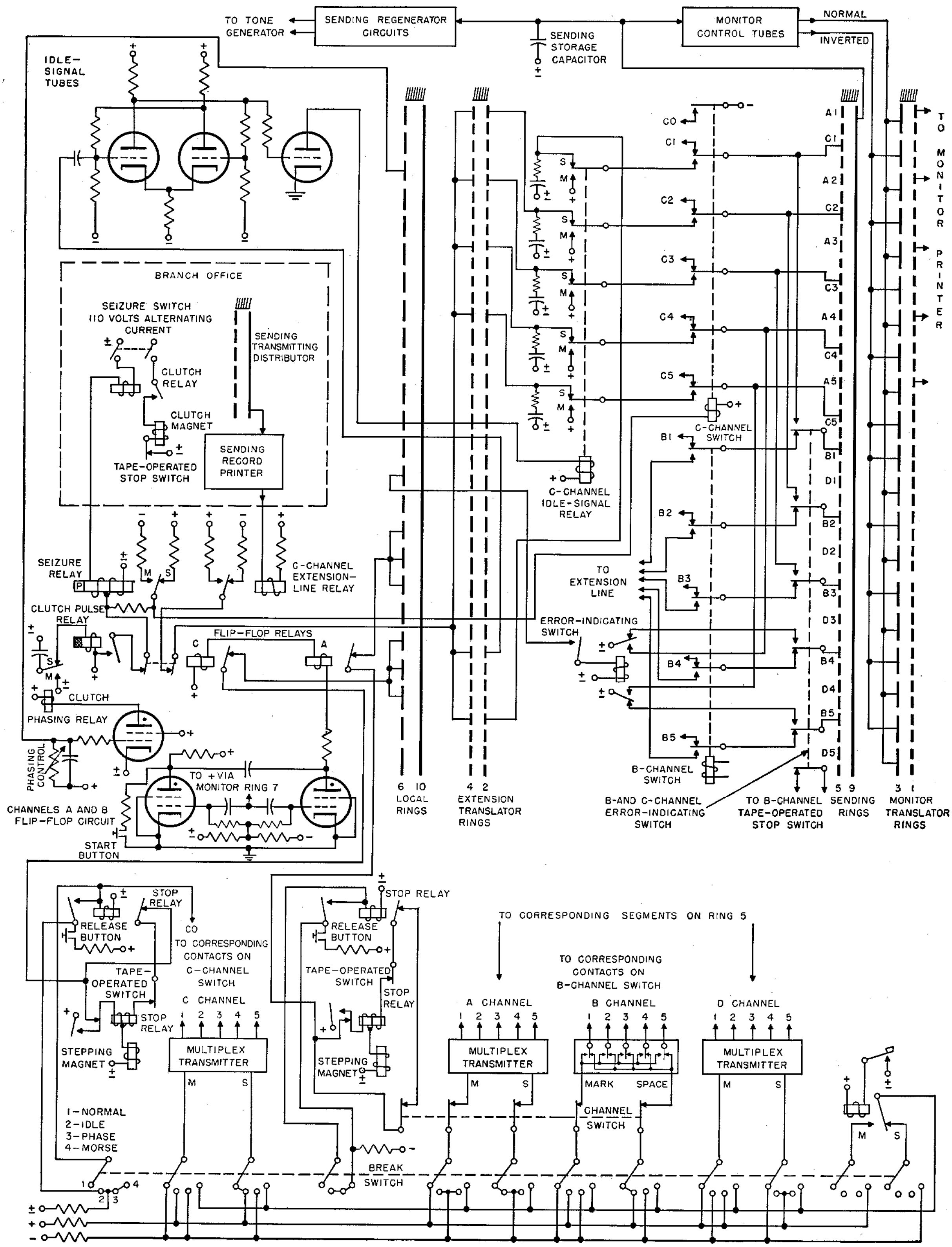

Figure 5—Sending-end circuits illustrating distributors, channel-switching system, and branch-office connections.

are passed through a negative-pulse clipper and the positive pulses pass to the primary of the input transformer. These pulses constitute accurate time markers, adjusted to occur at the approximate center of the signals from the sending ring. They sample the signal for polarity and initiate, with great accuracy of timing, a pulse of the polarity of the sampled signal. These polar pulses, brought out through the output transformer, are expanded into the regenerated polar signals in the polar-pulse expander. Due to the unbalanced ground connection across the primary of the output transformer, a small portion of the distributor signal as well as the polar pulses appears at the output of the converter, which facilitates monitoring.

The regenerator unit retimes the signals and supplies to the rotating distributor a fully regenerated signal; a signal that maintains a fixed phase relationship to the respective distributor faceplate segments and that has rigorously equal mark and space elements.

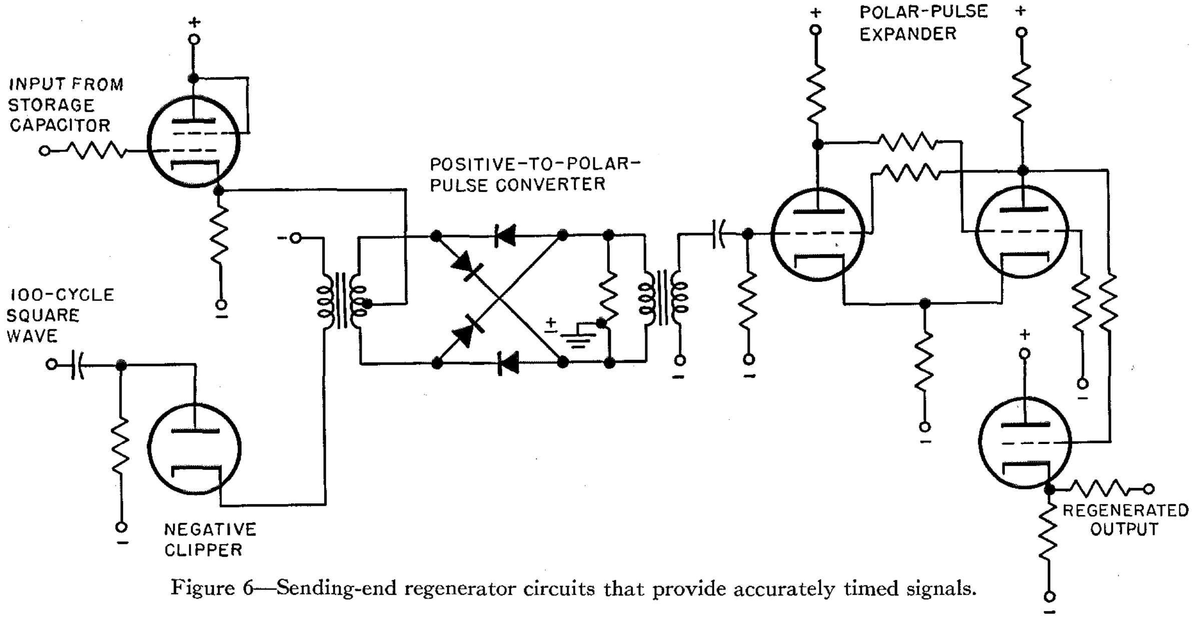

Figure 6—Sending-end regenerator circuits that provide accurately timed signals.

3. Receiving-End Circuits

3.1 General

Three main functions are to be performed by the receiving multiplex terminal equipment.

3.1.1 Regeneration of the Received Signal

The received radio signals are subject to distortions by various types of transmission irregularities. Before the signal can be distributed to the respective automatic printing devices, it is necessary to remove such distortions by a regeneration process.

3.1.2 Distribution of the Regenerated Signal

A rotary synchronous distributor distributes the regenerated signals to the proper channels and translates the five-unit Baudot code into start–stop-printer signals. The combination of gas and vacuum tubes provides the signal power for the receiving traffic and monitor printers, while both relay and vacuum-tube outputs are available for channel-extension circuits.

3.1.3 Detection of Errors

When error detection is desired, the sending end is made to repeat the same character on two transmissions. The error indicator will compare the two transmissions and when a discrepancy is detected will switch the corresponding receiving printer from the multiplex signal to an internal signal source. A sequence of start–stop signals will then be supplied to the printer to print a question mark (?) right after the erroneous character; thereupon the printer is returned at the proper instant to the signal circuit.

4. *Receiving Regenerator*

4.1 General

The receiving regenerator logically subdivides into four functional sections, the interrelation of which is illustrated in Figure 7.

The radio receiving station normally relays the received signal in the form of a keyed tone over regional facilities (land line or very-high-frequency link) to the operating office where the regenerator and associated terminal equipment are located. This keyed tone, after demodulation and shaping into a direct-current square-wave form, passes to both the synchronization and regeneration chassis.

The frequency-standard unit generates two separate sets of timing pulses, each at signal-element speed, which are supplied to the synchronization and regeneration units, respectively.

Comparison of the phase of one set of timing pulses with the timing of the radio-signal leading edges (crossover from space to mark) is made in the synchronization unit. Departures from the correct phase will result in synchronizing impulses being applied to the vacuum-tube fork oscillator in the frequency-standard unit.

In the regenerator unit, the local timing pulses sample the polarity of every signal element of the received signal to produce an output pulse, which when expanded to full signal-element duration is the regenerated signal.

In addition, an automatic margin indicator has been incorporated in the receiving-regenerator rack. This unit automatically indicates the degree of reliability of the regeneration process by lighting alarm pilot lamps should the signal be severely mutilated by transmission disturbances and it permits a range to be taken at any time without interfering with traffic.

4.2 Regeneration and Synchronization

The receiving-end regenerator rack has a precision vacuum-tube oscillator similar to the one used at the sending end. For proper synchronization, this receiving fork is adjusted to run faster than the sending-end fork by slightly more than the anticipated maximum speed difference between the two forks. With such an adjustment, the receiving end will always tend to run ahead of the sending end. A speed difference of about one part per hundred thousand was found adequate for an initial adjustment.

From the receiving-end fork, positive gating pulses are derived at signal-element speed which, of course, also tend to drift ahead of the received signal edges. By means of a manual phase-shifting device (synchronization phase shifter), these gating pulses are initially adjusted to lag slightly

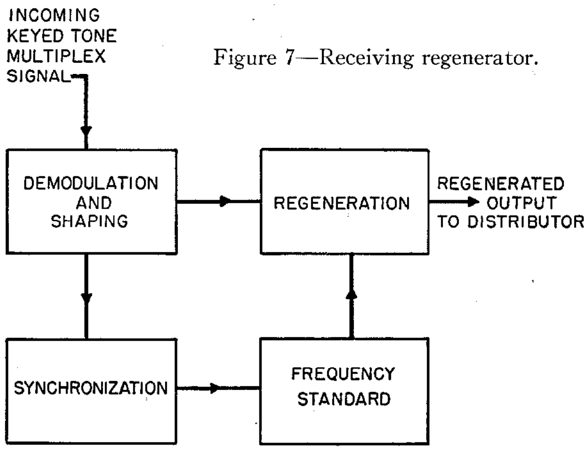

Figure 7—Receiving regenerator.

behind the signal edges. The gating pulses then will tend to drift into the edges of the radio signal. When the radio-signal leading edges (crossover from space to mark) and the gating pulses coincide, a synchronization pulse of a fixed magnitude and duration is applied to the fork. For the duration of the synchronizing pulse, the fork is slowed down by approximately 30 parts per hundred thousand. The synchronization marker pulses are thereby effectively moved back to some position lagging the signal edges. Once the synchronization pulse is over, the receiving fork will tend to resume its original rate, drift again into the signal edges, and the cycle will repeat itself.

It is to be understood that the fork is to synchronize to a radio signal subject to random transmission disturbances. The time the signals require to travel from a transmitter to a distant receiver is likely to vary from instant to instant due to multipath and other effects. The receiving fork will keep in step with the signal by assuming a phase relation to the signal edges, such that only some leading signal edges will arrive late enough and produce synchronization impulses. The majority of signal edges will have no effect on the frequency of the receiving fork.

It is desirable to keep the speed difference between the sending and receiving forks small. Each rate correction can then also be made small, and the fork will not follow each random phase variation of the signal. The receiving-end fork then runs essentially at constant speed except for minute rate corrections that cancel the speed difference between sending and receiving forks. Each correction reduces the speed of the receiving-end fork by approximately 30 parts per hundred thousand and lasts 0.1 second. This is equivalent to a phase shift of approximately 30 microseconds. Considering that a four-channel multiplex system operating at 50 words per minute per channel has a signal-element length of 10,000 microseconds, each individual correction represents indeed a minute phase shift.

The small effect of each individual correction also affords protection against over-synchronization. A signal may split due to severe multipath effect or noise. Such splits would simulate additional leading signal edges and might result in erroneous rate corrections. The effect of each correction being small, the receiving fork will not readily assume an incorrect phase with respect to the signal. In turn, should a few signals arrive exceptionally late, the phase of the receiving end will not be affected materially. As an additional protection against over-correction, the maximum repetition rate of the synchronizing pulses has been limited to 120 per minute.

The receiving regenerator unit operates on the same signal-sampling principle as the sending-end regenerator. Sampling pulses are derived at signal-element speed from the receiving-end fork and are manually phased to coincide with the approximate center of the received signal elements.

The *threshold* of a regenerator is defined as the minimum signal margin that the regenerator will accept and still regenerate faultlessly. Even if a perfect signal is supplied to the regenerator, it is to be understood that the margin will be less than 100 percent. Two main factors account for this: the width of the selection pulse and the synchronization-rate corrections.

Assuming that a perfect signal is received (four-channel operation), the range will be measured as 10 milliseconds minus the width of the sampling pulse. A further reduction of the margin reading results from the small retardations and accelerations to which the fork is subjected for synchronization.

At the sending end, where signals of only 3- to 4-percent distortion have to be regenerated, a high regenerator threshold is quite adequate and allows for circuit simplicity. At the receiving end, where signal mutilation can be severe, the regenerator has to be designed for a very low threshold. This is done by using an effective sampling pulse width of approximately 50 microseconds (0.5 percent of the signal element with four-channel operation). Since the effect of the individual synchronization impulse is also small, the threshold of this regenerator is of the order of 1.5 percent for four-channel operation.

4.3 Theory of Operation

Figure 8 shows a block diagram of the receiving regenerator. At the lower left, a vacuum-tube-driven fork supplies a 200-cycle wave to the synchronization phase shifter. This is a magnetic-type phase-shifting device that allows continuous control of phase shifting by means of a knob on the front panel. The phase-shifter output is frequency divided in the synchronization frequency-division stages, first to 100 and then to 50 cycles. Either 50- or 100-cycle outputs are available as pulses of about 50 microseconds width. These pulses initiate the synchronization timing pulses, which must occur at signal-element speed. Thus for four-channel 50-word-per-minute operation, the 100-cycle output of the frequency-division stages will be chosen. For two-channel 50-word-per-minute operation, the 50-cycle pulse output is used. A front-panel switch allows the selection of either output. Each pulse of the synchronization frequency divider will initiate a square-top positive pulse of 4 milliseconds duration through the gate.

The output of the input signal-shaping unit is a square-wave-type polar signal. This signal is differentiated by a short-time-constant resistance–capacitance network, the output of which will be a positive pulse for a leading signal edge and a negative pulse for a lagging signal edge. These signal edges and the 4-millisecond pulses are supplied to a coincidence gate. This stage is biased to cutoff so that neither the positive 4-millisecond pulses nor the positive signal edges alone will be sufficient to make the tube conduct. When the 4-millisecond pulses and the positive signal edges occur simultaneously, an output pulse will result from the coincidence gate. Obviously, the coincidence of a negative signal edge and a 4-millisecond pulse will not produce any output. Thus, a coincidence of the leading signal edges only with the 4-millisecond synchronization timing pulses is hereby achieved.

second width. The energy of these 0.1-second pulses will slow down the fork electromagnetically. To insure against over-correction, the maximum repetition rate of these 0.1-second rate corrections is limited to 120 per minute. A correction limiter renders the 0.1-second gate insensitive for an additional 0.4 second beyond the 0.1-second operation time of the gate. A front-panel neon lamp will flash every time correction pulses are applied to the fork.

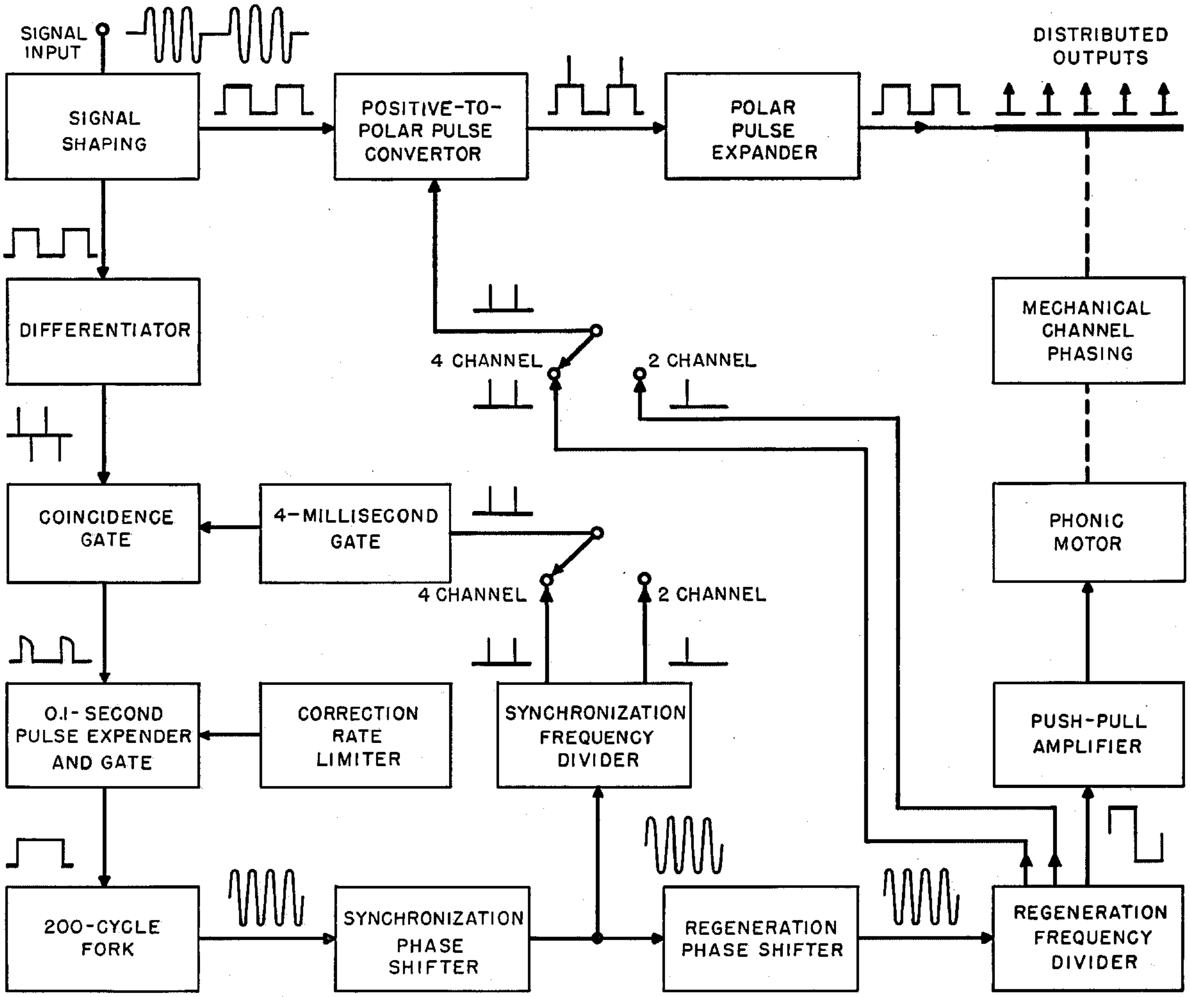

Figure 8—Details of the receiving regenerator. The circuit accurately retimes and reshapes the received signals.

The coincidence of a positive signal edge and a 4-millisecond pulse will apply a synchronization pulse to the 200-cycle fork. The output of the coincidence gate is a pulse of too short a duration effectively to slow down the fork. This pulse is therefore first expanded to 0.1 second width. This is accomplished by applying the output of the coincidence gate to a pulse generator. On reception of a coincidence pulse, the 0.1-second gate will generate a square-top positive pulse of 0.1

The synchronization phase shifter feeds also a phase-shifting unit termed a regeneration phase shifter. The regeneration frequency-divider following this phase shifter performs two functions: *A*) it supplies push-pull 50-cycle square-wave outputs, which after power amplification provide the driving energy for the phonic-wheel motor, and B) it provides scanning pulses at signal-element speed for regeneration.

Again, depending on whether two- or four-channel operation is desired, a switch allows the selection of either 100- or 50-cycle pulses from the regeneration frequency divider. These are positive pulses of 50-microseconds width and feed a converter stage simultaneously with the square-wave output from the signal-shaping unit.

Should a positive scanning pulse coincide with a positive signal element, the converter output will be a positive pulse. Coincidence of a positive scanning pulse with a negative signal element will result in a negative pulse output. The output of the converter will therefore consist of negative and positive pulses, the sequence of which will depend on the combination of positive and negative elements of the incoming signal. Since the polarity of these output pulses is determined by the signal polarity existing at the time at which scanning pulses are supplied to the converter, it is evident that the radio signal is hereby effectively sampled, signal element by signal element, by the scanning pulses.

The polar-pulse output of the converter is expanded to full signal-element length by means of a flip-flop circuit. The output of the flip-flop is of square-wave type. The crossover from one polarity to the other of the flip-flop can only occur at 10-millisecond intervals, as timed by the scanning pulses. This flip-flop output will therefore have a unity mark-to-space ratio. It will also maintain a constant phase with respect to the regeneration frequency divider. This flip-flop output is now fed to the distributor face plate for distribution. Since the distributor is synchronously driven by the regeneration frequency divider, the flip-flop output will also maintain a constant phase relationship to the distributor face-plate segments. Therefore, a fully regenerated signal is supplied to the distributor.

In order to have signal reversals on the air even when all channels are idle, channels A and D have been made positive or direct channels. Channels B and C are negative or inverted channels. Thus, at the receiving end, another inversion of channels B and C must be performed before distribution to the printers. To accomplish this, the flip-flop stage has two (symmetrical) outputs: one direct and one inverted output.

5. *Automatic Margin Indicator*

The automatic margin indicator provides an indication of the signal quality. The received radio signal is sampled by this unit into 3 classes.

A. Signals having more than 40-percent margin. Such signals are to be considered as good signals, and no indication will be given by the automatic margin indicator.

B. Signals having less than 40-percent margin but more than 20-percent margin. For each such signal, a white neon lamp will flash on the front panel.

C. Signals having less than 20-percent margin. Such signals are very poor, and the probability that error will occur is great. Should any one signal be received with less than 20-percent margin, a red neon lamp on the front panel will light and remain on until it is manually extinguished.

The automatic margin indicator distinguishes between forward and backward signal margin. Four neon-lamp indicators are therefore provided on the front panel of the unit: two white lamps for 40-percent backward and forward margin indications, respectively, and two red lamps for 20-percent backward and forward margin indications, respectively.

The regeneration frequency divider, as shown in Figure 9, supplies to the automatic margin indicator: A) positive pulses at signal-element speed, which are identical with those used for regeneration and are termed "scanning pulses," and B) positive pulses at signal-element speed similar to the scanning pulses but delayed by 5 milliseconds with respect to them.

The "scanning pulses" initiate two gates at signal-element speed. A gate A, which generates positive square-top pulses of 2 milliseconds width, and a gate H, which generates 1-millisecond square-top pulses. The scanning pulses that are delayed by 5 milliseconds initiate: A) a 2-millisecond gate C after an additional delay of 3 milliseconds, and B) a 1-millisecond gate D after a delay of 4 milliseconds. From Figure 11, it is seen that these four gates effectively surround the sampling pulse. Gates A and B are initiated at sampling time. Gates C and D are so timed that their lagging edges coincide with the sampling pulse.

The square-wave output of the radio signal-shaping unit is differentiated into positive and negative pulses. A full-wave rectifier converts all these polar pulses into positive pulses. Gates A

B, C, and D are fed into four coincidence stages, respectively. Each of these coincidence stages receives also the positive pulses derived from each radio-signal edge. Coincidence of a signal edge with any of the four gates will result in an output from the corresponding coincidence stage. Thus, a coincidence output from gate C indicates that the forward margin is less than 40 percent and a white neon lamp will flash for each coincidence pulse. In turn, coincidence of gate D and a signal edge indicates that the forward margin is less than 20 percent and the red neon lamp will light and will remain lit until manually extinguished. Similarly, coincidence outputs corresponding to the gating time of A and B will indicate back margins of less than 40 percent and 20 percent, respectively. These lamp indicators will change the scanning pulse phase with respect to the radio signal. Since the gates of the automatic margin indicator are initiated by the scanning pulses, they will move by the same amount as the scanning pulse. To take a margin with the automatic margin indicator, the regeneration phase shifter is manually advanced in phase until the forward 40-percent light just starts to flash. The phase shifter is then backed up slightly to a position where no flashes occur for, say, one minute. The regeneration phase shifter is then manually retarded in phase with respect to the signal and a similar reading is taken by observing the back 40-percent light. The difference between the two dial readings of the regeneration phase shifter plus 40 percent is equal to the signal margin. Since during this operation the scanning

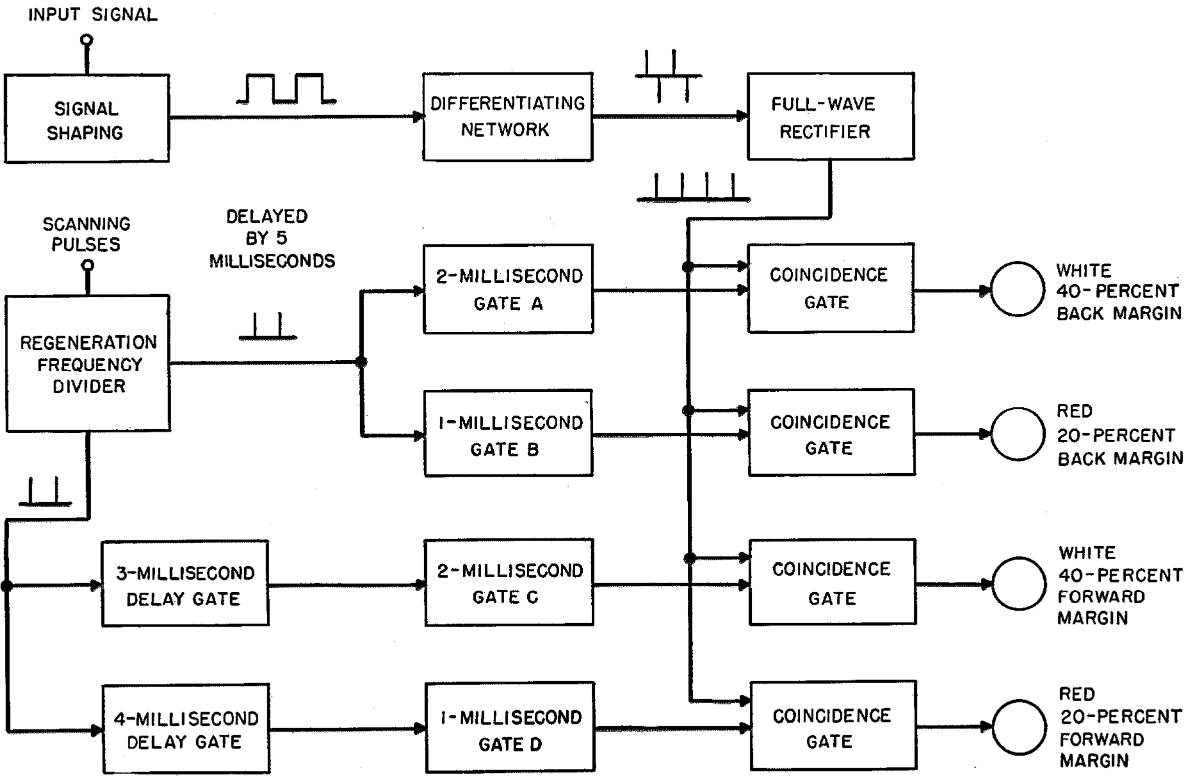

Figure 9—Block diagram of the automatic margin indicatior. Two percentages of both forward and backward margins are indicated by lamps.

therefore show the operator whether any adjustments of the circuit are required and indicate the reliability of the received signal.

The 40-percent indicator lamps also allow the margin to be taken without interruption of traffic. Manual rotation of the regeneration phase shifter pulse stays clear of the signal edges, no interference with traffic will result.

The automatic margin indicator unit is provided with front-panel test jacks for cathode-ray-oscilloscope observation. For quick monitoring, a composite wave pattern, similar to that shown

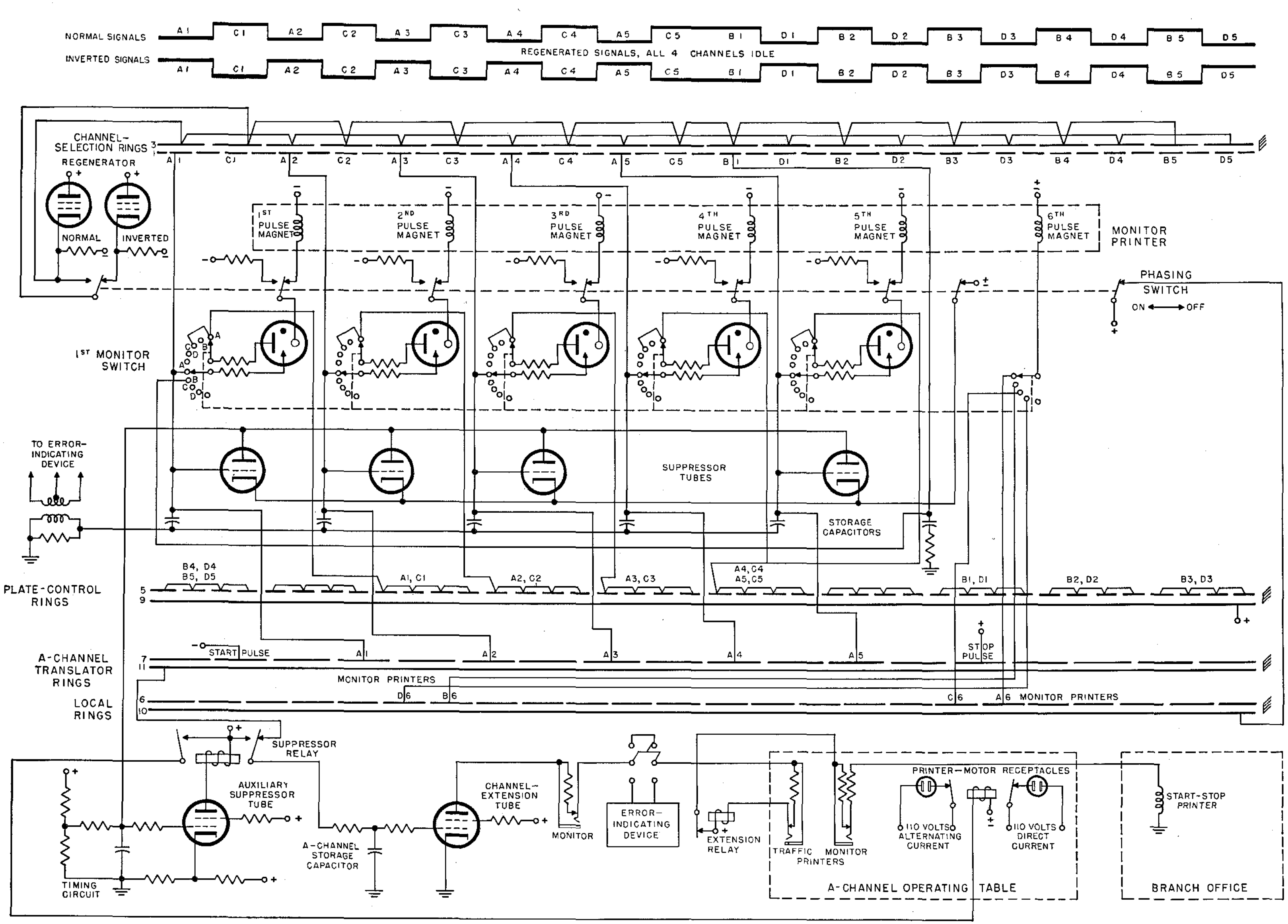

Figure 11—Receiving distribution, translation into start–stop signals, and monitoring are accomplished in these circuits.

in Figure 10, is provided, showing on the oscilloscope the time position of: *A*) the radio-signal edges, *B*) the scanning pulse, and *C*) the four gates of the automatic margin indicator. In addition, the 4-millisecond gate used for synchronization can be added to the cathode-ray pattern.

6. *Receiving Distributor Circuits*

6.1 General

The regenerated signals are supplied to the receiving distributor for channel distribution, for translation into start–stop signals, and for error detection as shown in Figure 11.

A- and *D*-channel signals are received from the normal regenerator output and those for *B* and *C* channels from the inverted output. These latter were sent inverted and must be "reinverted."

The brushes of the distributor are driven by a phonic motor that receives its power from the regenerator tuning fork and these brushes are adjusted so that they will be passing over one of the "live" segments of channel-selection ring 1 during approximately the center of the time interval for the reception of each signal pulse.

Channel phasing is accomplished by stepping the brushes backward on their drive shaft one line-segment interval at a time until the channels are properly phased. This is indicated by placing the phasing switch in the ON position and noting when a monitor printer on *D* channel receives the phasing-signal combination of all five marking pulses.

6.2. Translation Circuits

The signals are distributed through ring 1 to five storage capacitors for the signals of each channel. The monitor printers and the circuits that translate from multiplex to start–stop signals get their selection pulses from these storage capacitors. The start and selection pulses are of approximately 0.022-second duration.

For translation, the signals from the storage capacitors pass through a translator ring, the live segments of which are spaced so that the brush passes over successive segments at 0.022-second intervals, generating signal elements of this duration. Start and stop pulses are added through other segments at the proper intervals. Translation circuits are shown in Figure 11 for the *A* channel only; the signals of all channels are similarly translated.

The start and stop pulses being inserted independently of the signals, they would be present even when the circuit is idle. To prevent this, a suppressing means is made effective when the idle combination continues for more than 15 or 20 seconds.

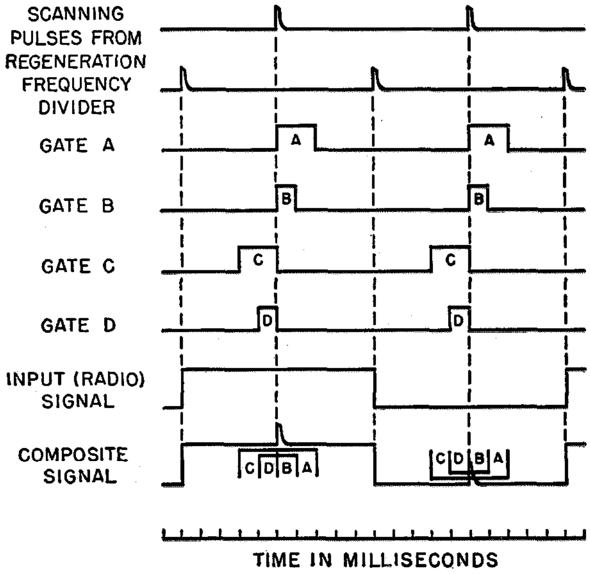

Figure 10—Automatic margin indicator timing and gating pulses related to input signal from radio receiver.

In normal operation, the marking pulses of the multiplex signals, as they appear across the storage capacitors, are applied to the grids of suppressor tubes. A timing capacitor is charged through a high resistance and discharged through the relatively low resistance of the plate circuits of the suppressor tubes. When these marker pulses are not received, the timing capacitor charges and, through an auxiliary suppressor tube, operates the suppressor relay to put a solid marking signal in the start–stop circuit. This action is reversed on reception of the first marking pulse when the circuit resumes operation.

6.3 Monitor-Printer Circuits

The monitor printers can be switched to any channel. Their selection pulses are obtained from the storage capacitors through thyratron tubes whose plate circuits are controlled from the plate-control rings.

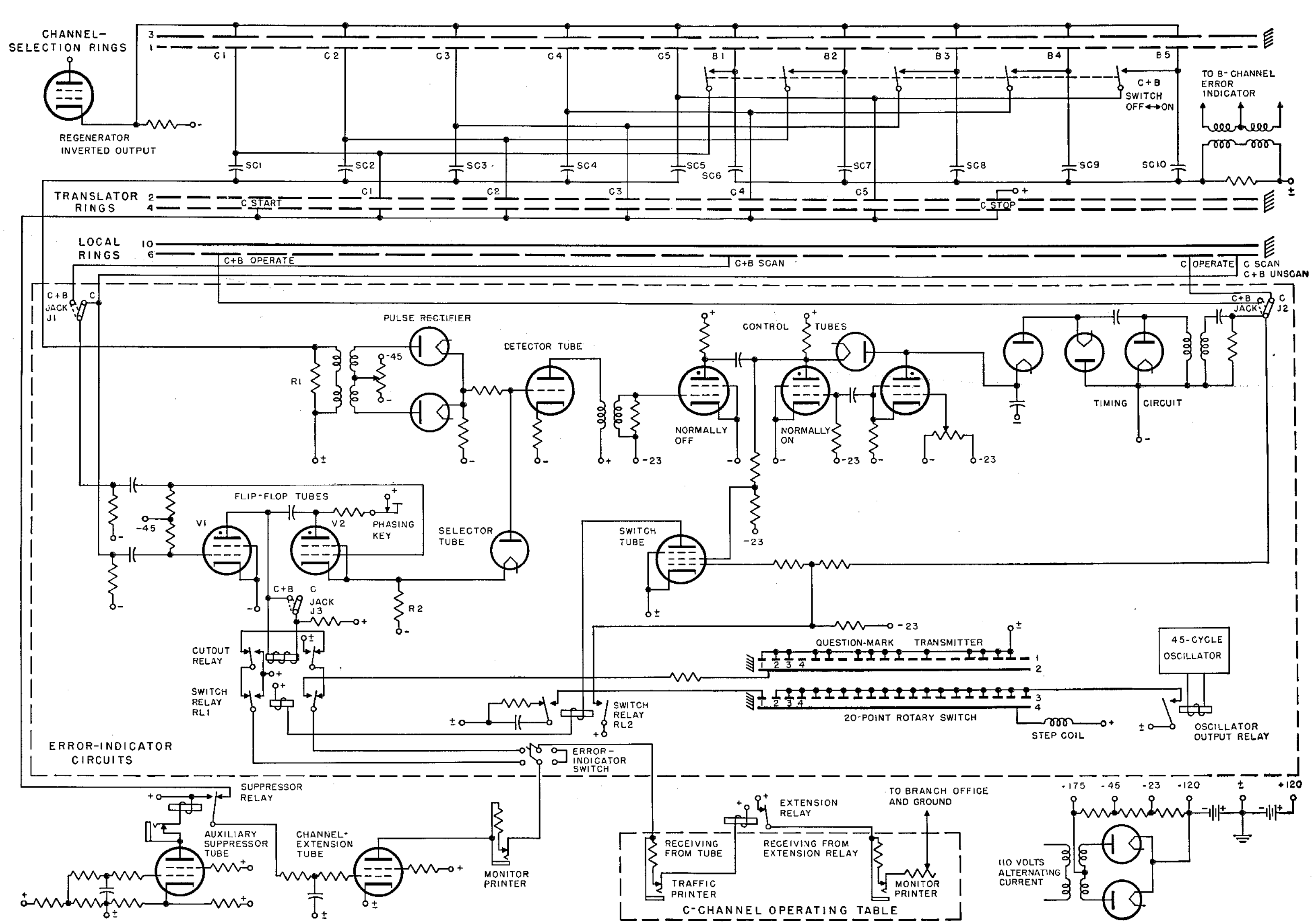

Figure 12—Error-indicator circuits. The circuit automatically prints a question mark if poor radio reception causes a faulty signal.

6.4 Error-Indicator Circuits

The equipment for error indication is mounted on a separate rack. When reception is on a four-channel basis, the signal combination for each character is received twice, on successive revolutions of the distributor brushes, and one is compared with the other. If there is any difference, normal reception is interrupted and a question mark is printed after the questionable character. Normal reception is then resumed. The circuits for this operation are shown in Figure 12 for *C* channel only.

During the first reception of a character, the five signal-storage capacitors *SC1–SC5* are charged through *R1* in parallel with the primary winding of the input transformer. Pulses from the secondary winding are rectified and pass through the selector tube to the flip-flop circuit. The path through the selector tube and *R2* is effectively a short-circuit as there is no plate current flowing through *R2*.

The flip-flop circuit is controlled by pulses from ring 6 of the distributor and is switched after each character is received. On this first character, *V1* of the flip-flop circuit is active and has energized the cutout relay, which disconnects the traffic printer from the channel-extension tube, thus making it inoperative.

After the five channel pulses have been received, the flip-flop circuit is pulsed through ring 6 and *V2* becomes active. The voltage drop across *R2* from the plate current flowing through *V2* acts as a cutoff bias on the selector tube and any pulses through *R1* will be transmitted to the grid of the detector tube.

When the detector tube is pulsed, the traffic printer is switched from the normal receiving circuit to the question-mark transmitter through operation of switch relay *RL1*. The timing circuit is put into operation through *RL2* so that the traffic printer will be reconnected to the normal circuits after the question-mark transmitter has sent the start–stop combinations for "figure shift," "question mark," and "letter shift."

The question-mark transmitter is a 20-point rotary switch driven from a 45-cycle-per-second oscillator.

The switch relays *RL1* and *RL2* are in the plate circuit of the switch tube, which is operated from two control tubes. One of these is coupled to the detector tube and initiates the operation while the other determines the length of operation by means of a pulse from ring 6 through a timing circuit, thus preventing the traffic printer from being switched during the printing of a character.

Figure 12 also shows the circuits used when the original transmission is on *C* channel and *B* channel is used for comparison. The *C*+*B* switch is thrown to the ON position and *J1*, *J2*, and *J3* are plugged into the *C*+*B* jacks. The flip-flop tubes are now controlled from two segments on ring 6. Each character combination is set up on the combined *C*- and *B*-channel storage capacitors during reception over *C* channel.

Printing and scanning for errors are done when the character is being repeated on *B* channel. The timing of the switch from normal to the question-mark transmitter is changed to accord with the new comparison period.

Interference in Multi-Channel Circuits*

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ON THE ASSUMPTION that the relation between output and input for any unit of the link can be represented as a power series of the input signal, suitably delayed, the harmonic content of the output for a pure-tone input signal is found. When a multi-channel signal is injected, it is found that the inter-channel interference falling in any particular channel consists of a number of separate terms, each bearing a functional relation to a corresponding harmonic component of the output when a pure-tone test signal is used as input. This relation is found in the form of an integral in which the root-mean-square amplitude of the pure-tone input occurs as a variable of integration. Hence, to find the interference level, the harmonics have to be measured over a range of values of input amplitude, and the interference components deduced by a somewhat involved analysis. Only in the case in which the second and third harmonics predominate is this procedure unnecessary, and there is then a direct and simple relation between interference and harmonics. This latter construction confirms the standard procedure for finding the interference level in this region. Otherwise, the paper is at variance with other published work, in that the latter neglects the effects of coherence, which are here considered in detail.

A particular case—distortion due to mismatches on a long feeder line—is investigated in detail, and it is shown that, for increasing feeder length, the interference and second harmonic at first increase together, but that, for long feeder lengths, the former decreases slowly while the latter passes through a maximum and then oscillates, its place as the dominant harmonic being taken first by the third, and then successively by higher harmonics. It is shown that, in this example, both the second and third harmonics could vanish for certain feeder lengths, although the interference could remain at an appreciable level, so that caution is necessary before accepting small absolute values of harmonics at their face value. In this connection, the importance of small variations of feeder length or of carrier frequency in determinations of interference levels, is stressed.

It is shown that the beat-frequency method of harmonic measurement, at present in use, although convenient from several points of view, is not exactly equivalent to a determination of the harmonic levels and cannot be used for the analysis of this paper. The two methods are equivalent, however, when the second and third harmonics can be shown to be the only significant ones, in which case, as already stated, the relationship with the interference level is straightforward.

• • •

1. Introduction

One of the most important characteristics of the performance of a multi-channel link is the inter-channel interference level. It is not normally possible to measure this directly in test, as a suitable multi-channel source is not likely to be available. Instead, a pure-tone signal can be injected, and the harmonic margin measured. This harmonic level is then usually taken as an indication of the interference level to be expected from the working link. The purpose of this paper is to examine in detail the connection between the harmonic level on the one hand, and the performance of the link on the other, and to suggest a precise test procedure for relating the two.

2. Harmonic Production

We shall confine ourselves, for the most part, to the multi-channel signal, which will be at "video" frequency. In this band, it will be assumed that the relation between output and input signals for any unit of the link can be represented by a power series

$$V = a_1 s + a_2 s^2 + \ldots, \tag{1}$$

where $s = s(t)$ is the input signal, either multi-channel, or test. There will in general be a time delay τ, so that s in (1) is really $s(t-\tau)$. This

* Reprinted from *Wireless Engineer*, v. 27, pp. 294–304; December, 1950.

delay can be ignored provided, of course, it is the same delay that occurs in each power of s. An exception can be made, however, for the term a_1s, which represents the recovered signal, and which will normally be much larger than the other (distortion) terms. Since we are concerned with the distortion level relative to the recovered-signal level, it is only the amplitude of the latter that is of interest so its phase can be ignored. But all of the distortion terms combine together to give the various harmonics, and hence their relative phase is important.

In what follows, it will be understood, to save repetition, that all harmonic and interference levels are referred to their respective signal levels.

If we put $s=x(2)^{\frac{1}{2}}\cos qt$, to represent a pure-tone test signal, into (1), x being the root-mean-square level, we get

$$V=a_1x(2)^{\frac{1}{2}}\{\cos(qt)+\sum_{2}^{\infty}f_n(x)\cos(nqt)\}. \quad (2)$$

The amplitude $f_n(x)$, which is a function both of the coefficients of (1) and the root-mean-square value of the signal x, is obtained in the appendix Section 12.1. It represents the harmonic margin of the nth harmonic of frequency nq, and is the quantity that can be measured in a straightforward way in test. For example, if with a certain input-signal level, $f_2(x)$ is measured as 10^{-3}, then the second harmonic is $20\log_{10}(10^{-3})=60$ decibels down on the recovered signal (which appears, of course, in a different part of the band).

The harmonic amplitudes $f_n(x)$ vary with the root-mean-square input-signal amplitude x and all obviously vanish at $x=0$. Their actual behaviour as x varies turns out to be important in relating them to the inter-channel-interference level.

3. *Multi-Channel Signal*

Let there be N channels (N large) equispaced in frequency by an amount p cycles per second, so that the "video" frequency of the nth channel is np. Then a multi-channel signal can be represented by

$$S=x(2/\alpha N)^{\frac{1}{2}}\sum_{1}^{N}\epsilon_n\cos(npt+\theta_n). \quad (3)$$

It is here assumed, for simplicity, that the channels start at $n=1$, but the ensuing analysis may be used if a small number at the bottom are missing, provided N is increased by a corresponding amount.

θ_n is a random phase angle, which ensures incoherence between different channels. ϵ_n is a quantity that is 1 or 0 according to whether the nth channel is working or not.

We assume $\sum_{1}^{N}\epsilon_n=\alpha N$, so that α is the fraction of channels actually transmitting at any given moment. (Approximately, half the channels will not be in use, owing to waiting, dialling, etc., while of those in use, roughly a half will be listening, and therefore not transmitting. α may therefore be taken to be about a quarter.)

Since $\epsilon_n{}^2=\epsilon_n$ (both being either 0 or 1), the root-mean-square value of the signal of (3) is

$$\{(2x^2/\alpha N)\sum_{1}^{N}\tfrac{1}{2}\epsilon_n{}^2\}^{\frac{1}{2}}=\{(2x^2/\alpha N)\tfrac{1}{2}\alpha N\}^{\frac{1}{2}}=x.$$

However, the signal level of a working channel is $x(2/\alpha N)^{\frac{1}{2}}$, and it is to this level that the interference will be referred.

4. *Generation of Inter-Channel Interference*

Let us confine our interest to the rth channel, and define a quantity $y=r/N$; y defines the position of the rth channel, is zero at the bottom, and 1 at the top of the band of channels.

Let us substitute in (1) the multi-channel signal of (3), multiply out, and convert products of cosine terms to sum-and-difference terms. All those whose resultant frequency falls in the rth channel can be picked out, and constitute the interference in that channel.

At first sight, it would appear that all such terms would be incoherent, thus adding up "power-wise" to give the total interference, but this is not the case. A detailed analysis reveals that they fall into groups with a certain amount of coherence within a group, and with no term in any one group coherent with any term in any other group. The amplitude of the nth group is denoted by $g_n(x)$ and is a function of the root-mean-square amplitude x of the multi-channel signal of (3). $g_n(x)$ is related, though not in a very simple way, to the function $f_n(x)$ discussed in Section 2. For this reason $g_n(x)$ can conveniently be referred to as the nth harmonic component of the interference. It should be understood, however, that this nomenclature is only by analogy with $f_n(x)$, and is not intended in any way to describe the method of formation of the group. As described above, each group is generated by beat frequencies of all orders and from all channels, and it is, perhaps, a matter of surprise that there is such a definite segregation of terms into groups.

5. *Relation Between $g_n(x)$ and $f_n(x)$*

We are interested in the rth channel, and in order to find the harmonics produced in this channel by a pure-tone input of frequency q, we take $q=\frac{1}{2}rp$ for the second harmonic, $q=\frac{1}{3}rp$ for the third, and so on. Then an input of $x(2)^{\frac{1}{2}}\cos(rpt/n)$ will produce in the rth channel its nth harmonic of amplitude $f_n(x)$ (relative to the fundamental, which appears, of course, in the channel r/n). It is this quantity $f_n(x)$ that will be related to that part of the component of interference $g_n(x)$ that falls in the rth channel. $g_n(x)$, as already stated, is referred to the level of the channel signal, so that, when it is found, the total interference margin $G(x)$ in the rth channel, can be found from $G^2(x)=\sum_2^\infty g_n^2(x)$. The power-wise addition follows from the mutual incoherence between the groups. $G(x)$ will, of course, also be a function of $y=r/N$, which gives the channel position, so that the interference will, in general, vary (smoothly) from channel to channel. Normally, only the first few harmonics $f_n(x)$ will be significant, so that the number of $g_n(x)$ to be summed is correspondingly few.

In Section 12.2 of the appendix, it is shown that

$$g_2(x)=\{4\alpha(1-y/2)\}^{\frac{1}{2}}\cdot\frac{1}{x^5}\int_0^\infty f_2(z)e^{-z^2/x^2}z^4dz. \quad (4)$$

4α can usually be replaced by unity, while the factor $(1-y/2)^{\frac{1}{2}}$ shows a variation of $0{\cdot}7:1\equiv 3$ decibels from the top to the bottom of the band of channels. For the rth channel, we simply insert the value of $y=r/N$. To calculate $g_2(x)$, which is the amplitude of the second-harmonic component of interference for the rth channel, it is first necessary to measure $f_2(z)$ over as wide a range of root-mean-square amplitude z as the experimental procedure may permit. A single tone of frequency $\frac{1}{2}rp$ is injected, and its root-mean-square value is varied as just described. The value of harmonic appearing in the channel r referred to the fundamental appearing in the channel $r/2$ is the function $f_2(z)$ required. This value must be inserted into (4), into which has also been put the correct value of x the root-mean-square value at which the total multichannel signal is intended to operate.

The integration may be performed graphically; or by fitting the measured $f_2(z)$ with a polynomial of the form $b_1z+b_3z^3+b_5z^5+\ldots$ only odd powers appearing. In either case, it is necessary to check that the largest value of z that has been reached is sufficient for the factor e^{-z^2/x^2} to have reduced the integrand to a small-enough value to justify neglecting the contribution from the rest of the range. It is easily shown that

$$\int_0^\infty z^{2n+1}e^{-z^2/x^2}dz=n!\,x^{2n+2}/2,$$

whence we have the following particular values.

$$\int_0^\infty z^5e^{-z^2/x^2}dz=x^6; \qquad \int_0^\infty z^7e^{-z^2/x^2}dz=3x^8;$$

$$\int_0^\infty z^9e^{-z^2/x^2}=12x^{10}; \qquad \int_0^\infty z^{11}e^{-z^2/x^2}dz=60x^{12};$$

$$\int_0^\infty z^{13}e^{-z^2/x^2}dz=360x^{14}; \quad \text{etc.}$$

Hence, when $f_2(z)=b_1z+b_3z^3+b_5z^5+\ldots$, we get

$$g_2(x)=\{4\alpha(1-y/2)\}^{\frac{1}{2}}(b_1x+3b_3x^3+12b_5x^5+\ldots). \quad (5)$$

In particular, when x is small enough, it is seen that, apart from the factor $\{4\alpha(1-y/2)\}^{\frac{1}{2}}$, the first terms of g_2 and f_2 are the same. Hence, for sufficiently small values of x we have

$$g_2(x)\approx\{4\alpha(1-y/2)\}^{\frac{1}{2}}f_2(x), \quad x\text{ small.} \quad (6)$$

The order of approximation is that the fourth and higher harmonics be negligible compared to the second. Unless this is so, (4) or (5) must be used in full. When (6) can be used, it is not necessary, of course, to measure $f_2(x)$ over a range of values since the integration of (4) is no longer called for.

The function $g_3(x)$ is similarly found to be given by

$$g_3(x) = \{4\alpha(1-y^2/3)/2\}^{\frac{1}{2}} \frac{1}{x^6} \int_0^\infty f_3(z) e^{-z^2/x^2} z^5 dz. \quad (7)$$

As before, $f_3(z)$ must be measured over a range of values of z, a fundamental of frequency $rp/3$ being used. $f_3(z)$ is obtained by referring the amplitude of the third harmonic appearing in the rth channel to the fundamental, which now appears in the channel $r/3$. $f_3(z)$ can be fitted by the expansion $b_2z^2+b_4z^4+b_6z^6+\ldots$, giving

$$g_3(x) = \{4\alpha(1-y^2/3)/2\}^{\frac{1}{2}} \times(3b_2x^2+12b_4x^4+60b_6x^6+\ldots). \quad (8)$$

When x is small enough, we get the approximation, from the first term only,

$$g_3(x) \approx \{3/(2)^{\frac{1}{2}}\}\{4\alpha(1-y^2/3)\}^{\frac{1}{2}} f_3(x), \quad x \text{ small.} \quad (9)$$

The order of the approximation is the same as for (6). When this is satisfied, the higher g_n are then negligible, so that we have the approximation

$$G(x) \approx [4\alpha\{(1-y/2)f_2^2(x) + (9/2)(1-y^2/3)f_3^2(x)\}]^{\frac{1}{2}}, x \text{ small.} \quad (10)$$

In general,

$$g_n(x) = [\{4\alpha h_n(y)\}^{\frac{1}{2}}/x^{n+3}] \int_0^\infty f_n(z) e^{-z^2/x^2} z^{n+2}\, dz,$$

with

$$h_n(y) = \frac{2^n}{\pi n!} \int_0^\infty \cos y\theta \left(\frac{\sin\theta}{\theta}\right)^n d\theta. \quad (11)$$

Particular values are

$$h_2(y) = 1-y/2,$$
$$h_3(y) = (1-y^2/3)/2,$$
$$h_4(y) = (8/9-y^2/3+y^3/12)/4,$$
$$h_5(y) = (46-12y^2+6y^4/5)/(24)^2.$$

$f_n(z)$ may be approximated by a polynomial commencing with the term $b_{n-1}z^{n-1}$, successive terms having exponents of z increasing by 2. The integrations can, of course, be done graphically.

6. *Comparison with Published Results*

Comparison will here be made with the paper by Brockbank and Wass,[1] which treats all orders of distortion in some detail. Although the approximate equation (10) above is in general agreement with the results of the same order in the above paper, there is no reference there to the semi-coherent groups $g_n(x)$ and their relation to $f_n(x)$. The reason can be found in paragraph (9.3) "Low-Order Products from High-Order Terms" in which it is stated: "It can be shown that, although the power in each such low-order product may exceed the power in each higher-order product, the number of products is so much lower when N is not small that the total distortion power in the higher-order products is considerably greater than the power in the lower-order products." They have, therefore, neglected these low-order products. What has not been realized is that these low-order products are *coherent* with other terms that are not negligible, and that these terms and the low-order products will therefore add up voltage-wise and not power-wise. The effect, as found here, is to make the effects of the low-order and high-order products of the same order of magnitude, so that agreement is limited to the region in which all such products can be neglected. This leads to (10).

As stated above, the occurrence of coherent groups is quite unexpected, and they can only be determined by a very detailed analysis. That they must occur is apparent when it is realized that such a term as, for example $(A+B-C)$ in a third-order product, is repeated identically in frequency *and phase* in a fifth-order product that gives such a term as $(A+B-C+D-D) = (A+B-C)$. The writer is not aware if such an effect has already been treated in the literature, but it seems necessary that, when higher-order harmonics have to be treated, the analysis follows closely that outlined here.

7. *Coherence with the Main Signal*

The odd-harmonic interference components are peculiar in that part of each of them undergoes a phase change of exactly ϕ when each individual term undergoes this same change. Thus if we consider the expanded form of the product $\cos(lpt+\phi)\cos(mpt+\phi)\cos(npt+\phi)$ one of the terms will be of the form $\cos\{(lpt+\phi)+(mpt+\phi)-(npt+\phi)\} = \cos\{(l+m-n)pt+\phi\}$, which is seen to have changed by the same amount ϕ. If the main signal also undergoes this same

[1] Brockbank and Wass, "Non-linear Distortion in Transmitting Systems," *Journal of the Institution of Electrical Engineers*, v. 92, Part 3, pp. 45–56; March, 1945.

change ϕ, it follows that part of the interference terms remain in phase with the main signal and hence will add up voltage-wise rather than power-wise as the signals are transmitted from repeater to repeater. *In any particular case, however, it must be checked whether the main signal really does alter by the same phase angle, as even a small variation will destroy the coherence effect.* In the next section, an example is treated in which a phase difference actually is introduced between the main signal and the interference components, such that the odd-harmonic components no longer give coherence with the main signal.

As an example of the separation of crosstalk components, let us investigate the third-harmonic component of interference $\bar{g}_3(x)$. Then it is shown in the appendix, Section 12

$$\bar{g}_3(x) = \{\alpha(1+2y-2y^2)\}^{\frac{1}{2}} \frac{1}{x^6} \int_0^\infty f_3(z) e^{-z^2/x^2} z^5 dz.$$

Hence

$$\bar{g}_3(x)/g_3(x) = \{(1+2y-2y^2)/2(1-y^2/3)\}^{\frac{1}{2}}, \quad (11)$$

a result that depends only on the position y of the channel. The right-hand side of (11) varies from 0·71 at $y=0$ to a maximum of 0·92 at $y=0{\cdot}7$ and down to 0·87 at $y=1$. Thus nearly the whole of $g_3(x)$ is of this in-phase type, and may add up voltage-wise from repeater to repeater.

8. *Distortion from a Mis-Matched Feeder*

Unless the analytic form of the output of (1) is known, the general analysis cannot be taken any further, and the results so far presented are intended for application when the various functions $f_n(x)$ have been determined experimentally.

However, a particular case that is capable of complete treatment is that caused by phase distortion due to a mis-matching at the end of a long feeder line. In a previous paper[2] on this subject the function $f_n(x)$ for this case was determined.

A frequency-modulated signal is transmitted from a long feeder, but owing to mis-matches at its ends, multiple reflections are set up within the feeder. These cause a phase distortion of the transmitted wave, and in (3) the expected output is shown to be given by

$$V = \Delta\omega \sin(\omega_a t) - \delta, \quad (12)$$

$$\delta = \frac{d}{dt}[kr_1r_2 \sin\{2\omega_c\tau - \theta_1 - \theta_2 + 2\tau\Delta\omega . \sin(\omega_a \overline{t-\tau}) \sin(\omega_a\tau)/\omega_a\tau\}]. \quad (13)$$

Here, ω_c is the radio-frequency carrier and $\Delta\omega \sin \omega_a t$ is the pure-tone input signal, which is recovered together with distortion terms; the latter are represented by the second term in (13). r_1 and r_2, and θ_1 and θ_2 are respectively the amplitudes and phase angles of the reflection coefficients at each end of the feeder. τ is the time delay along the feeder and equals l/v where l is the feeder length and v the group velocity of the waves. k is the attenuation along the double length of feeder and equals $e^{-2\alpha l}$, where α is the attenuation coefficient.

Equation (13) has to be compared with (1) with $s = \Delta\omega \sin \omega_a t$—the root-mean-square amplitude x being given by $\Delta\omega = x(2)^{\frac{1}{2}}$. It is seen that (13) as it stands is not capable of expansion in powers of s, owing to (A) the presence of $\sin(\omega_a\tau)/\omega_a\tau$, which contains ω_a a characteristic of the signal: (B) the occurrence of $t-\tau$ instead of t in the distortion terms, and (C) the operator d/dt. ω_a is the video frequency and is usually not very high so that unless the feeder run is so large that it becomes comparable with the wavelength of the video frequency, $\omega_a\tau$ will be small. Hence $\sin(\omega_a\tau)/\omega_a\tau$ can be replaced by unity without serious effect, and point (A) above is no longer a limitation.

Point (B) is covered by Section 2, which permits all powers of s above the first to contain a constant delay. However, since the recovered signal in (13) contains no delay, the distortion terms, although coherent with the main signal, will differ in phase from it by an angle $\omega_a\tau$. (That such a delay must occur is apparent when it is realized that the distortion terms arise from waves that have been reflected down the feeder and back again before transmission). Hence the in-phase components of odd-order interference harmonics, although coherent with the main signal, are not actually in phase with it, and the phase difference changes from repeater to repeater, giving the appearance of a random phase change. Thus for all practical purposes, this component behaves as if it were incoherent, and

[2] L. Lewin, J. J. Muller, and R. Basard, "Phase Distortion in Feeders," *Wireless Engineer*, v. 27, pp. 143–145; May, 1950: and *Electrical Communication*, v. 27, pp. 320–323; December, 1950.

there is no call to extract it from the rest of the interference terms for special treatment. It will not add up voltage-wise from repeater to repeater.

So far as the operator d/dt is concerned, if it were applied at the end of the analysis, no alteration would be called for. It would produce merely a factor, which for (13), would be $n\omega_a$ for the nth harmonic; and which, in the multi-channel case, would be rp for the rth channel. Hence, if we quote results in which d/dt has been applied at an earlier stage, the only change necessary is the replacement of $n\omega_a$ by rp.

The functions $f_n(x)$ for this problem are easily obtained from (13) by the Fourier-Bessel expansion and are given in (5) of the paper referred to. Replacing $n\omega_a$ by rp as discussed above we get

$$f_n(x) = kr_1r_2(rp)\frac{J_n\{x2(2)^{\frac{1}{2}}\tau\}}{x} \times\left\{(2)^{\frac{1}{2}} \begin{matrix}\sin\\ \cos\end{matrix} (2\omega_c\tau-\theta_1-\theta_2)\right\}. \quad (14)$$

The sin or cos occurs according to whether n is even or odd. Since the phase angle $2\omega_c\tau-\theta_1-\theta_2$ is indeterminate without a precise knowledge of the particular arrangement in use, we can, in considering over-all behaviour of many units, replace the terms in braces by their root-mean-square values, in this case unity. However, this is only an average effect, and for any particular feeder at any one repeater this term will be quite definite, and only if $2\omega_c\tau-\theta_1-\theta_2$ reduces to 45 degrees will the results be the same. But the effects of different repeaters, or of the same repeater under slightly different conditions of temperature, etc., can be simulated by a variation of $\omega_c\tau$. This can be done conveniently either by small changes of ω_c or by varying τ, by means of a line lengthener. In the ensuing analysis, the results will be averaged out by taking the root-mean-square values for the indeterminate phase term, but the above statement must be borne in mind in the interpretation of any single result.

Accordingly, we take for $f_n(x)$

$$f_n(x) = kr_1r_2(rp)J_n\{x2(2)^{\frac{1}{2}}\tau\}/x. \quad (15)$$

The integrations required by (11) can be performed by means of the following integral.[3]

[3] G. N. Watson, "A Treatise on the Theory of Bessel Functions," Second Edition, Cambridge University Press, London; 1944: p. 394, equation (4).

$$\int_0^\infty J_n(at)e^{-t^2/x^2}\, t^{n+1}\, dt = a^n\left(\frac{x^2}{2}\right)^{n+1} e^{-a^2x^2/4}.$$

Hence, using (11) and (15), we get

$$g_n(x) = \frac{kr_1r_2(rp)}{x^{n+3}}\{4\alpha h_n(y)\}^{\frac{1}{2}} \times\int_0^\infty J_n\{z2(2)^{\frac{1}{2}}\tau\}z^{n+1}\, e^{-z^2/x^2}\, dz$$
$$= k\{4\alpha h_n(y)\}^{\frac{1}{2}}\, r_1r_2(rp)x^{n-1}\,\tau^n\, 2^{n/2-1}\, e^{-2\tau^2x^2}. \quad (16)$$

The total crosstalk power in the rth channel is

$$G^2(x) = \sum_2^\infty g_n^2(x) = \frac{4\alpha}{\pi}(kr_1r_2rp\tau)^2e^{-4\tau^2x^2} \times\int_0^\infty \cos y\theta \sum_2^\infty \left(\frac{\sin\theta}{\theta}\right)^n \frac{(2\tau x)^{2n-2}}{n!}d\theta \quad (17)$$

on replacing $h_n(y)$ by its value as given by (11).

Summing, and taking the square root, we get for the total interference amplitude

$$G(x) = (\alpha)^{\frac{1}{2}}kr_1r_2(y\omega_N)\phi(\tau x)/2x,$$

where

$$\phi(\tau x) = \left[\frac{4}{\pi}\int_0^\infty \cos y\theta\{\exp\,(4\tau^2x^2\sin\theta/\theta) -1-4\tau^2x^2\sin\theta/\theta\}d\theta\right]^{\frac{1}{2}} e^{-2\tau^2x^2}. \quad (18)$$

Here, we have put

$$rp = \frac{r}{N}.Np = y\omega_N,$$

where $\omega_N = Np$ is the angular frequency (video) corresponding to the top of the multi-channel band.

The integral in (18) cannot be evaluated in closed form, but very good approximations for the top and bottom of the band ($y=1$ and 0) are found in the appendix, Section 12.3. The variation (for the integral) is always less than $1{:}2^{\frac{1}{2}}$ over the video band, and since the form is much simpler at the top, we will confine our attention there. However, it is seen from (18) that apart from this slow variation, $G(x)$ is also porportional to y, so that the interference *from the causes investigated in this example* vanish at the bottom channels of the band.

Using the approximations of the appendix, Section 12.3, for the top of the video band, we find the interference level in the top channel:

$$G(x)_{y=1} = G_1 \text{ (say)} \approx (\alpha)^{\frac{1}{2}} r_1 r_2 \omega_N \times \{1-(1+4\tau^2x^2)\exp(-4\tau^2x^2)\}^{\frac{1}{2}}/2x, \quad \tau x < 1. \tag{19}$$

For larger feeder lengths

$$G_1 \approx (\alpha)^{\frac{1}{2}} r_1 r_2 \omega_N \frac{1{\cdot}175}{2x}\left\{\frac{1-0{\cdot}32/(\tau x)^2}{\tau x}\right\}^{\frac{1}{2}}, \quad \tau x > 1. \tag{20}$$

For very short feeders, (19) gives

$$G_1 \approx (2\alpha)^{\frac{1}{2}} r_1 r_2 \omega_N \tau^2 x, \quad \tau x < 0{\cdot}3. \tag{21}$$

The actual curve as a function of τx is shown in Figure 1.

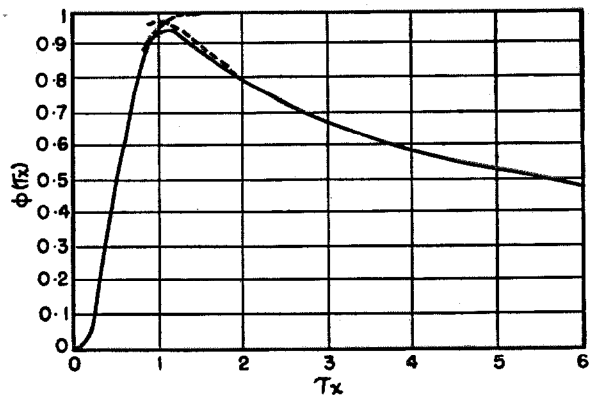


Figure 1—Inter-channel interference produced in a mismatched feeder. The solid curve is for the equation

$$\phi(\tau x) = \left[\frac{4}{\pi}\int_0^\infty \cos\theta\left\{\exp\left(4\tau^2x^2\frac{\sin\theta}{\theta}\right) - 1 - 4\tau^2x^2\frac{\sin\theta}{\theta}\right\}d\theta \times e^{-4\tau^2x^2}\right]^{\frac{1}{2}}$$

while the asymptotic form of (20) shown dotted is

$$\phi \approx 1{\cdot}175\left(\frac{1-0{\cdot}32/\tau^2x^2}{\tau x}\right)^{\frac{1}{2}}, \quad \tau x > 1{\cdot}5$$

and the series form of (19) shown dashed is

$$\phi \approx \{1-(1+4\tau^2x^2)e^{-4\tau^2x^2}\}^{\frac{1}{2}}, \quad \tau x < 1.$$

As an example, consider a feeder 55 feet long, with a system in which the root-mean-square frequency deviation is 2 megacycles per second. Assume the radio-frequency feeder is a waveguide in which the group velocity of the waves is 0·7 light velocity. Then $\tau x = 2\pi \times 55 \times 12 \times 2{\cdot}54 \times 2 \times 10^6 \times 1/(0{\cdot}7 \times 3 \times 10^{10}) = 1$. This is at the transition between (19) and (20) for which the two functions of τx at $\tau x = 1$ are respectively 0·95 and 0·97. The correct value, used in obtaining Figure 1, is 0·925 from the series expansion. It follows that, for this example, we can use the asymptotic form of (20) for runs longer than about 60 feet, the interference amplitude for such lengths dropping roughly as the inverse square root of the feeder length.

If ω_N is 0·8 megacycle per second and r_1 and r_2 correspond to standing-wave ratios of 0·95 and 0·8, we get, neglecting attenuation ($k=1$).

$$G = \left(\frac{1}{4}\right)^{\frac{1}{2}} \times \frac{1-0{\cdot}95}{1+0{\cdot}95} \times \frac{1-0{\cdot}8}{1+0{\cdot}8} \times \frac{0{\cdot}8}{4} \times 0{\cdot}925 = 0{\cdot}263 \times 10^{-3}.$$

Thus the interference is about 72 decibels down. This is some 7 decibels better than that obtained from (3) of Reference 2, from a consideration of the second harmonic only.

For $\tau x = 1$, the ratio of third to second harmonic, when each in turn is arranged to occur at the top of the band, is $J_3\{2(2)^{\frac{1}{2}}\}/J_2\{2(2)^{\frac{1}{2}}\} = 0{\cdot}28/0{\cdot}48 = 0{\cdot}56$, so that the second harmonic can no longer be considered dominant. For values of τx greater than 1; $J_2\{2(2)^{\frac{1}{2}}\tau x\}$ decreases, passing through zero when $\tau x = 1{\cdot}8$, while the third, and subsequently the fourth and higher harmonics "take over." The interference decreases monotonically for values of τx greater than 1, and there is no simple direct relation between it and the various harmonics, apart from the already-mentioned approximate equality to the second harmonic, when the latter is the dominant one.

It should be emphasized again that the behaviour described above is an *average* one, averaged for different values of the normally indeterminate phase angle $2\omega_c\tau - \theta_1 - \theta_2$. For a particular set-up, this quantity may, for example, be a multiple of π, so that

$$\sin(2\omega_c\tau - \theta_1 - \theta_2) = 0.$$

In this case, there would be no second harmonic at all—and the interference would be correspondingly reduced—but such an example would not adequately describe the characteristics of the unit, since the next one might well have a large second harmonic—and interference level—consequent on a different value of the phase. The only way to obtain significant results is, as mentioned previously, to conduct the test over a range of values of phase (by varying ω_c or l) and to take the average of the results. This should then lead to the *average* harmonic levels and interference levels of (15) and (19) respectively.

9. Recommended Test Procedure

In order to estimate the interference level in the rth channel, of video frequency rp, the following procedure is suggested:

A. Inject a signal of frequency $rp/2$ and root-mean-square amplitude z, and measure the level of harmonic appearing in the rth channel relative to the signal appearing in channel $(r/2)$. Vary z from zero to as large a value as the experiment may permit. Then the relative harmonic amplitude is $f_2(z)$ as defined in (2).

B. Calculate $g_2(x)$ from $f_2(z)$ by means of (4). In this derivation the root-mean-square level at which it is intended to work the total video signal in the link is presumed known. α may usually be taken as 1/4. The integration may either be performed graphically or by fitting $f_2(z)$ with a polynomial $b_1z+b_3z^3+b_5z^5+\ldots$ From (5), the value of $g_2(x)$ is $\{4\alpha(1-y/2)\}^{\frac{1}{2}}(b_1x+3b_3x^3+12b_5x^5+\ldots)$.

C. Repeat (A) but use a fundamental of frequency $rp/3$. The signal appears in channel $(r/3)$ and its third harmonic in channel r. The ratio of the amplitudes is $f_3(z)$.

D. Derive $g_3(x)$ from $f_3(z)$ using (7). If $f_3(z)$ is fitted by $b_2z^2+b_4z^4+b_6z^6+\ldots$ then, from (8)

$$g_3(x)=\{4\alpha(1-y^2/3)/2\}^{\frac{1}{2}}(3b_2x^2+12b_4x^4+60b_6x^6+\ldots).$$

E. Repeat the above measurements for the higher harmonics using a fundamental of frequency rp/n and deduce $g_n(x)$ from $f_n(z)$ by (11).

F. The total inter-channel interference power level G^2 is given by $G^2=g_2^2(x)+g_3^2(x)+g_4^2(x)+\ldots$ As explained in Section 7, it may be necessary to make a partial separation of the odd harmonics, to allow for the in-phase effect.

G. Repeat the above steps with slightly different carrier frequencies or feeder lengths, to simulate the effects of repeater variations.

If it can be shown that only the second and third harmonics are significant, then it is permissible to avoid most of the above analysis, and use simply (6) and (9) which give

$$\left.\begin{aligned} g_2(x) &\approx \{4\alpha(1-y/2)\}^{\frac{1}{2}}.f_2(x) \\ g_3(x) &\approx \frac{3}{2^{\frac{1}{2}}}\{4\alpha(1-y^2/3)\}^{\frac{1}{2}}.f_3(x) \\ G^2 &\approx g_2^2(x)+g_3^2(x). \end{aligned}\right\} \quad (22)$$

The measurement of $f_2(x)$ and $f_3(x)$ over a range of values of root-mean-square amplitude is not necessary here, and only the actual amplitude that it is intended to use in the working link is needed.

In deciding whether or not the second and third harmonics are the only ones of value, absolute smallness of these is *not in itself a reliable criterion*, although it may often be correct. For example, in the application given in Section 7, the second harmonic could be small either on account of the feeder phase being nearly a multiple of π, or because the length of feeder was actually such as to bring $x\tau$ of (15) into the region giving low second harmonic, the third or higher harmonics being dominant. In the first case, the smallness could be reflected in a low value of interference, albeit this effect would not be repeatable from repeater to repeater on account of the feeder phase angle not being exactly determinate. In the second case, the small value of second harmonic could be misleading. If we had $\tau x=1{\cdot}8$ and $2\omega_c\tau-\theta_1-\theta_2=n\pi+\pi/2$, both second and third harmonics would be zero, although the interference in this case would be near its maximum. On the other hand, if kr_1r_2 were very small (feeder well matched) the second and third harmonics would be small and this would be "genuine" in that the interference would be correspondingly small. Caution is therefore necessary before accepting low values of second and third harmonics at their face value.

10. Alternative Test Procedure

Since the bottom channel may, in practice, start somewhere around $r=15$ instead of $r=0$, it is not possible to carry out the above analysis too far, or over the whole of the band, as either the fundamental or its harmonics may be outside the range of the apparatus at one end of the band or the other. Hence an alternative method is in use in which a double-tone signal is injected and difference frequencies are looked for. This method has the advantage of being usable over the whole band. So long as only the second and third harmonics (of the single-tone test) are significant, the two methods give the same results, and the order of approximation is the same as that which leads to (22) above.

However, if higher-order harmonics than the third cannot be neglected, it is easy to see that the beat-frequency method no longer gives the various harmonics accurately, any more than $f_2(x)$ can give $g_2(x)$ directly, as in the approximations of (22). The order of errors is probably the same in both cases, so that it is pointless to try to use the preceding analysis without the use of the correct functions $f_n(x)$ {except, as already stated, to the approximation of (22)}. There is, of course, a relation between the harmonic amplitudes $f_n(x)$, on the one hand, and the beat-frequency measurements on the other, but they certainly are not of any simple form. It is hoped to investigate this at a later date. Meanwhile, if the full analysis of this paper is called for in any particular arrangement, there seems to be no alternative to the full procedure of Section 9 with its necessary limitations.

11. Conclusions

Although the general validity of existing test practice is confirmed in the case in which the second and third harmonics can be shown to be the only significant ones, caution is necessary in accepting low absolute values of these harmonics at their face value, especially when long feeder runs are involved. The complex analysis and harmonic method of testing outlined here seem to be essential when the higher harmonics are not negligible.

Further work will be required to bring out the relation between single-tone and double-tone methods of test.

12. Appendixes

12.1 Harmonic Generation

Let the output for an input signal s be given by

$$V=a_1s+a_2s^2+a_3s^3+\ldots \tag{23}$$

a_1s will be taken to be the recovered signal. This means that the terms of fundamental frequency produced by the higher powers of s are negligible; i.e., that $a_1 \gg a_n$. (This will certainly be so in the case of low distortion, which is the case of interest here).

Let $s=x(2)^{\frac{1}{2}}\cos\theta$, where x is the root-mean-square amplitude. Now

$$\cos^n\theta=\frac{1}{2^{n-1}}(\cos n\theta+{}^nC_1\cos\overline{n-2}\theta$$
$$+{}^nC_2\cos\overline{n-4}\theta+\ldots). \tag{24}$$

Substituting in (23) and picking out the various terms, we get (neglecting all $\cos\theta$ terms except $a_1\cos\theta$)

$$\begin{aligned}V=&\{x(2)^{\frac{1}{2}}a_1\}\cos\theta\\&+\{2x^2a_2.{}^2C_0.\tfrac{1}{2}+(2x^2)^2a_4.{}^4C_1.\tfrac{1}{8}\\&+(2x^2)^3a_6.{}^6C_2.\tfrac{1}{32}+\ldots\}\cos 2\theta\\&+[\{x(2)^{\frac{1}{2}}\}^3a_3.{}^3C_0.\tfrac{1}{4}+\{x(2)^{\frac{1}{2}}\}^5a_5.{}^5C_1.\tfrac{1}{16}\\&\qquad+\{x(2)^{\frac{1}{2}}\}^7a_7.{}^7C_2.\tfrac{1}{64}]\cos 3\theta+\ldots\end{aligned}$$

If we put $f_n(x)=$ ratio of nth harmonic to fundamental, we get

$$\left.\begin{aligned}f_2(x)&=\frac{x}{(2)^{\frac{1}{2}}a_1}\sum_1^\infty\frac{2n!}{2^{n-1}\,\overline{n-1}!\,\overline{n+1}!}a_{2n}x^{2n-2}\\f_3(x)&=\frac{x^2}{2a_1}\sum_1^\infty\frac{\overline{2n+1}!}{2^{n-1}\,\overline{n-1}!\,\overline{n+2}!}a_{2n+1}x^{2n-2}\\f_4(x)&=\frac{x^3}{2(2)^{\frac{1}{2}}a_1}\sum_1^\infty\frac{\overline{2n+2}!}{2^{n-1}\,\overline{n-1}!\,\overline{n+3}!}a_{2n+2}x^{2n-2}\\&\text{etc.}\end{aligned}\right\} \tag{25}$$

These formulae, which are relations between amplitudes, are obviously not affected if the term $a_1x(2)^{\frac{1}{2}}\cos\theta$ is replaced by $a_1x(2)^{\frac{1}{2}}\cos(\theta+a)$, so that the phase of the recovered signal is of no importance here.

12.2 Generation of Inter-Channel Interference

Let us consider a multi-channel signal of N channels, N being large. If ϵ_n represents a factor, which is 1 or 0 according to whether the nth channel is working or not, we get the form

$$\begin{aligned}S&=x(2/\alpha N)^{\frac{1}{2}}\sum_1^N\epsilon_n\cos(npt+\theta_n)\\&=x(2/\alpha N)^{\frac{1}{2}}.s\text{ (say).}\end{aligned} \tag{26}$$

Here, $\sum_1^N\epsilon_n^2=\sum_1^N\epsilon_n=\alpha N$. As explained in the text, α will be about a quarter, and the root-mean-square value of S is x.

If a power series of S is under consideration, we need to handle such terms as

$$\begin{aligned}s^n&=(\sum_1^N\epsilon_r\cos\phi_r)^n\\&=\underbrace{\sum_1^N\sum_1^N\ldots\sum_1^N}_{n\text{ times}}\epsilon\cos\phi_a.\cos\phi_b\ldots\cos\phi_n,\end{aligned}$$

where $\epsilon=\epsilon_a\epsilon_b\ldots\epsilon_n$ and $\phi_r=rpt+\theta_r$.

If the product of cosines is replaced by sum and different terms, it is seen that there can be all possible combinations of the form

$$\cos(\pm\phi_a \pm \phi_b \ldots \pm \phi_n).$$

Many of these terms will be repetitions of each other, and when like terms are collected together it is found that

$$s^n = \left(\sum_1^N \epsilon_r \cos\phi_r\right)^n =$$

$$2^{1-n} \underbrace{\sum_1^N \sum_1^N \ldots \sum_1^N}_{n \text{ times}} \epsilon\{\cos(\textstyle\sum_n \phi_r) + {}^nC_1 \cos(\textstyle\sum_{n-1}\phi_r) + {}^nC_2 \cos(\textstyle\sum_{n-2}\phi_r) + \ldots\}. \quad (27)$$

The series stops at the centre term of the binomial coefficients.

$\sum_{n-m}\phi_r$ means $(\phi_a+\phi_b+\ldots)-(\phi_\alpha+\phi_\beta+\ldots)$ with $n-m$ positive angles and m negative angles, $a, b \ldots, \alpha, \beta \ldots$ being the suffixes, which are each summed from 1 to N. The coefficients may be obtained by comparison with (24) to which (27) must reduce when either ϕ_r are all equal or $N=1$. (The restriction of N to large values does not yet apply.)

To what extent are terms in s^n repeated in s^m when m is greater than n? Obviously, m and n must be either both even or both odd, otherwise no term can be the same. Let us take them both even, and write $2n$ and $2m$ as the indices.

To get the rth term of s^{2n} from s^{2m} we need to choose $2n-r+1$ specific positive angles and $r-1$ specific negative angles from a total of $2m$ angles, the rest of the $2m$ angles cancelling out in identical pairs of positive and negative angles. The term in s^{2m} that has the requisite form, namely $2n-r+1+\frac{1}{2}(2m-2n)=m+n-r+1$ positive angles, has a coefficient ${}^{2m}C_{m+n-r+1}$. Provided N is large enough, the choice of angles can be made in ${}^{m+n-r+1}C_{2n-r+1}\times{}^{m-n+r-1}C_{r-1}$ ways, with the choice of m–n self-cancelling pairs in $(m-n)!$ ways. Hence the form of the rth term of s^{2n} is repeated in s^{2m} in ${}^{2m}C_{m+n-r+1}\times{}^{m+n-r+1}C_{2n-r+1}\times{}^{m-n+r-1}C_{r-1}\times(m-n)!$ different ways. This factor simplifies to ${}^{2n}C_{r-1}\ {}^{2m}C_{m-n}(m+n)!/2n!$ of which ${}^{2n}C_{r-1}$ is the same as the coefficient of the rth term of s^{2n}. The summation over the m–n self-cancelling pairs produces a further factor $\underbrace{\sum_1^N \ldots \sum_1^N}_{m-n \text{ times}} \epsilon_a^2\epsilon_b^2 \ldots = (\alpha N)^{m-n}$, while from (27) the expansions of s^{2n} and s^{2m} are seen to differ by an initial factor $2^{-(2m-2n)}$. Hence the entire s^{2n} is produced within s^{2m} with a factor $(\alpha N/4)^{m-n}\ {}^{2m}C_{m-n}\ (m+n)!/2n!$ If we use the full form S instead of s, there appears an additional factor $\{x(2/\alpha N)^{\frac{1}{2}}\}^{2m-2n}$, giving an over-all factor $\{x/(2)^{\frac{1}{2}}\}^{2m-2n}\{{}^{2m}C_{m-n}(m+n)!/2n!\}$.

The "multiplicity factor" in braces can also be obtained as follows. s^{2m} is the product of $2m$ factors s, of which any $2n$ can produce s^{2n}, while the constant term in the remaining s^{2m-2n} will provide, with the factor s^{2n}, some multiple of s^{2n}. The choice of $2n$ factors can be made in ${}^{2m}C_{2n}$ ways, while the constant term in s^{2m-2n} is seen, from (27) to be ${}^{2m-2n}C_{m-n}.2^{1-2m-2n}$ times the number of ways of extracting m–n self-cancelling pairs from $2m$–$2n$ suffixes, each running from 1 to N. This latter factor is $m\text{–}n!(\alpha N)^{m-n}$. Since ${}^{2m}C_{2n}.{}^{2m-2n}C_{m-n}.m\text{–}n! = {}^{2m}C_{m-n}.(m+n)!/2n!$ we have the same multiplicity factor as before. (In counting the number of self-cancelling pairs, such forms as $+a-a$ and $-a+a$ have been counted once only. The duplicity thereby omitted is made up for by the fact that the last term of the binomial coefficients appearing in (27) should carry a factor $\frac{1}{2}$ when n is even).

Of the two derivations, the first has the advantage, however, of being "exclusive" in the sense that it shows that no further terms in s^n occur in s^m, so that the groups of terms are now incoherent and add up power-wise.

Collecting all the terms associated with S^{2n}, we find that we have s^{2n} multiplied by

$$2^{1-2n}\{x(2/\alpha N)^{\frac{1}{2}}\}^{2n}\left\{\sum_1^\infty a_{2n+2m-2}\frac{x^{2m-2}}{2^{m-1}} \times\frac{(2n+2m-2)!}{(m-1)!(2n+m-1)!}\times\frac{(2n+m-1)!}{2n!}\right\}. \quad (28)$$

If this form is compared to the function $f_{2n}(x)$ of (25), they are seen to be very similar, the main difference being the occurrence in (28) of the factor $(2n+m-1)!/2n!$ multiplying the coefficient of $x^{2n+2m-2}$. Now $x^{2n+2m-2}(m+2n-1)!/2n!$

$$= (x^{2n+2m-2}/2n!)\int_0^\infty e^{-t}t^{m+2n-1}dt$$

$$= (1/2n!)\int_0^\infty e^{-t}(xt^{\frac{1}{2}})^{2n+2m-2}t^n dt$$

$$= (2x^{-2n-2}/2n!)\int_0^\infty e^{-y^2x^2}.(z)^{2n+2m-2}.z^{2n+1}dz$$

on putting $t=z^2/x^2$.

Using this result, the expression in (28) can be written

$$\frac{2a_1}{2n!}\cdot\frac{2^{\frac{1}{2}}}{x^{2n+2}}\frac{1}{(\alpha N)^n}\int_0^\infty e^{-z^2/x^2}f_{2n}(z)z^{2n+2}dz. \quad (29)$$

The extra power of z and the factor $a_1(2)^{\frac{1}{2}}$ appear because $f_{2n}(z)$, which is a relative amplitude, had been obtained by dividing by $a_1(2)^{\frac{1}{2}}z$ in (25), and this factor has had to be restored in (29).

In order to find the relative amplitude level of the interference arising from the terms of the form S^{2n}, we need to know the amplitude of s^{2n}. Denoting by ${}^{2n}A_r$ the resultant root-mean-square amplitude of those terms in s^{2n} that fall in the rth channel and dividing (29) by $a_1x(1/\alpha N)^{\frac{1}{2}}$, (the root-mean-square level of a working channel), we have the total interference level in the rth channel arising from S^{2n} terms. Denoting this level by $g_{2n}(x)$ we find

$$g_{2n}(x)=\frac{2(2\alpha)^{\frac{1}{2}}}{2n!}\left(\frac{{}^{2n}A_r}{\alpha^nN^{n-\frac{1}{2}}}\right)\frac{1}{x^{2n+3}}\times\int_0^\infty f_{2n}(z)e^{-z^2/x^2}z^{2n+2}dz. \quad (30)$$

A similar relation is found when the power of S is odd, so that, writing n for $2n$ in (30), we have the general relation for all n

$$g_n(x)=\frac{2(2\alpha)^{\frac{1}{2}}}{n!}\left(\frac{{}^nA_r}{\alpha^{\frac{1}{2}n}N^{\frac{1}{2}n-\frac{1}{2}}}\right)\frac{1}{x^{n+3}}\int_0^\infty f_n(z)e^{-z^2/x^2}dz. \quad (31)$$

There remains the calculation of ${}^nA_r$. In s^n, the term with n–m positive angles and m negative angles has a coefficient ${}^nC_m$. There are n–m! combinations of positive angles and m! combinations of negative angles each giving the same term when the suffixes are summed. Hence, if there are α_m combinations of suffixes giving a resultant in the rth channel, only $\alpha_m/(m!.n\text{–}m!)$ of them are independent and form an incoherent group. The mean-square amplitude is therefore $\frac{1}{2}({}^nC_mm!n\text{–}m)^2\ \alpha_m/(m!n\text{–}m!)$, the factor $\frac{1}{2}$ arising as the mean square of a cosine term. This expression simplifies to $n!\,{}^nC_m\alpha_m/2$. The total mean-square amplitude is thus $(n!/2)(\alpha_0+{}^nC_1\alpha_1+{}^nC_2\alpha_2+\ldots)$ terminating at the centre term of the binomial series. Now α_m is the number of combinations of suffixes, n–m positive and m negative, whose sum falls in the bands $+r$ and $-r$. This is the same as the number of n–m positive and m negative suffixes, plus the number of m positive and n–m negative suffixes whose sum falls in the band $-r$ (or $+r$). This is easily seen to be the coefficient of x^{-r} in

$$\left(\sum_1^N x^l\right)^{n-m}.\left(\sum_1^N x^{-l}\right)^m+\left(\sum_1^N x^l\right)^m.\left(\sum_1^N x^{-l}\right)^{n-m}.$$

The amplitude sum therefore reduces to the coefficient of x^{-r} in

$$\tfrac{1}{2}(n)!\sum_1^n {}^nC_m\left(\sum_1^N x^l\right)^{n-m}.\left(\sum_1^N x^{-l}\right)^m$$
$$=\tfrac{1}{2}(n)!\left(\sum_1^N x^l+\sum_1^N x^{-l}\right)^n.$$

Summing the geometrical progression, we get

$$\tfrac{1}{2}(n)!\left(\frac{1-x^{N+1}}{1-x}+\frac{1-x^{-N-1}}{1-x^{-1}}\right)^n=\tfrac{1}{2}(n)!\left\{\frac{1-x^{N+1}}{1-x}(1+x^{-N})\right\}^n.$$

When N is large, we can replace $N+1$ by N without much error, so that we can write

$$({}^nA_r)^2\approx(\alpha^nn!/2)\times\text{coefficient of } x^{-r} \text{ in } \left(\frac{x^{-N}-x^N}{1-x}\right)^n. \quad (32)$$

The factor α^n arises from the effects of ϵ_n, which have so far been neglected. α is actually the probability that any particular term is present, and with n independent terms occurring in the summations leading to $({}^nA_r)^2$ the factor α^n accordingly appears.

Taking the square root of (32) and expanding both numerator and denominator by the binomial theorem we get

$${}^nA_r=\alpha^{\frac{1}{2}n}[\tfrac{1}{2}(n!)\times\text{coefficient of } x^{-r} \text{ in } (1-x)^{-n}\{x^{-nN}-{}^nC_1x^{-(n-2)N}+{}^nC_2x^{-(n-4)N}\ldots\}]^{\frac{1}{2}}.$$

If we assume that r is large enough for such products as $r(r+1)$ to be replaced by r^2, we get the following fairly simple form

$${}^nA_r\approx\alpha^{\frac{1}{2}n}[(n/2)\{(nN-r)^{n-1}-{}^nC_1\{(n-2)N-r\}^{n-1}+{}^nC_2\{(n-4)N-r\}^{n-1}\ldots\}]^{\frac{1}{2}}, \quad (33)$$

the terms stopping when the coefficient of N in any bracket is less than 1.

An alternative formulation is given by putting

$$\left(\frac{x^{-N}-x^{+N}}{1-x}\right)^n \approx \left(\frac{x^N-x^{-N}}{x^{\frac{1}{2}}-x^{-\frac{1}{2}}}\right)^n,$$

writing $x=e^{i\beta}$, and finding the coefficient of $e^{-ir\beta}$ by Fourier analysis. This gives

$$({}^nA_r)^2=\frac{\alpha^n n!}{4\pi}\int_0^{2\pi} e^{ir\beta}\left(\frac{\sin N\beta}{\sin \beta/2}\right)^n d\beta.$$

For N and r both large, and $y=r/N$ finite and less than 1, only the range near $\beta=0$ and π is important in the integrand. Changing the variable from β to $\theta=N\beta$, the range for θ can be taken to ∞. With $\sin\beta/2\approx\beta/2$ at the bottom of the range, this gives the asymptotic form, for large N

$$({}^nA_r)^2\approx\frac{n!2^nN^{n-1}\alpha^n}{2\pi}\int_0^\infty \cos y\theta\{\sin(\theta)/\theta\}^n d\theta. \quad (34)$$

Equation (34) leads to the same result as (33) with $r=yN$.

It should be pointed out that although terms of the form s^n have been extracted from terms of the form s^m for m greater than n, when s^m is itself considered, no allowance has been made for the extracted terms. To this extent the approximations are of the same order as those made by Brockbank and Wass, and discussed in the text in Section 6, namely, the neglect of low-order products. But this is permissible here, since s^m with the low-order products removed is incoherent with $s^n (m>n)$, and since the interference terms of different orders are here treated as incoherent groups, the error of re-including the low-order products in s^m is at most of order $1/N$, an order of approximation that has been used elsewhere, for example in (34). But the inclusion of the low-order products of s^m in s^n is essential on account of their coherence. (Thus if $a\gg b$, b^2 can be neglected in the incoherent form a^2+b^2, but in the coherent form $(a+b)^2=a^2+2ab+b^2$, the additional term $2ab$ is not necessarily negligible. Brockbank and Wass treat all the low-order products in the first way.)

To find the in-phase component of the odd harmonics, we need only those terms of ${}^nA_r$, say ${}^nB_r$, that are formed with $(n+1)/2$ positive and $(n-1)/2$ negative terms. This gives

$$\begin{aligned}({}^nB_r)^2&=\alpha^n[{}^nC_{(n-1)/2}\{(n-1)/2\}!\{(n+1)/2\}!]^2\\ &\quad\times\alpha_{(n-1)/2}/[2\{(n-1)/2\}!\{(n+1)/2\}!]\\ &=\alpha^n(n!/2)\ {}^nC_{(n-1)/2}\alpha_{(n-1)/2}. \quad (35)\end{aligned}$$

$\alpha_{(n-1)/2}$ is the coefficient of x^r in

$$\left(\sum_1^N x^l\right)^{(n+1)/2}\left(\sum_1^N x^{-l}\right)^{(n-1)/2}.$$

Summing the geometrical progression, and making the same approximations as before, for large N, we get

$$\begin{aligned}\alpha_{(n-1)/2}&\approx\text{coefficient of } x^r \text{ in}\\ &\quad\left(\frac{1-x^N}{1-x}\right)^{(n+1)/2}\left(x^{-N}\frac{1-x^N}{1-x}\right)^{(n-1)/2}\\ &=\text{coefficient of } x^r \text{ in } \left(\frac{1-x^N}{1-x}\right)^n x^{-N(n-1)/2}\\ &=\text{coefficient of } x^r \text{ in } (1-x)^{-n}\\ &\quad\times\{x^{-N(n-1)/2}-{}^nC_1\,x^{-N(n-3)/2}+\ldots\}\\ &\simeq\frac{1}{(n-1)!}[\{\tfrac{1}{2}N(n-1)+r\}^{n-1}-\\ &\quad{}^nC_1\{\tfrac{1}{2}N(n-3)+r\}^{n-1}+\ldots],\end{aligned}$$

the series stopping at the term ${}^nC_{(n-1)/2}\,r^{n-1}$.

$$\begin{aligned}\therefore\quad {}^nB_r&\approx\alpha^{\frac{1}{2}n}\{\tfrac{1}{2}n\,.\,{}^nC_{\frac{1}{2}(n-1)}[\{\tfrac{1}{2}N(n-1)+r\}^{n-1}\\ &\quad-{}^nC_1\{\tfrac{1}{2}N(n-3)+r\}^{n-1}+\ldots]\}^{\frac{1}{2}}\\ &=\alpha^{\frac{1}{2}n}\{\tfrac{1}{2}n\,.\,{}^nC_{\frac{1}{2}(n-1)}2^{1-n}[\{Nn+(2r-N)\}^{n-1}\\ &\quad-{}^nC_1\{N(n-2)+(2r-N)\}^{n-1}+\ldots]\}^{\frac{1}{2}}.\end{aligned}$$

Comparing with (33), we see that ${}^nB_r$ is obtained from ${}^nA_r$ by multiplying by $({}^nC_{(n-1)/2}2^{1-n})^{\frac{1}{2}}$ and replacing r by $N-2r$ (or, alternatively, replacing y by $1-2y$). Thus instead of $h_3(y)=\frac{1}{2}(1-y^2/3)$ in (10) of Section 5 we have

$$\begin{aligned}\bar{h}_3(y)&={}^3C_1 2^{-2}.\tfrac{1}{2}\{1-(1-2y)^2/3\}\\ &=(1+2y-2y^2)/4.\end{aligned}$$

Similarly

$$\begin{aligned}\bar{h}_5(y)&={}^5C_2.2^{-4}\{46-12(1-2y)^2\\ &\quad+(6/5)(1-2y)^4\}/(24)^2\\ &=2(11+12y-6y^2-12y^3+6y^4)/(24)^2, \text{ etc.}\end{aligned}$$

When the in-phase components of interference $g_{2n+1}(x)$ have to be treated separately, the above functions $\bar{h}_{2n+1}(y)$ are used instead of $h_{2n+1}(y)$ in (10) of Section 5. The difference functions $h_{2n+1}(y)-\bar{h}_{2n+1}(y)$ must still be used to form a modified $g_{2n+1}(x)$ when the power-wise components of the interference are summed.

12.3 Approximation to $\phi(\tau x)$

Let

$$F(y,\lambda)=\int_0^\infty \cos y\theta(e^{\lambda \sin\theta/\theta}-1-\lambda\sin\theta/\theta)d\theta.$$
$$(\lambda=4\tau^2x^2.)$$

At the top and bottom of the video band, $y=1$ and 0 respectively.
We define

$$F_0=F(0,\lambda)=\int_0^\infty (e^{\lambda\sin\theta/\theta}-1-\lambda\sin\theta/\theta)d\theta, \quad (36)$$

$$F_1=F(1,\lambda)=\int_0^\infty \cos\theta(e^{\lambda\sin\theta/\theta}-1-\lambda\sin\theta/\theta)d\theta. \quad (37)$$

Integrating by parts, we get

$$\begin{aligned}F_0&=\theta(e^{\lambda\sin\theta/\theta}-1-\lambda\sin\theta/\theta)_0^\infty\\&\quad-\lambda\int_0^\infty \theta(\cos\theta/\theta-\sin\theta/\theta^2)(e^{\lambda\sin\theta/\theta}-1)d\theta\\&=\lambda\int_0^\infty\cos\theta(e^{\lambda\sin\theta/\theta}-1)d\theta\\&\qquad+\lambda\int_0^\infty\frac{\sin\theta}{\theta}(e^{\lambda\sin\theta/\theta}-1)d\theta\\&=-\lambda\int_0^\infty\cos\theta(e^{\lambda\sin\theta/\theta}-1-\lambda\sin\theta/\theta)d\theta\\&\quad-\lambda^2\int_0^\infty\frac{\cos\theta\sin\theta}{\theta}d\theta\\&\qquad+\lambda\frac{\partial}{\partial\lambda}\int_0^\infty(e^{\lambda\sin\theta/\theta}-1-\lambda\sin\theta/\theta)d\theta\\&=-\lambda F_1-\lambda^2\frac{\pi}{4}+\lambda\frac{\partial}{\partial\lambda}F_0\end{aligned}$$

$$\therefore\quad F_1=\frac{\partial F_0}{\partial\lambda}-\frac{F_0}{\lambda}-\lambda\frac{\pi}{4}=\lambda\frac{\partial}{\partial\lambda}(F_0/\lambda)-\lambda\frac{\pi}{4}. \quad (38)$$

This relation gives F_1 in a simple way when F_0 is found. To find F_0, expand (36) and integrate.

$$F_0=\sum_2^\infty\frac{\lambda^n}{n!}\int_0^\infty(\sin\theta/\theta)^n\,d\theta. \quad (39)$$

The following values are easily obtained (e.g., Whittaker and Watson, p. 123, example 13)—

$$\int_0^\infty(\sin\theta/\theta)^2d\theta=\frac{\pi}{2}$$
$$\int_0^\infty(\sin\theta/\theta)^3d\theta=\frac{\pi}{2}\cdot\frac{3}{4}$$
$$\int_0^\infty(\sin\theta/\theta)^4d\theta=\frac{\pi}{2}\cdot\frac{4}{6}$$
$$\int_0^\infty(\sin\theta/\theta)^5d\theta=\frac{\pi}{2}\cdot\frac{5}{8}\cdot\left(\frac{23}{24}\right)$$
$$\int_0^\infty(\sin\theta/\theta)^6d\theta=\frac{\pi}{2}\cdot\frac{6}{10}\cdot\left(\frac{11}{12}\right)$$
$$\int_0^\infty(\sin\theta/\theta)^8d\theta=\frac{\pi}{2}\cdot\frac{8}{14}\cdot\left(\frac{151}{180}\right),\text{ etc.}$$

From this we get the approximation

$$\int_0^\infty(\sin\theta/\theta)^nd\theta\approx\frac{\pi}{2}\frac{n}{2(n-1)},$$

which is exact to $n=4$, and still nearly correct at $n=8$. Thus a good approximation in the neighbourhood of the origin is

$$F_0\approx\frac{\pi}{4}\sum_2^\infty\frac{\lambda^n}{n!}\frac{n}{n-1}=\frac{\pi}{4}\lambda\sum_1^\infty\frac{\lambda^n}{n.n!}=\frac{\pi}{4}\lambda\int_0^\lambda\frac{e^\lambda-1}{\lambda}d\lambda.$$

Jahnke and Emde define (page 2)

$$\overline{Ei}(x)=\log\gamma x+\int_0^x(e^t-1)/t\ dt.$$

Hence

$$F_0\approx(\pi/4)\lambda\{\overline{Ei}(\lambda)-\log\gamma\lambda\}\ (\log\gamma=0{\cdot}577). \quad (40)$$

The probable limit of validity of (40) is when the eighth term of the exponential series becomes significant, which is about $\lambda=4$. Since $\lambda=4\tau^2x^2$ this gives $\tau x<1$ as the upper limit. From (38)

$$F_1=\frac{\pi}{4}\lambda(e^\lambda-1)/\lambda-\lambda\pi/4=\frac{\pi}{4}(e^\lambda-1-\lambda). \quad (41)$$

To get a form usable in the range $\lambda\gg4$, we need an asymptotic form for $\int_0^\infty(\sin\theta/\theta)^nd\theta$ for large n. Now when n is large, only the range near $\theta=0$ is of value in the integrand. The forms $e^{-n\theta^2/6}(1-n\theta^2/180)$ and $(\sin\theta/\theta)^n$ are the same up to powers of θ^4, and can be shown to give

$$\begin{aligned}\int_0^\infty(\sin\theta/\theta)^nd\theta&\approx\int_0^\infty e^{-n\theta^2/6}(1-n\theta^4/180)d\theta\\&=\frac{\pi}{2}\left(\frac{6}{\pi n}\right)^{\frac{1}{2}}\left(1-\frac{3n}{20}\right)\approx\frac{\pi}{2}\left\{\frac{6}{\pi(n+3/10)}\right\}^{\frac{1}{2}}.\end{aligned}$$

This last form is very good, even for so low a value as $n=2$. However, it cannot be used directly as the resulting series cannot be easily handled. Now

$$\int_0^{\pi/2} (\cos\theta)^r d\theta = \frac{1}{2}\frac{\Gamma\frac{1}{2}\Gamma(r+1)/2}{\Gamma(r+2)/2}.$$

The form taken when r is large is easily found by using the asymptotic form for the gamma functions. With r replaced by $n-1/5$ we get

$$\int_0^{\pi/2} (\cos\theta)^{n-1/5} d\theta \approx \frac{\pi}{2}\left\{\frac{2}{\pi(n+3/10)}\right\}^{\frac{1}{2}}.$$

Hence

$$\int_0^{\infty} (\sin\theta/\theta)^n d\theta \approx (3)^{\frac{1}{2}} \int_0^{\pi/2} (\cos\theta)^{n-1/5} d\theta.$$

Using this result, it is found that

$$F_0 \approx (3)^{\frac{1}{2}} \int_0^{\pi/2} (e^{\lambda\cos\theta} - 1 - \lambda\cos\theta)(\cos\theta)^{-1/5} d\theta. \quad (42)$$

The first term of the expansion of (41), which is the correct one, is equal to the first term of the expansion of (42) within 10 per cent. The succeeding terms in (42) rapidly approach their correct values.

Equation (42) could also have been obtained, albeit in a rather superficial manner (on account of the path of integration for ϕ), by the substitution $\sin\theta/\theta = \cos\phi$ directly in (36), and using series expansions of θ and $\cos\theta$ in terms of ϕ.

It remains to find the asymptotic form of (42). For large λ, only the term $e^{\lambda\cos\theta}$ is important. Expanding to the first few powers of θ, we get

$$F_0 \approx (3)^{\frac{1}{2}} \int_0^{\pi/2} e^{\lambda(1-\theta^2/2+\theta^4/24)}(1+\theta^2/10)d\theta$$

$$\approx (3)^{\frac{1}{2}} e^{\lambda} \int_0^{\pi/2} e^{-\lambda\theta^2/2}(1+\theta^2/10+\lambda\theta^4/24)d\theta.$$

Putting $\phi=\lambda\theta^2/2$, and taking the upper limit of ϕ to ∞, we find

$$F_0 \approx e^{\lambda}\{3/(2\lambda)\}^{\frac{1}{2}} \int_0^{\infty} e^{-\phi}\phi^{-\frac{1}{2}}\left(1+\frac{\phi}{5\lambda}+\frac{\phi^2}{6\lambda}\right)d\phi$$

$$= e^{\lambda}\{3\pi/(2\lambda)\}^{\frac{1}{2}}\{1+1/(10\lambda)+1/(8\lambda)\}$$

$$= e^{\lambda}\{3\pi/(2\lambda)\}^{\frac{1}{2}}\{1+9/(40\lambda)\}.$$

From (38) we find, for the asymptotic form of F_1

$$F_1 \approx e^{\lambda}\{3\pi/(2\lambda)\}^{\frac{1}{2}}(1-1{\cdot}275/\lambda).$$

This form is valid for values of λ greater than about 4; i.e., $\tau x>1$.

Recent Telecommunication Development

Miniature Toroidal Inductor

In these days of miniature components, an interesting development item is a high-Q toroidal inductor manufactured by Standard Telephones and Cables, Limited, which is shown in the accompanying photograph.

The core is pressed from radio-frequency iron powder as a single ring 0.106 inch (2.7 millimeters) thick with outside and inside diameters of 0.375 and 0.187 inch (9.5 and 4.7 millimeters). Its effective permeability is about 14.

An inductance of 27±0.5 microhenries with a Q of approximately 120 between 2 and 4 megacycles per second is achieved by the application of a single layer of size 38 Steel Wire Gage (0.0060-inch-diameter) enameled wire.

The coil is machine wound and can be produced with a tap taken from the center of the winding. Its weight is 0.82 gram.

Television Scanner for Slides

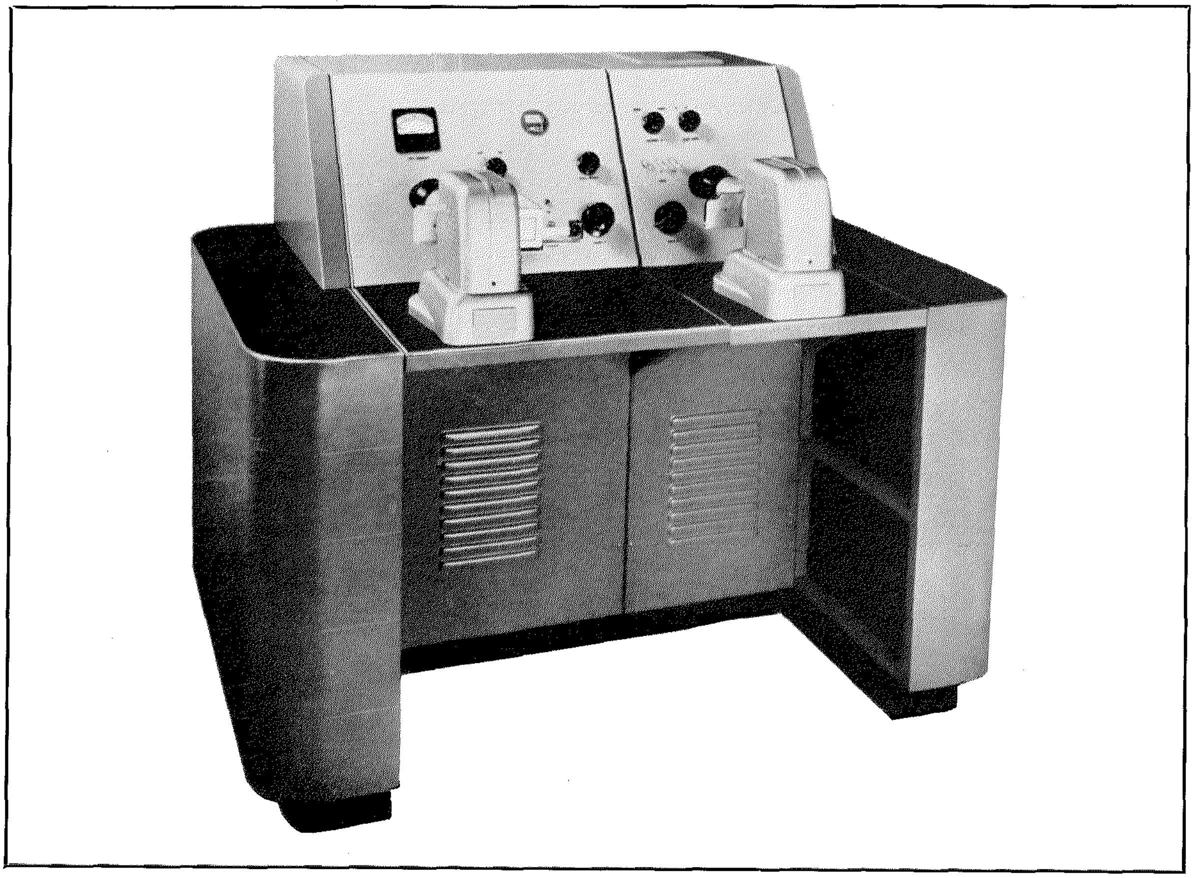
TELEVISION broadcasting stations frequently transmit test and station-identification patterns and other "still" material such as spot news announcements and messages from the sponsors of the programs. It is uneconomical to use the regular pick-up equipment for these transmissions. They can be recorded on standard 35-millimeter photographic film and converted into a corresponding electrical television signal by means of a compact consol-mounted unit recently developed by Federal Telecommunication Laboratories.

The basic unit will handle from 1 to 36 double-frame slides of the 2-inch-square size, which may be shown in or out of sequence. The signal is automatically blanked out while the slide is in motion.

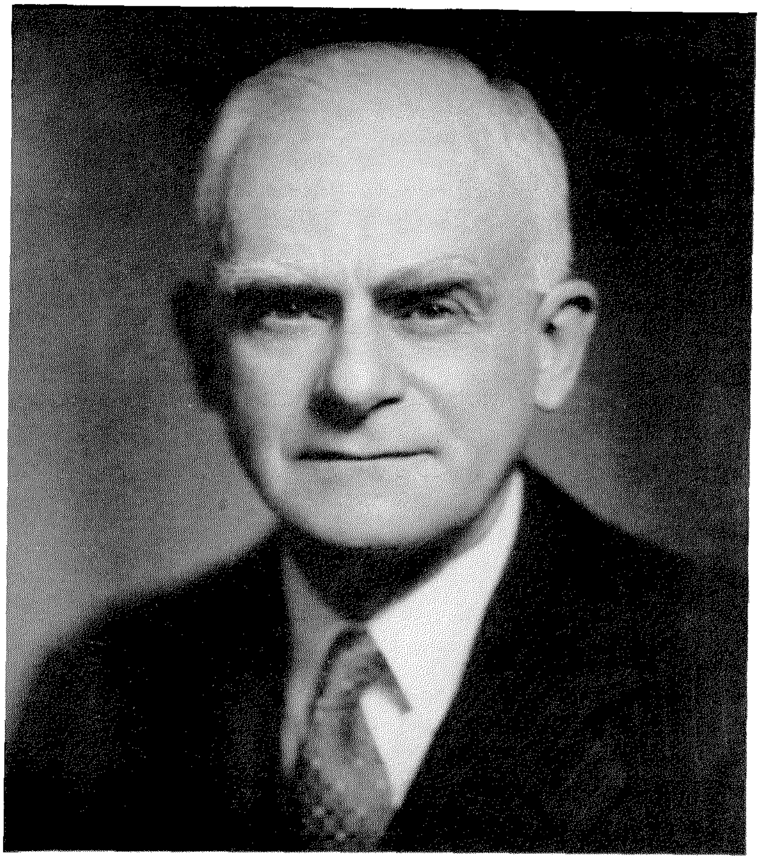
A second "projector" may be added and permits lap dissolves, fades, and other effects such as montages of two slides or of one slide and a "live" program. All types of superposition may be effected including the insertion of spot announcements into programs.

The equipment is based on the flying-spot type of scanning and produces a video-frequency signal having a horizontal resolution of at least 600 lines, a signal-to-noise ratio of not less than 36 decibels, and a contrast range of 30 to 1 or better. The geometric distortion will not exceed 2 percent. The output signal conforms to the television transmission standards of the United States.

Dual flying-spot scanner for developing television signals corresponding to the images on photographic-film slides.



In Memoriam



HERMAN T. KOHLHAAS

HERMAN T. KOHLHAAS was born at Brooklyn, New York, on December 28, 1882. He received the B.S. degree in electrical engineering from Cooper Union in 1907 and later was awarded engineering degrees by that institution and by Brooklyn Polytechnic Institute.

From 1905 to 1922, he served in the laboratories of the Western Electric Company as an engineer, section head, and finally as a division head in the physical laboratory. In 1922, he was appointed executive personnel assistant to the systems engineer of Bell Telephone Laboratories.

He was transferred to the Western Electric Company headquarters in 1924. The following year, he was assigned to statistical studies and the editing of *Electrical Communication* for the International Western Electric Company. Through the purchase of that company, he became a member of the staff of the International Telephone and Telegraphic Corporation.

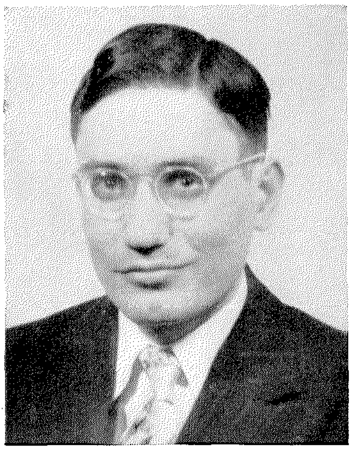
Mr. Kohlhaas edited *Electrical Communication* at London from 1937 through 1939 and, while there, served as publicity representative of the International Telephone and Telegraph Corporation and International Standard Electric Corporation.

He was appointed an assistant vice president of International Telephone and Telegraph Corporation in 1945, and retired at the end of 1947. He served as a consulting editor of the magazine for the following year.

He was a Fellow of the American Institute of Electrical Engineers, a Senior Member of the Institute of Radio Engineers, and a Member of the Norwegian Chamber of Commerce.

Mr. Kohlhaas died in an automobile accident that occurred in Florida on April 24, 1951.

Contributors to This Issue



WILLIAM DITE

WILLIAM DITE was born at New York City on March 2, 1919. He received the B.E.E. degree from the College of the City of New York in 1940.

From 1940 to 1943, he was with the Signal Corps Laboratories, Fort Monmouth, New Jersey, working on sound-ranging and radar equipment.

Since 1943, he has been associated with Federal Telecommunication Laboratories, Nutley, New Jersey, where his work has been largely concerned with communication systems employing pulses.

Mr. Dite is a member of the American Institute of Electrical Engineers.

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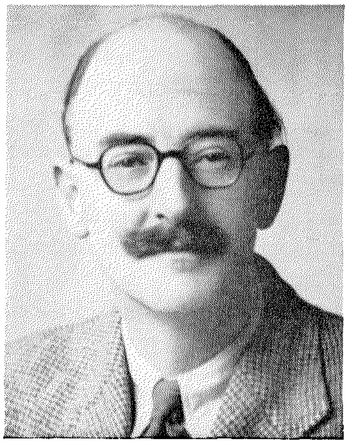

C. C. EAGLESFIELD

C. C. EAGLESFIELD was born on June 28, 1906, at Swansea, Wales. He received the B.A. degree in mechanical science from the University of Cambridge in 1928.

From 1929 to 1932, he was a research engineer in the laboratories of Standard Telephones and Cables in London and Paris. He then entered the research laboratories of the Mullard Radio Valve Company, where he specialized in the television and high-frequency fields. Since 1947, he has been in the valve division of Standard Telephones and Cables at Ilminster, where he is in charge of a group working on valve development.

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ROBERT W. HUGHES was born on March 21, 1922, at New York City. He received the B.S. degree in electrical engineering from Cornell University in 1943.

He served in the United States Signal Corps during the next three years and introduced the first Army pulse-time-modulation equipment in the European and Pacific theaters. From 1946 to 1948, he worked on radar plotting boards for Electronic Associates.

Since 1948, Mr. Hughes has been with Federal Telecommunication Laboratories working on pulse-time modulation and is now a project engineer.

Mr. Hughes is a Member of the Institute of Radio Engineers.

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LEONARD LEWIN was born in 1919 at Southend-on-Sea, England. He studied mathematics with particular reference to transcendental functions and the electromagnetic theory of radiation.

During the war, he did research work on antennas and mirrors at the British Admiralty and in 1945 served as chairman of the Inter-Service Committee on Radar Camouflage.

He joined the engineering staff of Standard Telecommunication Laboratories in London in 1946.

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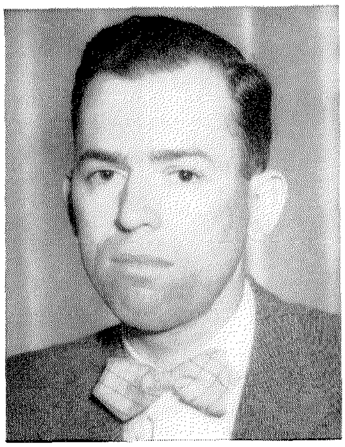
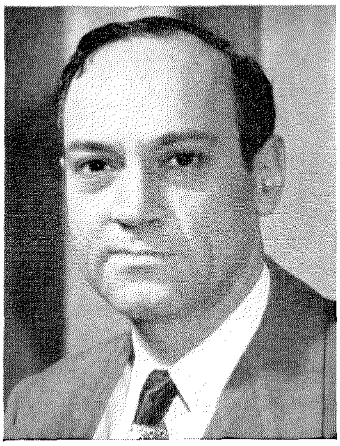


ROBERT W. HUGHES

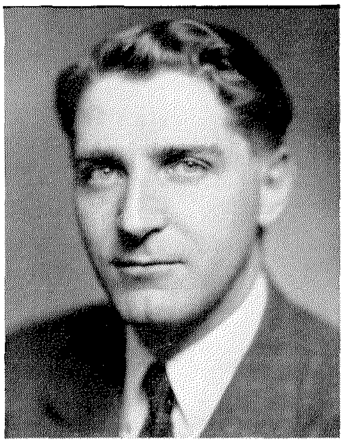
HORACE G. MILLER was born on July 16, 1907, at Kansas City, Kansas. He received a degree in electrical engineering from Kansas State College in 1936.

In 1928, he joined the radio engineering department of Westinghouse Electric Corporation, where he worked on television. During 1930, he was with Jenkins Television Corporation. From 1931 to 1936, he was engaged in the development of television and facsimile equipment for Radio Inventions. He returned to Kansas State College that summer and received his degree.

In 1936, Mr. Miller joined the television research department of Philco Corporation. In 1938, he went to A. B. DuMont Laboratories to design tele-



HORACE G. MILLER



ANATOLE MINC

vision receivers and wideband oscilloscopes. From 1940 to 1942, he was with Panoramic Radio working on panoramic frequency-scanning receivers.

Since 1942, Mr. Miller has been a project engineer at Federal Telecommunication Laboratories on the development of portable radar, missile controls, pulse-time modulation, and television systems.

Mr. Miller is a Senior Member of the Institute of Radio Engineers.

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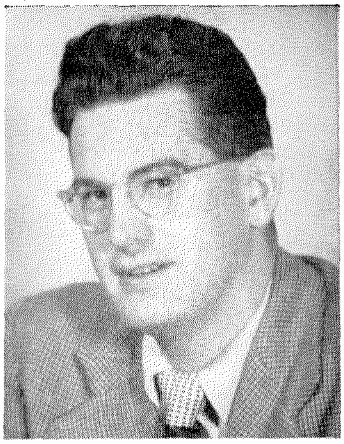
ANATOLE MINC was born on August 22, 1918 at Rostov-on-Don, Russia. He received the baccalaureate in philosophy from the University of Paris in 1937. After studying in several European universities, he came to America and was awarded a B.S. in electrical engineering by Columbia University in 1943. He served as a teaching assistant at Columbia during 1944.



SCOTT NEVIN

From 1944 to 1946, he was engaged in electronic work for the United States Navy. In 1946, he joined the engineering staff of Mackay Radio and Telegraph Company where he has been active in developing terminal equipment for radio and cable systems.

Mr. Minc is a Member of the Institute of Radio Engineers.

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SCOTT NEVIN was born on September 11, 1924, at Ithaca, New York. He received the B.S. degree in mechanical engineering from Cornell University in 1945.

He joined the Capehart–Farnsworth Corporation in 1946 and has worked on photocathodes and secondary-emission devices.

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PHILIP F. PANTER was born in 1908 in Poland. After early schooling in Tel-Aviv, Palestine, he later received from McGill University, Montreal, Canada, the following degrees: B.Sc. in 1933, B.Eng. in electrical engineering in 1935, and Ph.D. in physics in 1936. He continued research in spectroscopy at McGill for an additional year.

After teaching mathematics and physics in Palestine for a year, he returned to Canada as assistant professor of mathematics and physics in the evening division of Sir George Williams College in Montreal. He served also on the staff of the physics department of McGill University as instructor in physics and later as part-time lecturer, until the end of 1945.

Early in 1941, Dr. Panter joined the transmitter department of the Canadian Marconi Company in Montreal. In October, 1945, he was appointed senior engineer, responsible for the development of frequency-modulation broadcast equipment, at Federal Telephone and Radio Corporation. He later transferred to Federal Telecommunication Laboratories and is now in charge of the theoretical group of the communications division.

Dr. Panter is a member of the Institute of Radio Engineers and the Radio Club of America.

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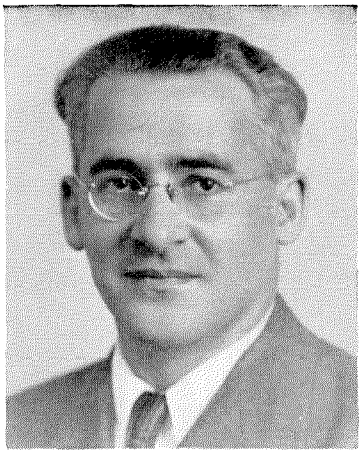

P. F. PANTER

W C. PETERMAN was born on March 28, 1889 at Linfield, Pennsylvania. He received the B.S. degree in electrical engineering in 1911 from Lehigh University.

From 1912 to 1928, he was engaged in ocean-cable development for Western Union Telegraph Company. He continued in this field for All America Cables and Radio and other associates of the International Telephone and Telegraph Corporation until his retirement in 1950. He holds a number of patents including that on the rotary regenerative repeater for automatic telegraphy.

Mr. Peterman is a Member of the American Institute of Electrical Engineers and of the Franklin Institute.

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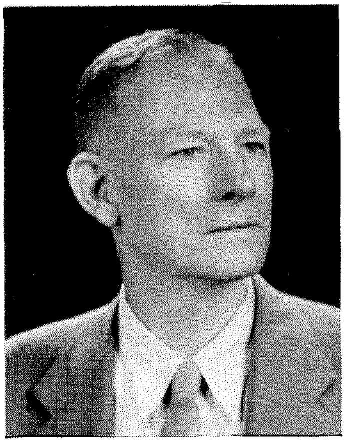
W. C. PETERMAN



George T. Royden

George Taylor Royden was born at Fort Clark, an army post in Texas, on June 20, 1895. He received a B.A. degree in 1917 and an engineering degree in 1924 from Stanford University.

He was employed by Federal Telegraph Company at Palo Alto, California, on part time in 1916 and full time after graduation from college. He was engaged in the design of arc transmitters of powers up to 1000 kilowatts.

From 1919 until 1925 he was at Mare Island Navy Yard. His duties included work on Navy radio stations in San Diego, Hawaii, and Alaska.

He returned to Federal Telegraph Company in 1925 to do research work on broadcast receivers for operation on alternating current.

In 1927, he joined the newly organized Mackay Radio and Telegraph Company becoming division engineer. From 1936 to 1946, he was with Federal Telegraph Company in Newark, New Jersey. He then returned to Mackay Radio and Telegraph Company.

Mr. Royden is a Fellow of the Institute of Radio Engineers and a Member of the American Institute of Electrical Engineers and of Sigma Xi.

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Hans Salinger was born at Berlin, Germany, on April 1, 1891. He received the Ph.D. degree in 1915 from the University of Berlin.

From 1919 to 1925, he was a research associate at the Reichspostzentralamt and from 1929 to 1935, a professor at the Heinrich Hertz Institute, both in Berlin.

In 1936, he joined Farnsworth Television, now Capehart–Farnsworth Corporation, where he is engaged in research work. He is also doing part-time teaching at Purdue University and has published numerous technical papers.

Dr. Salinger is a Fellow of the Institute of Radio Engineers and a member of the American Physical Society, American Association for the Advancement of Science, and Sigma Xi.

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Leo Staschover was born on May 8, 1922, at Saarbrücken, Saar Territory. He received the B.S. degree in electrical engineering in 1943 from the College of the City of New York and the M.S. degree from Polytechnic Institute of Brooklyn in 1948.

Hans Salinger

Nelson Weintraub

From 1943 to 1946, he was a research assistant at Polytechnic Institute of Brooklyn, working on microwaves and teaching electrical engineering.

In 1946, he joined Federal Telecommunication Laboratories becoming a senior engineer and was engaged in the development of radio links for communication, frequency modulation, and television applications. In 1951, he entered the television department of Paramount Pictures Corporation.

Mr. Staschover is a member of Tau Beta Pi, Eta Kappa Nu, and Sigma Xi, and an Associate of the Institute of Radio Engineers.

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Nelson Weintraub was born at New York City on April 9, 1923. He received a B.S. degree in electrical engineering from the College of the City of New York in 1944 and a master's degree from Polytechnic Institute of Brooklyn in 1946.

He has been with Federal Telecommunication Laboratories since 1944 and has worked on color television. At present he is a senior engineer on pulse-time-modulation systems.

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INTERNATIONAL TELEPHONE AND TELEGRAPH CORPORATION

Associate Manufacturing and Sales Companies

United States of America

International Standard Electric Corporation, New York, New York

Federal Telephone and Radio Corporation, Clifton, New Jersey

International Standard Trading Corporation, New York, New York

Capehart-Farnsworth Corporation, Fort Wayne, Indiana

Flora Cabinet Company, Inc., Flora, Indiana

Thomasville Furniture Corporation, Thomasville, North Carolina

Great Britain and Dominions

Standard Telephones and Cables, Limited, London, England

Creed and Company, Limited, Croydon, England

International Marine Radio Company Limited, Croydon, England

Kolster-Brandes Limited, Sidcup, England

Standard Telephones and Cables Pty. Limited, Sydney, Australia

Silovac Electrical Products Pty. Limited, Sydney, Australia

Austral Standard Cables Pty. Limited, Melbourne, Australia

New Zealand Electric Totalisators Limited, Wellington, New Zealand

Federal Electric Manufacturing Company, Ltd., Montreal, Canada

South America

Compañía Standard Electric Argentina, Sociedad Anónima, Industrial y Comercial, Buenos Aires, Argentina

Standard Electrica, S.A., Rio de Janeiro, Brazil

Compañía Standard Electric, S.A.C., Santiago, Chile

Europe and Far East

Vereinigte Telephon- und Telegraphenfabriks Aktiengesellschaft Czeija, Nissl & Co., Vienna, Austria

Bell Telephone Manufacturing Company, Antwerp, Belgium

China Electric Company, Limited, Shanghai, China

Standard Electric Aktieselskab, Copenhagen, Denmark

Compagnie Générale de Constructions Téléphoniques, Paris, France

Le Matériel Téléphonique, Paris, France

Les Téléimprimeurs, Paris, France

C. Lorenz, A.G. and Subsidiaries, Stuttgart, Germany

Mix & Genest Aktiengesellschaft and Subsidiaries, Stuttgart, Germany

Süddeutsche Apparatefabrik Gesellschaft m.b.H., Nuremberg, Germany

Nederlandsche Standard Electric Maatschappij N.V., The Hague, Netherlands

Fabbrica Apparecchiature per Comunicazioni Elettriche, Milan, Italy

Standard Telefon og Kabelfabrik A/S, Oslo, Norway

Standard Electrica, Lisbon, Portugal

Compañía Radio Aérea Marítima Española, Madrid, Spain

Standard Electrica, S.A., Madrid, Spain

Aktiebolaget Standard Radiofabrik, Stockholm, Sweden

Standard Telephone et Radio S.A., Zurich, Switzerland

Telephone Operating Systems

Compañía Telefónica Argentina, Buenos Aires, Argentina

Compañía Telegráfico-Telefónica Comercial, Buenos Aires, Argentina

Compañía Telegráfico-Telefónica del Plata, Buenos Aires, Argentina

Companhia Telefônica Nacional, Porto Alegre, Brazil

Compañía de Teléfonos de Chile, Santiago, Chile

Compañía Telefónica de Magallanes S.A., Punta Arenas, Chile

Cuban American Telephone and Telegraph Company, Havana, Cuba

Cuban Telephone Company, Havana, Cuba

Compañía Peruana de Teléfonos Limitada, Lima, Peru

Porto Rico Telephone Company, San Juan, Puerto Rico

Shanghai Telephone Company, Federal Inc. U.S.A., Shanghai, China

Radiotelephone and Radiotelegraph Operating Companies

Compañía Internacional de Radio, Buenos Aires, Argentina

Compañía Internacional de Radio Boliviana, La Paz, Bolivia

Companhia Radio Internacional do Brasil, Rio de Janeiro, Brazil

Compañía Internacional de Radio, S.A., Santiago, Chile

Radio Corporation of Cuba, Havana, Cuba

Radio Corporation of Porto Rico, San Juan, Puerto Rico

Cable and Radiotelegraph Operating Companies

(Controlled by American Cable & Radio Corporation, New York, New York)

The Commercial Cable Company, New York, New York[1]

Mackay Radio and Telegraph Company, New York, New York[2]

All America Cables and Radio, Inc., New York, New York[3]

Sociedad Anónima Radio Argentina, Buenos Aires, Argentina[4]

[1] Cable service. [2] International and marine radiotelegraph services.
[3] Cable and radiotelegraph services. [4] Radiotelegraph service.

Laboratories

Federal Telecommunication Laboratories, Inc., Nutley, New Jersey

International Telecommunication Laboratories, Inc., New York, New York

Laboratoire Central de Télécommunications, Paris, France

Standard Telecommunication Laboratories, Limited, London, England