ELECTRICAL COMMUNICATION

Technical Journal of the
International Telephone and Telegraph Corporation
and Associate Companies

SPEECH-INPUT EQUIPMENT OF BRUSSELS BROADCASTING HOUSE

MONITOR FOR FREQUENCY-MODULATION BROADCASTING

MEDIUM-POWER MULTICHANNEL COMMUNICATION TRANSMITTERS

MODERNIZATION OF INTERNATIONAL TELEPHONE SERVICE

SHUNTED-AMPLIFIER INPUT ADMITTANCE

PHYSICAL PROPERTIES OF CRYSTALS AND THEIR SYMMETRY

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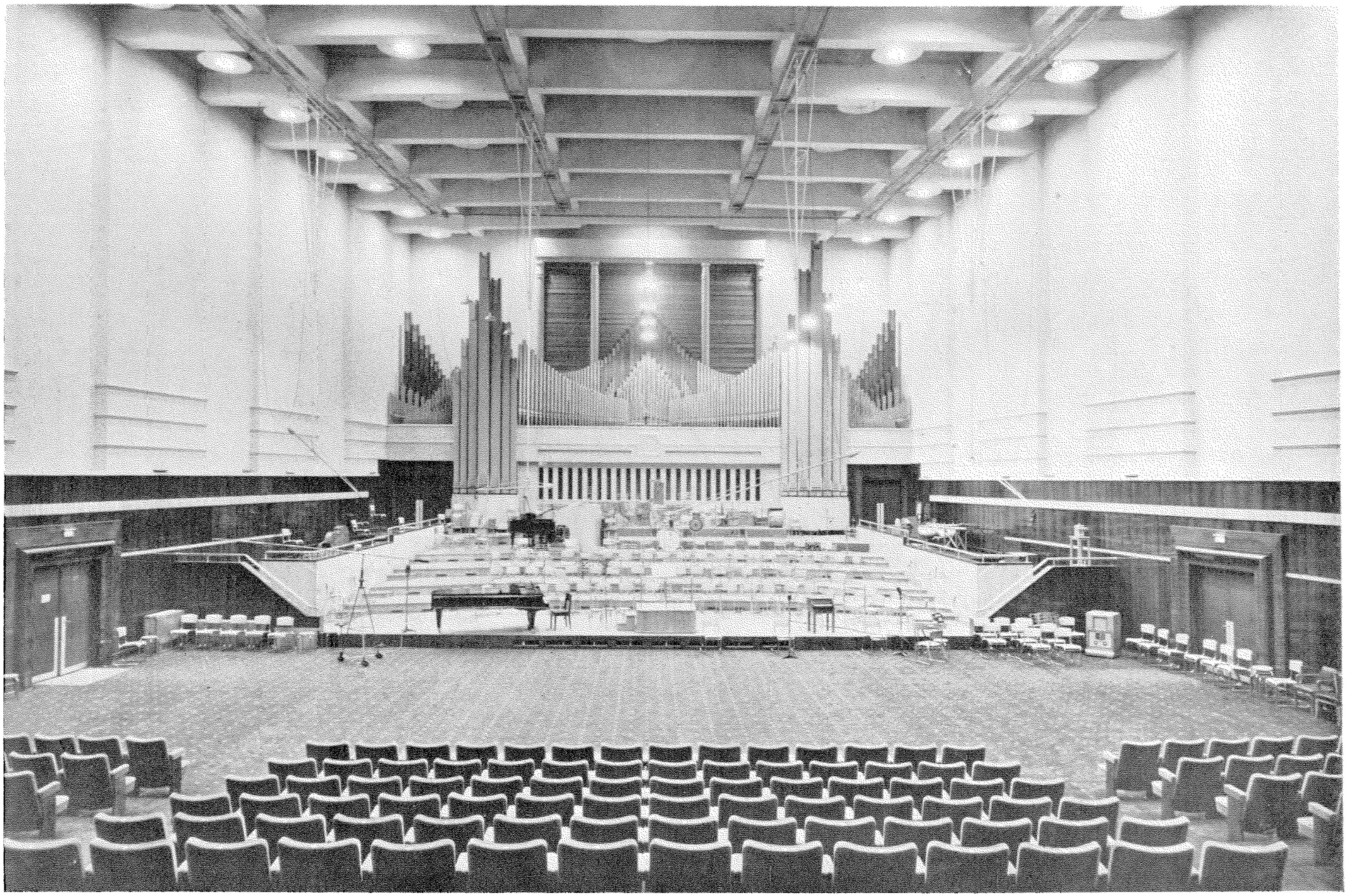


The great concert hall in Broadcasting House of the Institut National Belge de Radiodiffusion in Brussels, Belgium.

Speech-Input Equipment of Brussels Broadcasting House

By FERNAND MORTIAUX

Institut National Belge de Radiodiffusion, Brussels, Belgium

SHORTLY before the start of World War II, the broadcasting headquarters of the Institut National Belge de Radiodiffusion were completed in Brussels. A new building, shown in Fig. 1, provides for 19 studios, 5 recording rooms, and all of the communication, signaling, and supervisory accessories essential to a modern broadcasting system. A view of the concert hall is shown in the frontispiece.

This installation provides for two simultaneous main programs and one colonial program; each main channel supplies about 4500 hours of broadcasting time per year. A studio includes an associated announcing room and a control room from which all studio activity is visible. All announcements are made audible in the studio and all studio material is reproduced in the announcing room. The control room personnel, of course, hear everything.

Each channel, which comprises all equipment from the microphone or pickup to the outgoing line to the radio transmitter, is flat within ±2 decibels over a frequency band from 30 to 14,000 cycles per second, has less than 1-percent harmonic distortion, and has minimum phase distortion. Noise of all types, including that from vacuum tubes, resistance thermal agitation, and disturbances from alternating-current supply circuits as a result of inadequate filtering or induction, is at least 80 decibels below a reference level of 1.55 volts in a 40-ohm circuit, which corresponds to 100-percent modulation of the radio transmitter.

The number of studios is determined not only by the programs being broadcast but by the time needed for rehearsals. This usually amounts to three or four times the actual broadcasting requirement and, to provide two national and one colonial program, the studio facilities are often engaged up to 80 percent of their maximum possible utilization.

Smaller installations have found it convenient to centralize all technical equipment in a single room. Such an arrangement would have led to considerable complexity, and continuity of operation, flexibility, and supervision would have been extremely difficult. The decentralized system adopted groups the amplifiers and control equipment near the studio with which they are associated. The switching means for connecting the various studio outputs to the radio transmitter, recorders, or other terminals, are concentrated in separate rooms to which only the broadcasting supervisors have access. This decentralization introduces a certain amount of automatic switching to enable the required connections to be set up without damaging delay.

A studio, together with its announcing and control rooms and all necessary technical equipment to supply an outgoing signal at the reference level corresponding to full modulation of the radio transmitter, is designated as an "alpha." It is a complete source of program material. Alphas designed for speech only do not have announcing rooms and those for reproducing recorded material have no studios. An alpha is under the supervision of a single operator who is responsible for its technical performance, including the outgoing volume level. There are 24 alphas in the installation.

A "lambda" is a destination that receives the outgoing signal from an alpha. The 10 lambdas consist of three lines to radio transmitters, five recording equipments, and two general mixing desks. The lambdas are controlled by the broadcast supervisor who is responsible for technical quality, volume control, and continuity of transmission. Switching means are provided whereby each alpha may be connected to any one lambda.

All incoming and outgoing telephone lines terminate in a room called the "lines alpha." All outside programs, including relays of foreign broadcasts, are distributed from the lines alpha. Similarly, programs from within the building, intended for retransmission abroad, leave from this room. All outgoing lines from alphas and lambdas are brought out on jacks on a central switchboard. Lines alpha contains a manual telephone exchange as well.

The automatic program distribution service permits any "subscriber" inside the building to listen to any alpha by plugging in headphones or an amplifier and loudspeaker and dialing the designated number.

Fig. 1—Broadcasting House of Institut National Belge de Radiodiffusion in Brussels.

1. Alpha

A studio alpha is a basic source of program material and consists of a studio and its associated announcing and control rooms. It may obtain program material from four different sources: the studio, the announcing room, a circuit from one of the recording rooms, and a transmission from outside the alpha. This fourth source may be the retransmission of a foreign program, a program originating outside the building (outside broadcast), material transmitted from any other alpha, or a time signal. These four sources are connected to preamplifiers, mixed, and then passed through a second amplifier, which is called the line amplifier, for transmission to the lambdas at reference level.

A technician and an operating musician who has charge of microphone placement and general acoustic balance of the program are located in the control room. They may listen to the program either from a loudspeaker or headphones. The producer and announcer will be in the announcing room and will hear the program from a loudspeaker. Automatic switching is provided to silence a speaker when the microphone in the same room is in service. The control desk is illustrated in Fig. 2.

In monitoring, the volume variation at the microphone, which may reach 80 decibels or so, is reduced to approximately 40 decibels for transmission to the lambdas.

The Comité Consultatif International Téléphonique recommends a limit of 1.55 root-mean-square volts for transmitting program material over a telephone line. This value has been adopted as reference level and will modulate the radio transmitters fully.

The minimum voltage is governed by the noise generated in the transmitter and is about 60 decibels below the 100-percent-modulation level. The noise level from the alpha is reduced to more than 80 decibels below reference level. Modern amplifiers permit this low noise level even with alternating-current operation of vacuum-tube heaters. The value of this wide volume range is evident when it is remembered that a large symphonic orchestra may produce a sound level varying over a range of 60 to 100 decibels.

The characteristic impedance of all program transmission lines was standardized at 40 ohms. This low impedance makes the line quite susceptible to induced magnetic disturbances but reduces the effects of capacitance, thus permitting a wide frequency band to be transmitted on some rather long circuits within the large building.

1.1 VOLUME CONTROLS

The microphone volume controls are of the symmetrical series type to minimize losses. Because of the small electroacoustic efficiency of the microphones, the load resistance may drop below the internal resistance without producing harmonic distortion. This control has an attenuation of 2 decibels for each of 30 steps and a maximum of 110 decibels in the "out" position.

The mixing operation for the studio microphones is the only one done at low level. Experience had shown that contact and induced noises would not be damaging if the microphone lines were reasonably short and were separated effectively from power lines. Low-level mixing avoids the use of an amplifier for each microphone, thus simplifying operation and maintenance.

The amplifier volume controls are of the *L* type and are connected in series to permit fading in and out of programs from different sources, such as studio and announcing microphones and reproducing pickups. The load resistance at the output of the amplifier is higher than its output impedance. The attenuation steps are the same as for the microphone control.

The third volume control in the channel is the master or main control. It is of the ladder type, but the resistance normally connected in series with the rotating arm has been omitted. Although this affects the constancy of impedance with attenuation, it reduces the insertion loss to a minimum. Starting with the minimum attenuation position, the first 25 steps are of 2 decibels each, the next four steps give, progressively, 53, 58, 67, and 84 decibels, and the off position, 100 decibels of attenuation.

1.2 RECORDED MATERIAL

Turntables and pickups for reproducing recorded material are located in the control room and may be seen in Fig. 3. The output of the pickup passes through a 40-ohm volume control of the *T* type and an equalizing network. Low-pass filters cutting off at either 4800 or 6000 cycles may be switched into the circuit for noise suppression. These program sources are particularly useful for sound effects and incidental music in plays and, of course, for completely recorded musical programs.

A synchronous motor operating at 1000 revolutions per minute drives a turntable weighing approximately 11 kilograms (24 pounds) through a speed reducer and a planet gear. An oil buffer is located immediately beneath the turntable to dampen vibrations from the drive system. The

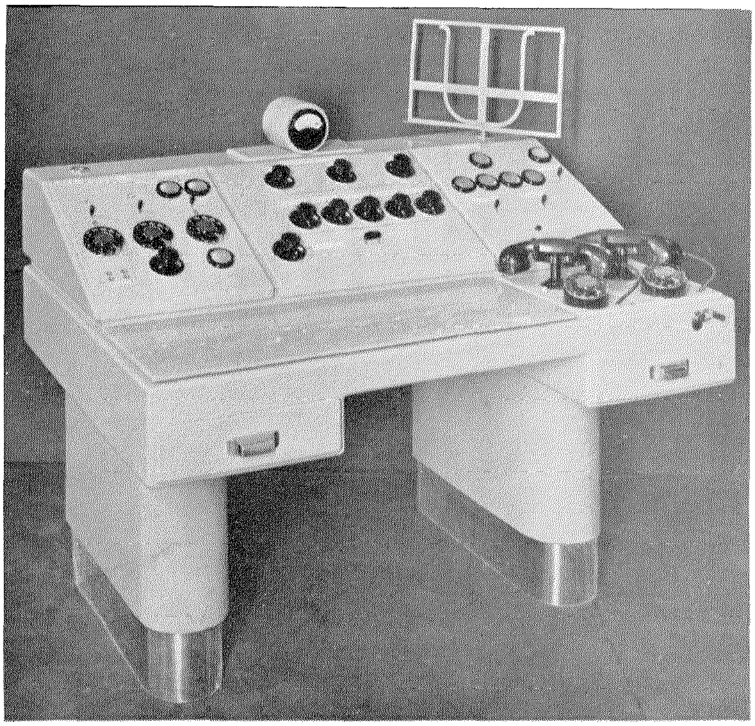


Fig. 2—Alpha control desk.

instantaneous variations in turntable speed are less than 0.5 percent. It is possible to shift the relative position of the turntable and spindle under operating conditions to synchronize two records and obtain a perfectly smooth transition from one to the other. Thus, lengthy orchestral selections can be transmitted without interruption.

The pickup arm has large inertia and is balanced by a counterweight. An indexing device enables the needle to be set in any predetermined groove on the record.

1.3 TALK-BACK

The microphones in either the control or announcing rooms may be connected to a loudspeaker in the studio, the necessary switching being provided to disconnect the studio microphone. This permits the studio personnel to hear the announcements.

The use of the microphone in the control room permits instructions to be given to the performers in the studio, which is particularly valuable during rehearsals.

In conjunction with the automatic program distribution service, talk-back from elsewhere in the building is a particularly important facility when several studios are involved in a single broadcast.

1.4 SIGNALING

Signal lights are installed in the different units of an alpha to facilitate operation and reduce the possibilities of errors. The fading-in of a studio microphone automatically energizes the corresponding signal lights through auxiliary contacts on the volume control. These contacts also control those loudspeakers that must be silenced to avoid singing. Special keys are provided to disable these auxiliary controls under special circumstances. To keep a permanent check on the more important signal lamps, a small pilot lamp, mounted near the control, is in series with the main lamp.

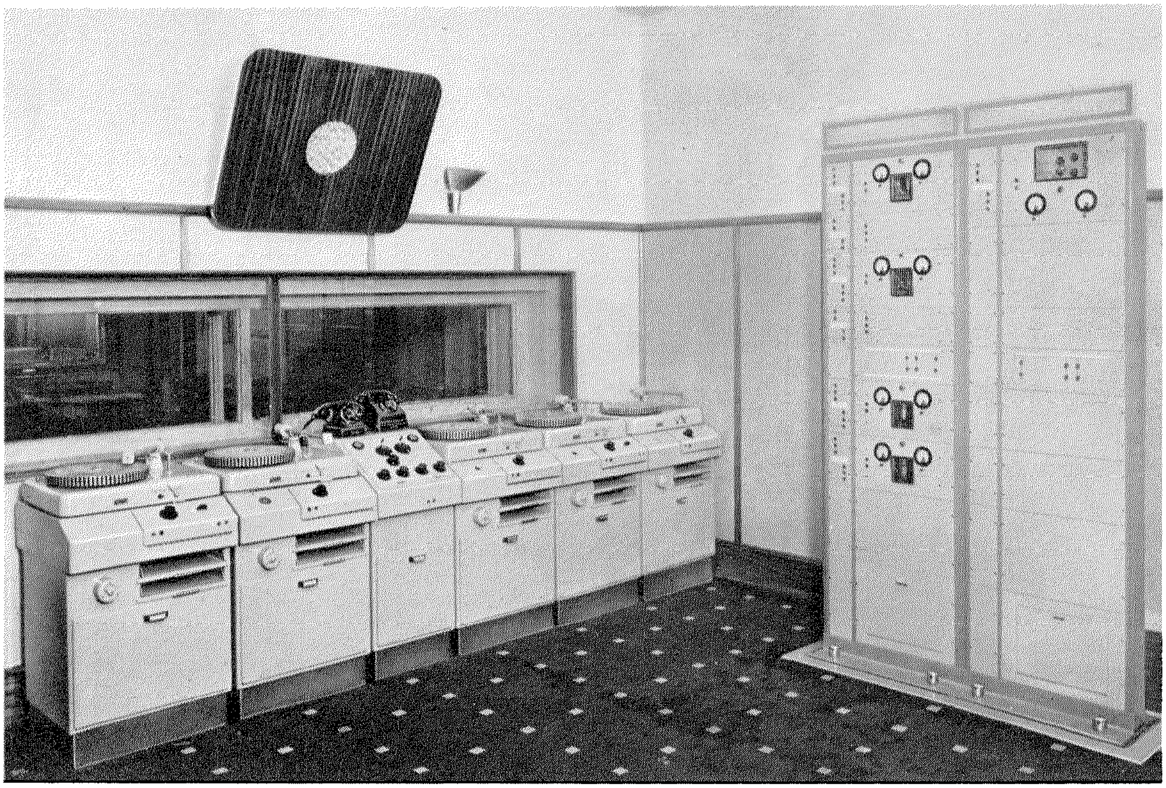


Fig. 3—Turntable desks and equipment racks in an alpha control room.

1.5 ALPHA-LAMBDA SIGNALING AND INTERCONNECTION

An automatic switching system, normally referred to as "antenna selection" or "lambda selection," is provided to permit rapid connection of any alpha to any lambda.

The alpha technician depresses a key, which sets up a connection to the antenna-selection equipment and operates an indicator lamp. A standard telephone dial is then manipulated to send impulses to a five-level selector of the step-by-step type, individual to each alpha. At the end of this operation, the selector is in the position corresponding to the required lambda. A green light then appears on the control desk of the alpha and on the corresponding panel of the lambda.

If the desired lambda is free, the branching relays operate immediately and this is indicated by red lights in the alpha and lambda. If, however, the lambda is already engaged by an alpha, a vertical locking device on the switchboard prevents the branching of the new alpha. This is a waiting position and is indicated by a green light in the alpha and lambda. A third condition is when the lambda is engaged and another alpha is in the waiting position. Then, a new alpha and all following alphas will receive a flashing green signal.

In this latter case, the broadcast supervisor in the lambda can intervene and by means of a release key any alpha may be disconnected temporarily. It is very exceptional to have more than two alphas in the waiting position.

To inform the technician of an alpha that his demand for a particular lambda has been correctly transmitted, the step-by-step selector switch is connected to the program multiple of the program distribution system. Thus, dialing of a desired lambda operates a selector in the program distribution room and lights a white signal on the control desk in the alpha when the latter is connected to the lambda. This indicates unmistakably that the antenna selection has been correctly made and transmitted.

To disconnect an alpha from a lambda, the alpha technician returns the "antenna selection" key to its original position. A waiting alpha is then automatically connected to the lambda and its technician is informed by the green signal changing to red.

1.6 CABLING

For transmission purposes, each alpha is linked to 10 pairs of wires. One pair is the normal outgoing line, a second pair operates a program meter, three trunk pairs go to the lines alpha, four pairs are for the automatic program distribution service, and the final pair is for talk-back. The signaling circuits are contained in a cable consisting of one quad and 15 pairs. These cables build up what is called the "multiple" of the alphas in the lambdas and terminate in the lines alpha.

2. Lambda

The lambdas have branch relays for connection to the alphas. Equipment is also provided to permit the broadcast supervisor to monitor the program being transmitted.

The channel from an alpha terminates on a switchboard in each of the lambdas. A second line, connected to the output of the program meter of the alpha, also terminates on the general switchboard of the lambdas. This results in a large switchboard of the crossbar type, connections being made at the intersections of horizontal lines representing the alphas and vertical lines representing the lambdas. Contact is established by two special branching relays which are operated automatically by a call from an alpha if the lambda is free.

The flexibility of automatic branching was considered necessary to insure rapid switching to a series of different alphas which might be required for the transmission of certain programs by a given lambda.

As a precaution against failure, the single relay in the transmission channel proper may be bridged by hand.

To avoid the generation of noise, each contact spring of the branching relays is equipped with two independent contact points in parallel. In addition, these contacts are "wetted" by a special circuit arrangement that automatically sends a very weak direct current through the contacts as they close. This current is gradually decreased before the relay releases, thus eliminating any electrical disturbances from its sudden interruption.

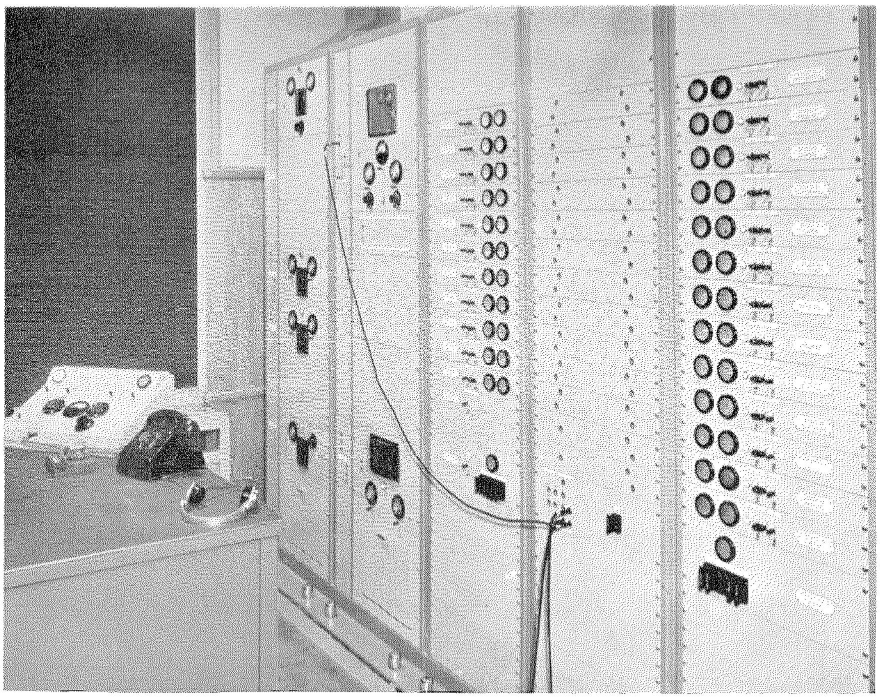


Fig. 4—Transmitter lambda. The rack at the left is for signaling apparatus and the next rack contains transmission equipment. The remaining three racks are for the alphas, each of which uses a full panel and the adjacent two jacks, which connect to the transmission channel and the program meter.

2.1 Transmitter Lambda

To permit the broadcast supervisor to check the technical performance of a broadcast channel and radio transmitter, a high-quality fixed-tuned receiver is provided. A program meter bridged across the output of the receiver permits ready comparison with the corresponding meter from the alpha originating the program.

Fig. 4 shows the equipment racks of the transmitter lambda; the broadcast supervisor's control box is on the desk. The indicating instrument is connected to the program meter of the alpha in service. An orange lamp at the left indicates that the oscillator for transmitting the time signal is in operation and the time signal will be sent within three minutes. The time signal is transmitted automatically unless the supervisor operates a disabling switch.

2.2 Alpha-Lambda for Recording

Recording has assumed a role of very great importance in modern broadcasting. The decentralized recording rooms have the same general access to lambdas for originating programs as do the alphas. Conversely, the recorder may take the place of a lambda transmitter and all alphas have access to the recorders.

As originally installed, there were five recording rooms. Two employed steel tape or "magnetophone" equipment, one recorded on cellulose-varnish discs, and another used wax masters. There was one spare room. This installation was seriously damaged during the war and is in the process of being rebuilt.

2.3 Alpha-Lambda for Mixing

Two large mixing desks are used for complex broadcasts in which a large number of studios or other originating sources must be linked together or the outputs from which must be mixed. Their automatic switches differ from those in the lambdas in the absence of vertical locking devices. This enables the simultaneous branching of all alphas. In addition to having lines to all the alphas in the building, there are 15 junction lines for handling outside broadcasts received through the lines alpha.

On dialing, an alpha is connected immediately through its branching relay to the mixing lambda and receives confirmation of this through a green signal light. The alphas connected to the mixing lambda are automatically indicated by their numbers appearing in lights on a number board in the mixing lambda as shown in Fig. 5.

The connection from an alpha terminates at a jack on the mixing desk. The operator connects a mixing volume control to each of these jacks and gives a starting signal to the alpha desired simply by turning up the mixer control. This operation illuminates the red lamp in the alpha and changes the green signal on the number board to red.

Before operating the mixer volume control on a given alpha, the operator can listen in on that circuit by plugging his headphones into a jack and pressing a key located next to the volume control involved. This greatly facilitates the operation of the system and prevents errors.

This same system is used in mixing a program from an outside source. For convenience, the signals are also reproduced at the lines alpha from where an operator issues instructions by telephone to the remote point.

3. *Lines Alpha*

The automatic switching system is limited to transmission channels completely within the building. Programs originating outside the building must be received on manual equipment, which is concentrated in the lines alpha. For convenience, the automatic telephone switchboard for the public telephone system is also housed in this room.

To enable all possible interconnections between outside sources and the alphas and lambdas, the output circuits of all alphas and a large number of interconnecting trunks are brought into the lines alpha.

Certain auxiliary facilities are also located in this room and include equipment for receiving foreign programs, time-signal equipment, isolating amplifiers, and oscillators for checking program meters. A view of one section of lines alpha is shown in Fig. 6, the entire installation encompassing about 35 racks of equipment.

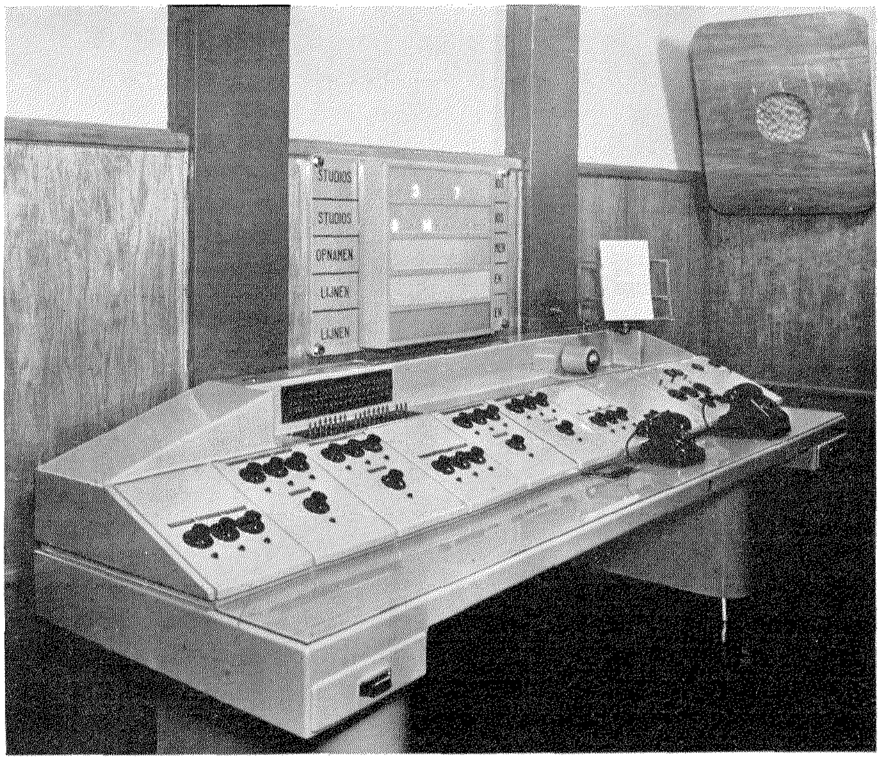


Fig. 5—Mixing-lambda desk. The number board indicates the connected alphas in green and the alpha in service in red.

3.1 Broadcast Receiving Sets

Seven broadcast receiving sets are maintained in the lines alpha. Two of these are for high-frequency reception. A common antenna is used and consists of a single wire suspended about 15 meters (50 feet) above the roof of the building. It is connected to a broad-band amplifier, the output of which supplies signals to the seven receivers and also to the fixed-tuned receivers for monitoring the broadcast radio transmitters. The output of each receiver passes through two band-eliminating filters, which are particularly effective in suppressing certain types of interference.

The output of the receivers can be connected to the automatic program distribution system.

3.2 Time Signals

Three equipments for time signals are provided.

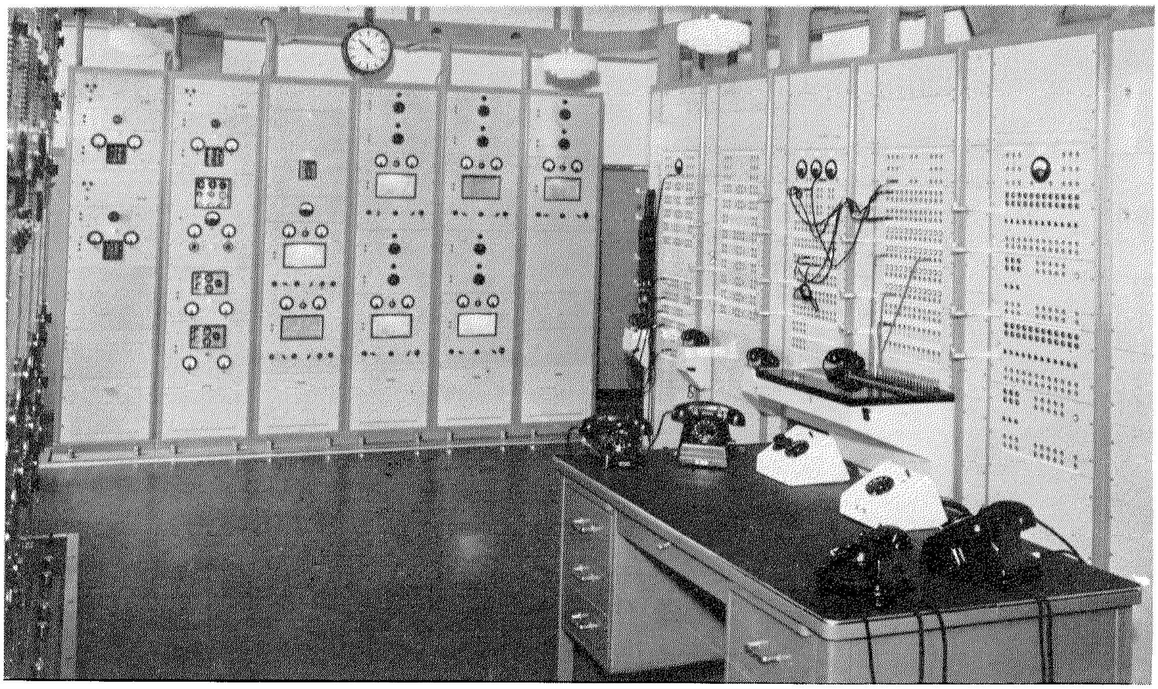


Fig. 6—Part of lines-alpha room.

These are of the "music box" type and are permanently in operation.

Being connected to the program distribution system, their output may be obtained from the listening headphone jack in each alpha. For broadcasting, the technician of the alpha connects the input of an amplifier to the proper jack and so inserts the signal into the channel.

3.3 Isolation Amplifiers

Isolation amplifiers produce no gain but are employed to prevent disturbances in circuits connected to their outputs from affecting the input circuits. About 30 of these are in constant use. For instance, all signals supplied to the program distribution system are passed through isolation amplifiers.

3.4 Program Meters

Five program meters are distributed throughout the lines alpha room to permit the program level on any transmission line to be measured. Each is provided with several indicating instruments. As each meter must be checked before it is used, two oscillators have been provided which have an output of exactly 1.55 root-mean-square volts at 1000 cycles.

4. *Amplifiers, Program Meter, and Line Equalizer*

4.1 AB Amplifier

The *AB* amplifier is the basic element of the speech-input installation. It consists of two push-pull resistance-capacitance-coupled stages employing triode AC2 vacuum tubes. The fine characteristics of this amplifier are a result of the care given to the individual components and the over-all design.

4.1.1 Noise

There are three principal causes of noise in the first stage of an amplifier. One of these, Johnson noise, is the result of thermal agitation of the electrons in the input resistance of the amplifier. The theoretical value of this noise voltage is equivalent for the *AB* amplifier to a noise voltage of 0.0678 inserted in the input circuit.

The electronic noise is a result of the instantaneous variations in the liberation of electrons from the cathode of the vacuum tube. This noise, converted to an equivalent voltage input to the amplifier, would be 0.0202 microvolt.

The mains hum is caused by the use of alternating current for operating the indirectly heated cathodes. For the AC2, an average hum voltage of 3 microvolts is produced for the first push-pull stage. This corresponds to a hum voltage at the input of the amplifier of 0.0428 microvolt.

The total noise voltage is equivalent at the input of the amplifier to 0.082 microvolt. As the amplifier produces less than 1-percent harmonic distortion with an input of 5 millivolts, a volume range of 95 decibels may be applied to the input.

The input transformer required particular attention from the viewpoint of minimizing noise. The transformation ratio between the line and grids directly influences the volume range of the amplifier; to reduce the noise level, the impedance ratio has been made very high, 40 to 200,000 ohms. Through careful design, the screening of this transformer at the frequencies normally giving the most trouble has been made very high.

The use of alternating current for cathode heating introduces a mains transformer. This transformer is separated from the other elements of the amplifier by a screen consisting of an aluminum sheet 2 millimeters (0.079 inch) thick sandwiched between two silicon steel sheets each 1 millimeter (0.039 inch) thick.

4.1.2 Microphonics

The problem of microphonics is essentially of a mechanical nature, the disturbances being transmitted normally either by simple mechanical shocks or by acoustic fields. The susceptibility to vibrations of the elements in the vacuum tube used in the first stage of an amplifier is the most important single factor. Although the AC2 has a very small tendency to microphonics, the wide volume range of the amplifier dictated special precautions to keep such disturbances below the noise level of the amplifier.

The effect of shocks can be eliminated by suspending the vacuum tubes individually. In the case of the *AB* amplifier, superior results were obtained when the four tubes and their coupling elements were suspended as a unit. In Fig. 7, the design of this suspension is evident. A very heavy mounting plate is supported on four long adjustable springs. Stops are provided to prevent excess motion and the entire assembly may be blocked for shipping.

Protection against acoustic fields is more difficult. It was considered necessary to put the tubes of the first stage in individual hermetic enclosures. Primarily to avoid the generation of acoustic fields within the amplifier, the transformers are impregnated with an insulating compound to hold the windings and the permalloy core laminations rigidly. It is not necessary to mount the transformers elastically.

4.1.3 Distortion

The distortion in the amplifier is almost entirely produced in the vacuum tubes of the output stage.

The push-pull output transformer is capable of operating with a 30-percent unbalance of plate currents of the output tubes with only a negligible contribution to distortion. The output transformer has an impedance ratio of 50,000 ohms to 40 ohms. Operating at a higher level than the input transformer, it requires no elaborate screening.

4.1.4 Performance

The response-frequency curve of the amplifier is flat to within ±1 decibel from 20 to 14,000 cycles. The distortion with a signal output of 1.55 volts, which is the maximum output under normal operation, is less than 0.3 percent. The distortion with an input of 5 millivolts is less than 1 percent. The phase shift is given in Fig. 8. The noise level is 94 decibels below the maximum output signal.

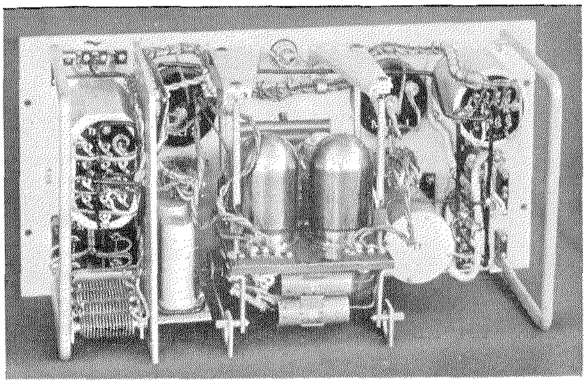


Fig. 7—*AB* amplifier showing spring suspension of tubes and coupling elements. The input tubes are in hermetically sealed cases. These precautions minimize microphonic noises.

4.2 Program Meter

The program meter fulfills the requirements set up in 1935 by the International Broadcasting Union. These requirements are that over a range from 30 to 10,000 cycles the input impedance shall be greater than 10,000 ohms and the error in reading shall not exceed 1 decibel for impulses having a minimum duration of 10 milliseconds.

The amplifier for the meter is comparable to the *AB* amplifier except that the second stage uses power triodes. The output tubes have directly heated cathodes and the noise level is 65 decibels below maximum signal. The output of the amplifier charges a capacitor through a double diode. The capacitor discharges through a dry rectifier and a resistance that is higher than that of the rectifier. The voltage across the rectifier is applied to the grid of a triode, the plate current of which is a measure of modulation depth. The resistance of the rectifier varies logarithmically with voltage giving this desirable characteristic to the program meter.

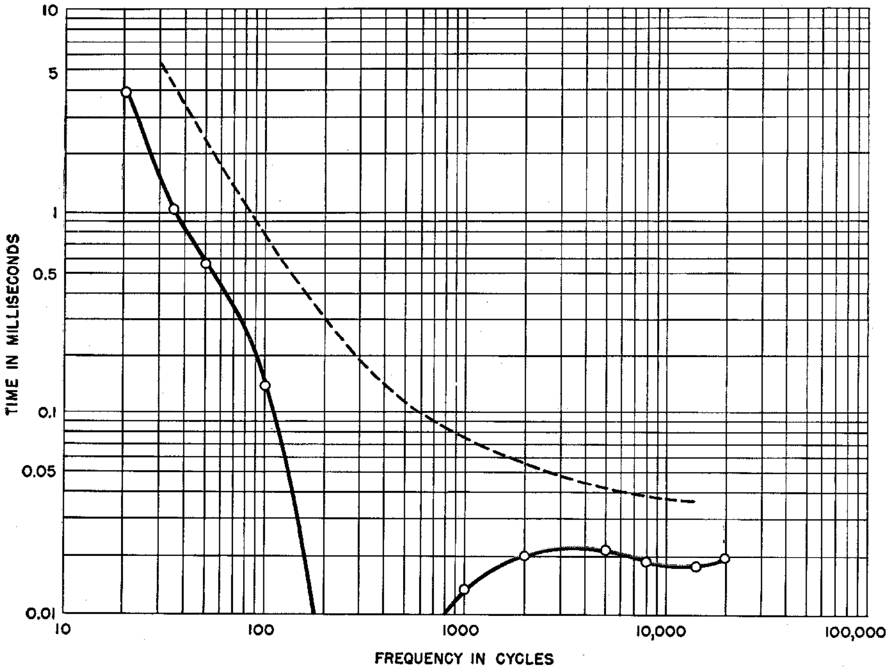


Fig. 8—Phase delay versus frequency for the *AB* amplifier for an output load of 40 ohms. The dotted curve is the maximum permissible phase delay. The solid curve is the measured values.

An important feature of this design is the use of uranium dioxide as the material for the resistor in series with the rectifier to provide temperature compensation. This makes unnecessary the control of temperature required in previous designs. Fig. 9 shows a front view of the program meter.

4.3 Line Equalizer

Line equalizers are required to modify the transmission characteristics of the telephone lines over which program material is transmitted to the building. These lines may be nonloaded cable or open-wire circuits. They may be loaded lines having upper cut-off frequencies varying between 2800 and 10,000 cycles. Also, they may be loaded and repeatered lines of the type used for international circuits or provided especially for broadcasting.

As all of these external circuits have a characteristic impedance of approximately 600 ohms, the equalizer has been designed for this value. A transformer converts this impedance to 40 ohms to permit the insertion of an *AB* amplifier to bring the signal to the standard value of 1.55 volts.

For the higher frequencies, a constant-impedance equalizer of the shunted-*T* type is used. The reactive circuit in the series arm consists of an inductance and capacitance in series, each of which is adjustable in 10 steps. The conjugate values of impedance provided by a parallel resonant circuit in the shunt arm are automatically selected by the switch that controls the series-arm elements.

For the lower frequencies, a comparable circuit is employed but only a simple inductive or capacitive reactance

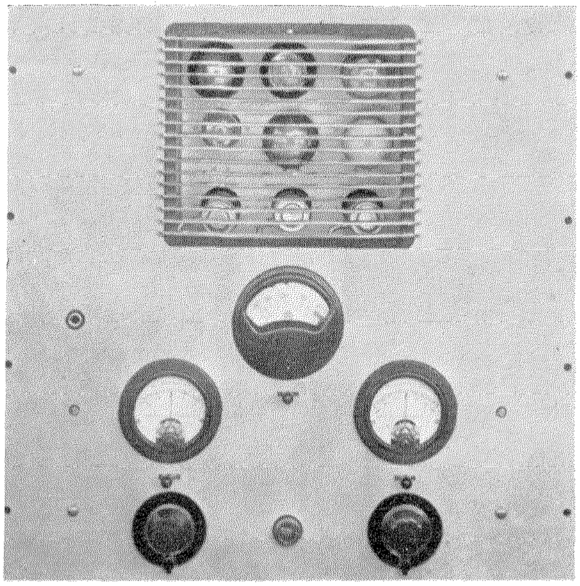


Fig. 9—Program meter.

is used instead of the resonant circuit. The reactive elements are adjustable in four steps. A switch that reverses the inductive and capacitive elements in the series and parallel arms permits either a rising or falling frequency characteristic to be obtained.

A constant-impedance attenuator having a range from 0 to 20 decibels, is provided for adjusting the input level to the *AB* amplifier.

The high- and low-frequency equalizers being independent and of constant impedance produce no interaction and the over-all equalization is the sum of the two individual equalizations. Lines may be equalized within ±1 decibel from 30 cycles to about 85 percent of the cut-off frequency of the line, and to 10,000 cycles if there is no cut-off frequency.

A line equalizer is shown in Fig. 10. The inductors use iron-dust cores. The capacitors use paper dielectric for the high-capacitance values and mica for the low values.

5. *Standardization*

In such an installation, much is to be gained by standardizing on a few basic pieces of equipment and using them wherever possible. The number of spare equipments is reduced, servicing and maintenance are simplified, and continuity of operation is improved. Consequently, considerable attention was given to these features.

5.1 CIRCUIT UNITS

The *AB* amplifier is used as a microphone amplifier, line amplifier, and in conjunction with the equalizers on incoming lines. It is also used as a reproducing amplifier in the recording rooms.

A single type of program meter is used throughout the entire installation.

A 7-watt loudspeaker-amplifier is used for monitoring programs. The amplifier is also used for recording.

Only one type of line equalizer is used.

A single type of isolating amplifier is used in the program distribution system and also for the branching of external lines.

The same scratch filter is used in reproducing records in the alphas and in the recording rooms.

5.2 VACUUM TUBES

Only three types of vacuum tubes are used extensively in the entire installation. The few other types that are employed meet special requirements that these three standardized types do not fulfill.

5.3 TRANSFORMERS

Only six types of audio-frequency transformers are used in the various amplifiers. These are the input transformer of the *AB* amplifier, the output transformer of the *AB* and of the isolating amplifiers, the output transformer of the loudspeaker amplifier, the output transformer of the program meter, the input transformer of the

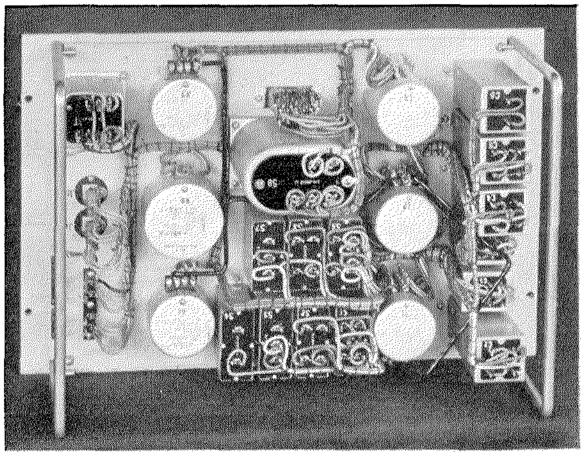


Fig. 10—Rear view of line equalizer. The control knobs are on the panel side.

isolating and loudspeaker amplifiers and of the program meter, and the input transformer of certain recording amplifiers.

In addition, only two types of mains supply transformers are used. The supply circuits of the *AB* and isolating amplifiers and of the fixed-tuned receivers are identical.

It should also be noted that all assemblies are mounted on standardized panels 480 millimeters (18.9 inches) by a multiple of 40 millimeters (1.6 inches).

6. *Continuity of Service*

Continuity of service is extremely important in broadcasting and much thought was given to protection against failure of equipment.

At least two microphones are available in every studio, one of which serves as a spare.

An operating amplifier in a control room can be replaced by a spare amplifier in the brief time required to shift plugs from one set of jacks to another. The spare amplifiers are normally used for secondary purposes and are thus always ready for service.

A spare volume control is provided on each control desk. If the master or main volume control becomes defective, it can be cut out of service and its function temporarily taken over by an amplifier volume control elsewhere in the channel.

Because of expense, only one program meter is installed in each alpha. In case of its failure, the alpha technician may receive instructions from the lambda to which his program is connected.

Spares are normally provided for monitor receivers, oscillators, and similar units having only single functions.

Microphone lines are always duplicated and the operator of lines alpha can readily replace a main interconnection line by one normally used for secondary functions.

7. *Program Distribution System*

There are a large number of individuals involved in rehearsing and broadcasting a program. Among others, they include the technical operator, producer, author, and broadcast supervisor.

There are approximately 50 sources within the building from which program material may originate. These include alphas, lambdas, recording rooms, broadcasting receivers, and talk-back and other telephone circuits. If the number of "subscribers" to the program distribution system amounts to 150, it is evident that there are 7500 possible interconnections. This is far too many to permit adequate manual switching and so automatic equipment, based on the same principles as the automatic telephone exchange, has been installed.

Each subscriber has a small desk with a calling key, signaling lamp, telephone dial, and volume control, together with a cabinet containing an amplifier and loudspeaker.

This system differs from a simple automatic telephone exchange in that most of the programs are connected to the distribution system through isolating amplifiers.

The necessity of reducing cross talk to a value 70 decibels below normal level on these circuits was a real difficulty. In a well-built telephone exchange, a cross-talk figure of more than 40 to 50 decibels is rarely attained. By using twisted and screened wires and careful arrangement of the multiple cable on the automatic switching bays, all cross-talk figures were at least 80 decibels down. In addition, all lines have been carefully balanced to earth by resistors to eliminate clicks which normally would be caused by switching operations.

8. *Cabling*

Broadcast transmission circuits are carried in flexible cables of single-screened pairs. Those from each alpha are bundled and terminate successively in the lines of the 10 lambdas in the lines-alpha room where they are balanced to earth by resistors.

The signaling cable of an alpha contains one quad and 15 pairs and goes directly to the automatic program distribution room. All other signaling circuits from the nontechnical rooms are directed to the nearest terminal strip from where they go by multiple-pair cables to the automatic program distribution room. The lambdas are interconnected by a signaling multiple consisting of two 15-quad cables for the green and red lamps of the alphas and two 25-

quad cables for switching relays in the transmission channels.

The automatic program distribution room is the central point of the entire cabling plant and a general distributing frame has been installed there.

For design purposes, the signaling circuits are considered as sources of disturbances and the transmission circuits as being susceptible to interference. A 25-millimeter (1-inch) twisting step was chosen for the transmission channel circuits; the twisting step for the signaling circuits, which should be a multiple of the other, was fixed at 100 millimeters (4 inches). No wire crossings or single-conductor circuits have been used.

Induced static charges have been eliminated by screening all wires and terminal strips. Screens are always earthed on the output side only to avoid setting up loop circuits.

Transmission and signaling cables are laid in separate metallic ducts, all of which are electrically linked and earthed.

In the control rooms of the alphas, microphone circuits, which are run in steel tubing, are maintained at a minimum distance of 1.5 meters (5 feet) from lighting and power circuits. The outputs of the microphone amplifiers are considered as operating at an average level and may be run in the transmission ducts without further precautions. However, they are kept as well separated as possible from cables that are operated at the reference level of 1.55 volts.

Although the entire installation is operated from alternating-current mains, the noise level is so low that the workable volume range for broadcasting is about 80 decibels. Between the large majority of the lines, cross-talk figures of over 114 decibels have been measured.

Several separate earthing systems are employed for the high-voltage power equipment, low-voltage power circuits, telephone installation, signaling circuits, and transmission circuits.

9. *Electricity Supply*

Two independent underground cables, one of which is a spare, supply power at 5250 volts to the substation of Broadcasting House. A voltage regulator maintains this supply within narrow limits. A 220-volt synchronous motor drives an alternator, which supplies power to the entire broadcasting system in the building. This arrangement provides a stabilized 50-cycle 220-volt output free from the parasitic disturbances normally associated with power delivered from a central station.

In case of failure of the city mains, the motor-generator will be driven by a Diesel engine, which starts up and couples itself automatically. During the change-over, the supply voltage to the broadcasting equipment decreases only by 10 percent for about 4 seconds. A very heavy flywheel on the shaft of the synchronous motor is responsible for this fine performance.

10. *Acknowledgment*

Started on June 8, 1935, Broadcasting House was completed on July 25, 1938. The project was directed by a Committee of Works composed of members of the Managing Council of the Institut National Belge de Radiodiffusion presided over by Mr. Marcel Malderez, at present Secretary General of the Belgian Ministry of Communications. The consulting services of Mr. Raymond Braillard, then Manager of the Control Centre of the International Broadcasting Union, were available.

I am particularly grateful to my many collaborators, among whom will be mentioned Messrs. Georges Hansen, Chief Engineer-Assistant General Manager; Gaston De Lafonteyne, Georges Gourski, and Charles Van Loo, Managing Engineers; Léo Wallenborn, now Secretary General of the International Broadcasting Organization; Eugène De Keyser and Raymond Henderickx, Senior Engineers; and Fernand Viart, Engineer.

To Bell Telephone Manufacturing Company of Antwerp was assigned the important task of constructing and installing the speech-input equipment.

Monitor for Frequency-Modulation Broadcasting*

By M. SILVER

Federal Telecommunication Laboratories, Incorporated, Nutley, New Jersey

ADEQUATE technical performance of a frequency-modulation broadcast transmitter can be obtained only through constant monitoring of the emitted wave. The center transmission frequency and percentage of modulation must be indicated at all times, and an overmodulation alarm must be provided. In addition, it must be possible to check for noise and audio-frequency distortion.

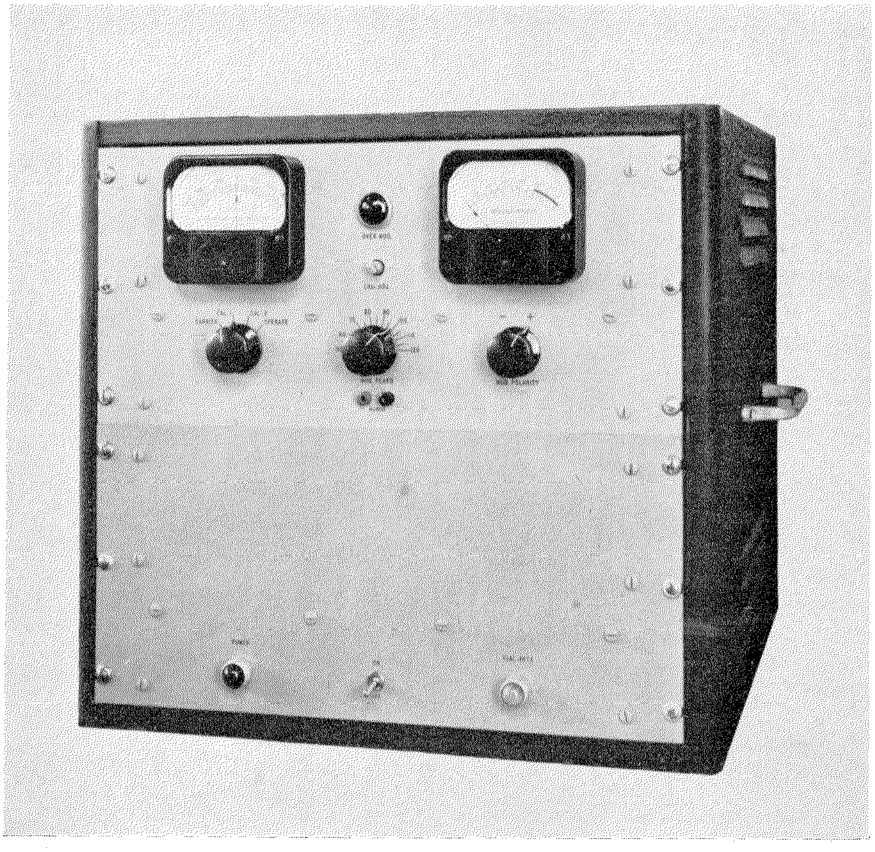
A monitor designed for this service uses a resistance-capacitance discriminator that acts as a counter to indicate the center frequency, independent of modulation deviation. Audio-frequency output from the discriminator is applied to a vacuum-tube voltmeter for modulation measurement and the operation of an alarm on overmodulation. All requirements of the Federal Communications Commission are met by this monitor.

• • •

The Federal Communications Commission requires that each frequency-modulation broadcast station in the U.S.A. have a suitable monitor for checking the frequency and other important characteristics of the transmitted wave. Frequency must be measured to within 1000 cycles per second in the band from 88 to 108 megacycles. Modulation must be measured to within ±5 percent and overmodulation must be indicated by some sort of an alarm.

Although not explicitly stated, the monitor should be capable of measuring noise and distortion of the transmitted wave. Transmitter noise must not exceed a level at least 65 decibels below 100-percent modulation and audio-frequency distortion must be less than 1 percent. The inherent noise level specified for the monitor is 75 decibels below 100-percent modulation and the inherent distortion must not exceed 0.25 percent. Another requirement is that the audio-frequency amplitude characteristic must follow a standard de-emphasis curve of 75 microseconds to within 0.5 decibel.

Several output impedance levels are required: a high-impedance output for noise and distortion measurements and 600- and 150-ohm outputs for acoustic monitoring. Both meters and overmodulation indicators should have provision for remote operation. The monitor must be self-calibrating.



Monitor for frequency-modulation transmitter operating in the 88–108-megacycle band.

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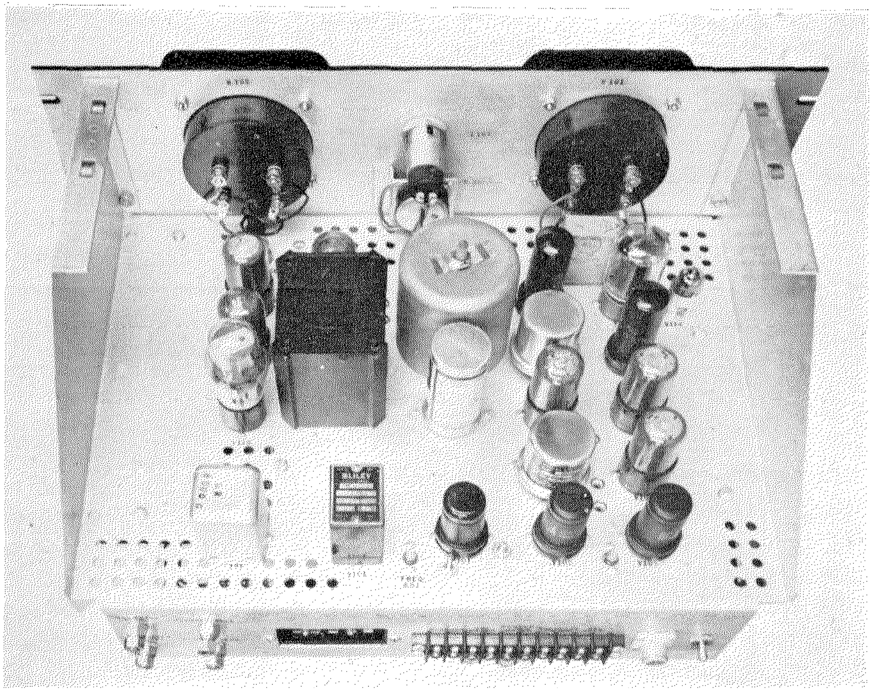
* Presented, Institute of Radio Engineers National Convention, March 5, 1947, New York, New York.



Upper chassis of monitor. The lower chassis contains the various power supplies.

1. *Discriminator*

The discriminator is the basic element in a monitor. In general, a discriminator operating at the transmission frequency will have a greater effective band width and be better suited to modulation monitoring than if it is designed for a lower frequency. Conversely, low-frequency discriminators can be made more sensitive and stable. This suggested the desirability of using separate discriminators for these two functions.

Tuned driscriminators of both single and balanced types were studied for modulation measurements directly at the transmission frequency. Satisfactory designs were developed and were capable of measuring noise and distortion of very low orders of magnitude.

A discriminator operating at 210 kilocycles was developed separately for frequency monitoring. A resistance-capacitance discriminator, it performs as a simple integrating counter. It has the necessary linearity to prevent the integration of the modulation from producing a direct-current component of sufficient amplitude to appear as an error in frequency measurement during modulation. The counter is capable of measuring frequency to within 100 cycles under conditions of 100-percent modulation.

The linearity of the counter-discriminator was far in excess of that required for modulation monitoring. The only significant problem in using it for both measurements was that of separating the 210-kilocycle intermediate frequency signal from the 15-kilocycle audio-frequency modulation. A suitable filter, having an attenuation of 60 decibels at 30 kilocycles, was devised. The final design of the monitor, therefore, uses only a single untuned discriminator.

2. *Counter*

A square wave is applied to the input grid circuit of the counter tube shown in Fig. 1. This square wave is derived from a series of cascaded clippers and limiters. When the negative square wave is applied to the grid, the counter-tube plate current is completely cut off. The capacitor *C*1 is charged during this period through rectifier 1 to the peak value of the plate voltage. During the positive half cycle the tube is conductive and the capacitor discharges through rectifier 2

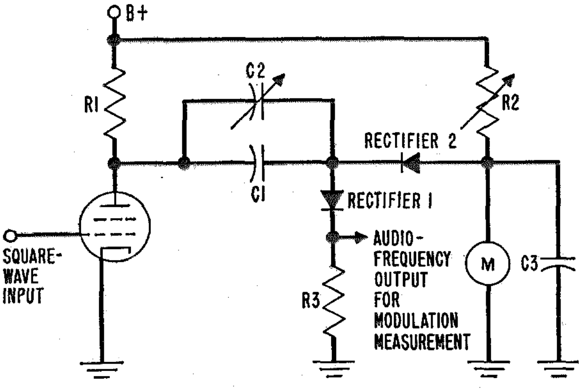


Fig. 1—Counter-discriminator circuit.

and the counter tube. The average value of current flowing through the meter circuit is proportional to the rate of discharge of the capacitor and is, therefore, a measure of frequency. In addition, the voltage across resistor $R3$ is a measure of frequency and is proportional to the modulation. This permits the same discriminator to be used for monitoring frequency and modulation.

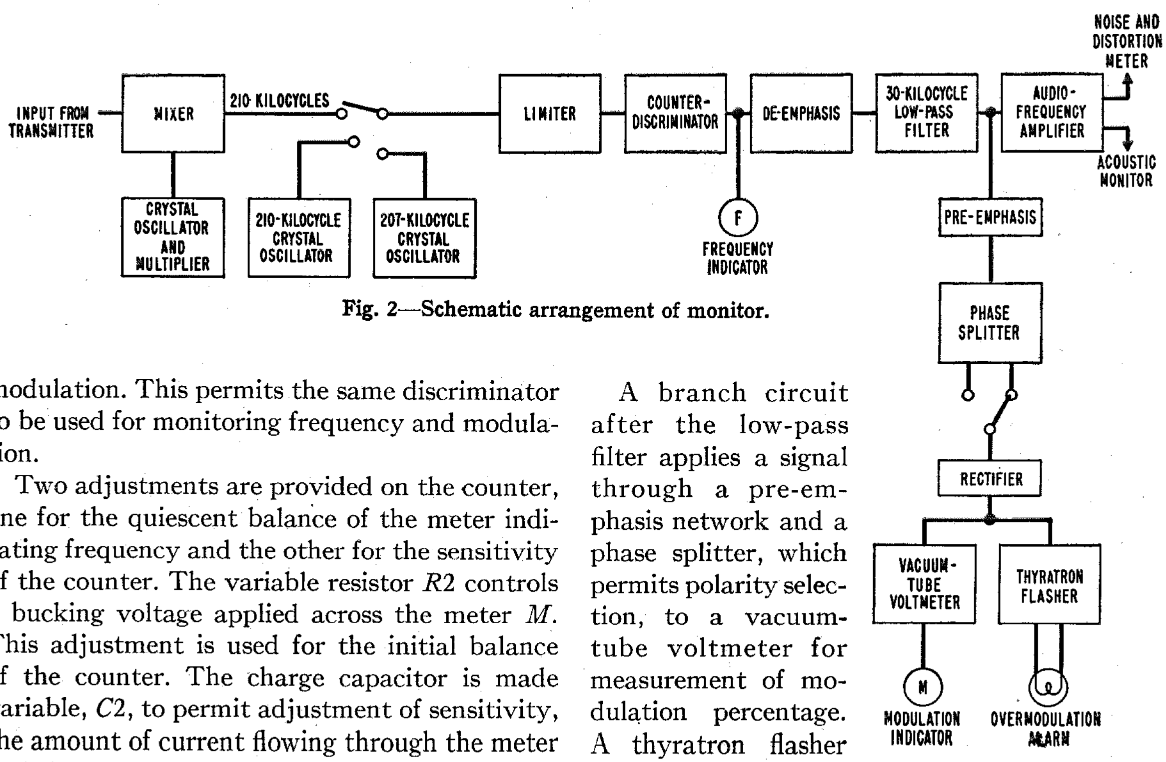


Fig. 2—Schematic arrangement of monitor.

Two adjustments are provided on the counter, one for the quiescent balance of the meter indicating frequency and the other for the sensitivity of the counter. The variable resistor $R2$ controls a bucking voltage applied across the meter M. This adjustment is used for the initial balance of the counter. The charge capacitor is made variable, $C2$, to permit adjustment of sensitivity, the amount of current flowing through the meter M being proportional to capacitance.

3. *Monitor*

Basically, the monitor consists of cascaded limiters, counter-discriminator, filter network, calibrating circuits, and modulation-indicating devices as indicated in Fig. 2. In operation, energy from the transmitted wave is mixed with a crystal-controlled oscillation to produce a difference frequency of 210 kilocycles. The crystal oscillator is temperature stabilized and is the operating standard for frequency measurements.

The 210-kilocycle output of the mixer is applied to the counter-discriminator tube through a series of cascaded limiter circuits, which square and limit the voltage. Frequency is indicated by the meter in the counter circuit.

The output across $R3$ of Fig. 1 is applied through a standard de-emphasis network to a cathode-follower operating into a low-pass filter having a cut-off frequency of 30 kilocycles. The signal then passes through an amplifier having 600- and 150-ohm balanced outputs for acoustic monitoring and a high-impedance termination for a noise and distortion analyzer.

A branch circuit after the low-pass filter applies a signal through a pre-emphasis network and a phase splitter, which permits polarity selection, to a vacuum-tube voltmeter for measurement of modulation percentage. A thyratron flasher circuit indicates overmodulation.

Two crystal oscillators are provided for calibrating the discriminator. One supplies a 210-kilocycle signal to permit zero adjustment of the counter-discriminator for frequency measurement. The second produces a 207-kilocycle signal for adjusting the sensitivity (discriminator slope) of the counter circuit.

3.1 LIMITER

The limiter circuit is conventional in all respects but one. If all circuits are of the plate-limiting type, the limiter is a high-gain amplifier and subject to oscillation. To avoid this possibility, a clipper circuit was used in the first stage followed by a conventional plate-limiting second stage.

3.2 Modulation Indicator

Fig. 3 is a schematic diagram of the modulation-indicator circuit. The audio-frequency signal from the counter-discriminator is rectified and applied to the grid of a cathode-follower, which serves as a branch of a bridge circuit. A quiesent balance is obtained by adjusting *R*3. Under modulation, the balance is disturbed and a current, which is proportional to the amplitude of modulation, flows through the meter circuit. The sensitivity of indication may be varied by *R*2.

The same rectifier is used to actuate the thyratron flasher circuits. Alternating voltage is applied to the plate of the thyratron so that plate current is interrupted on the negative alternations. The characteristics of the rectifier circuit are such that the charge time is much shorter than the discharge time and peak indications persist for an appreciable period after a modulation peak has passed. When the voltage on the grid of the thyratron exceeds a gate value determined by the adjustments of *R*4 and *R*5, the thyratron tube becomes conductive causing a lamp in the plate circuit to glow. *R*1 is a sensitivity control.

The voltage to the filaments of the bridge and thyratron tubes is regulated by a ballast tube. This stabilizes the modulation indicator under line-voltage fluctuations. In addition, the screen circuit of the thyratron is arranged to compensate for changes in plate supply voltage as a function of line voltage.

To calibrate the modulation-indicating circuits, a 60-cycle signal is applied to the input of the rectifier circuit. The amplitude of this alternating voltage is set at the factory to correspond to 100-percent modulation. Provision is made for checking the amplitude of this voltage in operation.

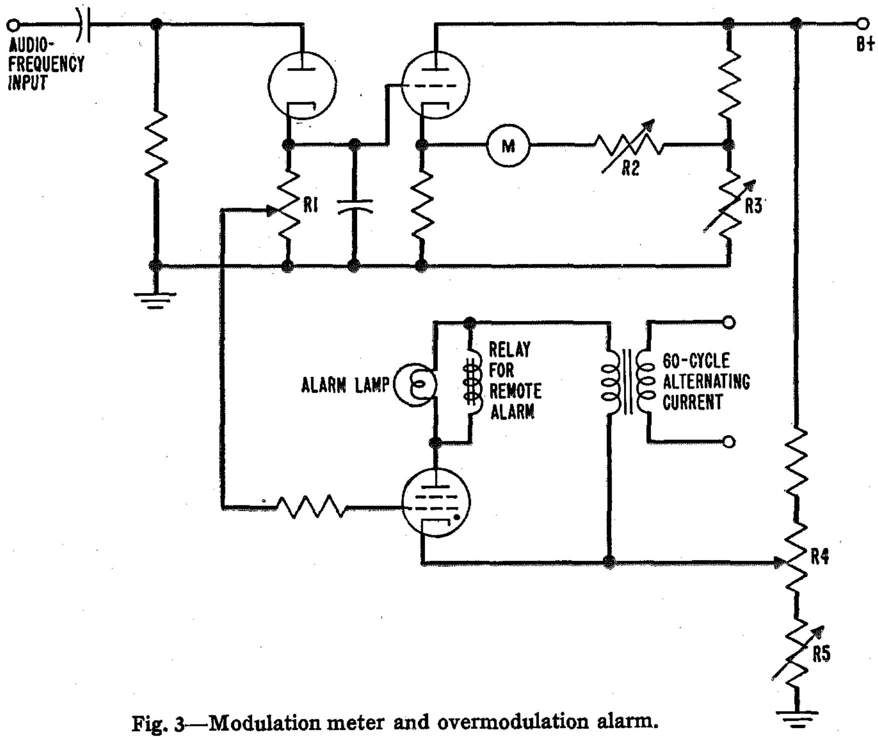


Fig. 3—Modulation meter and overmodulation alarm.

4. Measurement of Monitor Characteristics

Measuring the characteristics of a monitor is somewhat of a problem, and it was thought better to use indirect means of measurement rather than build a super monitor for the purpose. Suitable standards are available to measure the accuracy of frequency calibration, so this feature will not be discussed.

The most difficult characteristics to determine are inherent noise and distortion. Consider the measurement of inherent noise. With the output of the monitor connected to a suitable noise and distortion analyzer, there was applied to the monitor input a signal that was frequency modulated at a 400-cycle rate with a deviation of ±75 kilocycles. A broadcast transmitter was used as a signal source. This establishes a reference level for 100-percent modulation on the noise and distortion analyzer. If there is substituted for the signal source, a crystal oscillator using direct current for plate and filament supply, the residual noise of the monitor may be obtained. Fortunately, such a circuit exists within the monitor in the form of the 210-kilocycle calibration oscillator. The noise level measured in this manner is 80 decibels below that of the reference level, and is better than that specified.

Determining the inherent distortion is somewhat more complex. The distortion inherent in the counter circuit and in the audio-frequency amplifier circuits must be known. The counter distortion was computed by means of a static plot of frequency versus output direct voltage of the counter. With an accurate potentiometer, voltages were measured to four places. The distortion in the counter was found to be in the order of 0.05 percent. The distortion inherent in the audio-frequency amplifier was measured by the use of a noise and distortion analyzer using an audio-frequency oscillator whose distortion is 0.1 percent. A distortion of 0.2 percent was measured at 50 cycles, the frequency at which distortion was maximum. The distortion of the discriminator being so much lower than that of the audio-frequency amplifier, the inherent distortion of the monitor was determined as 0.2 percent, which is below the permitted value.

Summing up the characteristics of the monitor, the inherent noise is 80 decibels below a reference of 100-percent modulation. The inherent distortion is 0.2 percent. The error in frequency indication under conditions of 100-percent modulation is ±100 cycles, assuming proper calibration of the reference crystal oscillator. The error in modulation percentage and overmodulation indication is ±5 percent.

It is interesting to note that a plate power supply with a ripple level of less than 2 millivolts was required. A hard-tube regulator circuit with additional feedback was used. It was also necessary to use direct current for the filament circuits to reduce the noise level to the low values measured. The filament supply uses selenium rectifiers. The complete equipment, including all power supplies, is contained in two chassis.

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Medium-Power Multichannel Communication Transmitters

By B. T. ELLIS

Federal Telephone and Radio Corporation, Clifton, New Jersey

POINT-TO-POINT and ground-to-aircraft communication normally requires two or more channels in different parts of the frequency spectrum for uninterrupted service throughout the day and year. To avoid uneconomical duplication of equipment, a transmitter has been developed that has one power supply and modulator but two radio-frequency units. Thus, a radiotelephone and radiotelegraph or two radiotelegraph circuits can be operated simultaneously at two different radio frequencies. Four additional radio-frequency units may be added to permit any two of six channels to be so utilized. Output powers of 400, 500, and 200 watts are produced in the bands from 200 to 540 kilocycles, 2 to 20 megacycles, and 108 to 140 megacycles, respectively.

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It is well known that in the high-frequency spectrum the optimum operating frequency for a given point-to-point circuit varies widely with such changing factors as season of the year, time of day, local atmospheric conditions, and the degrees of utilization of the available channels. Transmitting equipment, therefore, should be capable of convenient frequency change or separate transmitters may be maintained for each of the operating frequencies. A very effective solution to this problem is the use of a multichannel transmitter in which several radio-frequency units, each permanently adjusted to a single frequency, are operated from a common power supply. Flexibility and speed of frequency change are thus combined with economy and efficiency.

The *FTR*-3 was the first transmitter built on the multichannel principle. Radio-frequency units, power supplies, and modulators are constructed in separate cabinets, which can be set up side-by-side in a sectionalized frame to meet individual requirements. Radio-frequency units and modulators can be controlled remotely, the desired facilities being selected by operating a telephone dial. This philosophy has been widely followed by other manufacturers.

The *FTR*-3 is rated at an output of 3 kilowatts from each radio-frequency unit. A similar transmitter, the *FTR*-5, is rated at 5 kilowatts per channel. Transmitters 184, 185, and 186, which are described here, provide output powers in the fractional-kilowatt range.

1. General Design

The new transmitters depart from the horizontal side-by-side arrangement of their predecessors. As lower power permits much smaller basic components, the units may be positioned vertically as well as horizontally, with a saving in floor space. Cabinets have been designed to house several combinations and to provide the necessary ventilation, metering, and interconnecting circuits.

Transmitter output is nominally rated at 400 watts for the 200- to 540-kilocycle unit, 500 watts for the 2- to 20-megacycle unit, and at 200 watts for the 108- to 140-megacycle unit. All units are capable of 100-percent amplitude modulation. Primary power is 220 volts, 50 or 60 cycles, single phase.

All major units, except high-voltage rectifiers, are constructed on movable chassis, which may be slid part way out of the cabinet for inspection. Flexible cables maintain electrical connections to units thus withdrawn. Supply voltages and control circuits are brought to each unit by a cabinet cable assembly that includes a large terminal board for connection of external control equipment. Links and jumpers, adjusted by the user, connect each radio-frequency unit for telephone or continuous-wave operation, the choice of which may be made either directly or remotely.

2. Cabinets

The basic cabinet assembly is the left-hand half of the transmitter shown in the illustration. The high-voltage rectifier is on the floor of the cabinet, the filter components are on a sliding shelf over the rectifier, and a modulator chassis slides into place over the filter. Two radio-frequency units fit side-by-side in the top section. Space is provided between the modulator and radio-frequency units for control equipment and low-voltage and bias rectifiers.

The basic assembly permits simultaneous operation on two channels, either with two telegraph or with one telegraph and one telephone circuit. Only one modulator is provided, thus limiting telephony to a single channel at a time. Two identical units will double this service. Another arrangement is shown in the illustration. It uses a single power supply and modulator with up to six radio-frequency units, any two of which may be operated simultaneously.

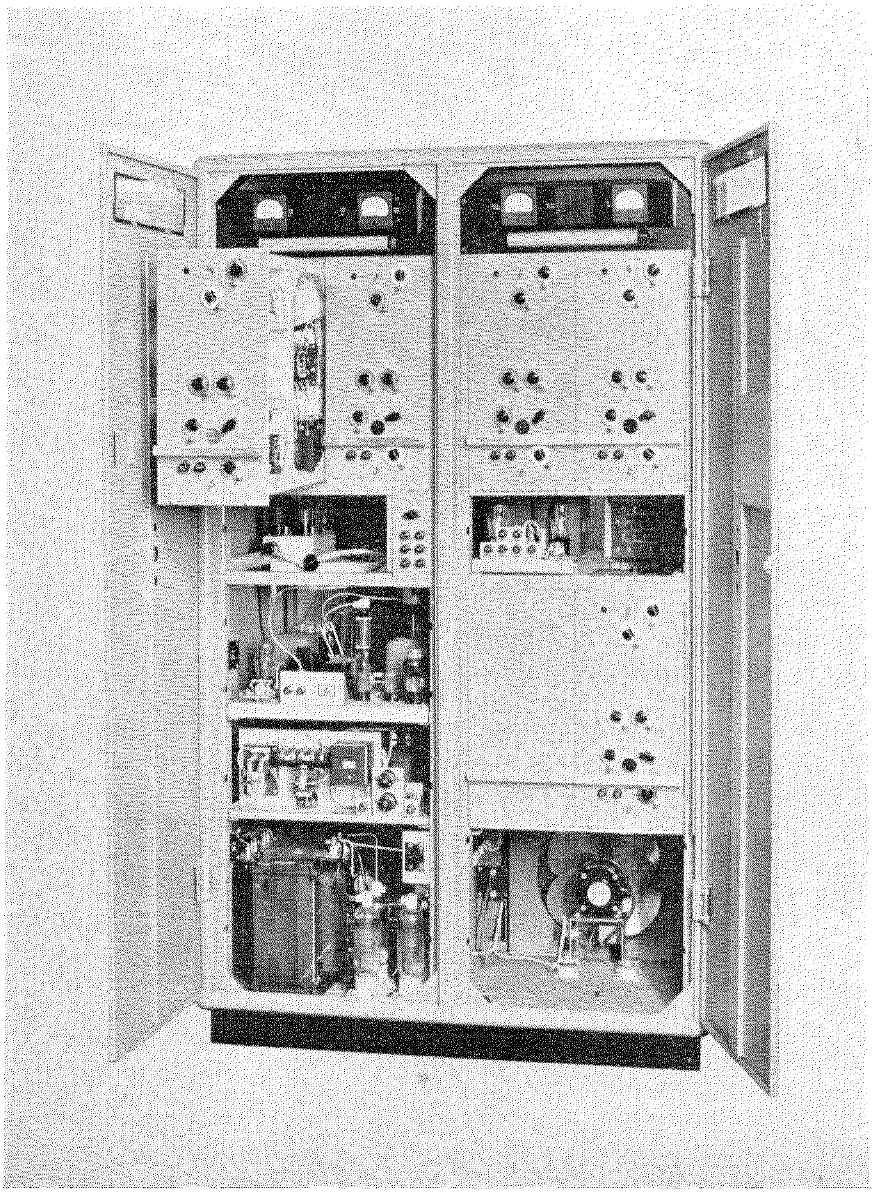
3. Power Supply

Two radio-frequency units, one of which may be voice modulated, can be operated simultaneously from one power supply. When more than two radio-frequency units are associated with a single power supply, only two may be on the air at the same time.

The main high-voltage supply delivers 1500 volts. This voltage was selected as a compromise between high plate efficiency and size of components. The rectifier is a center-tapped bridge circuit using F-872A tubes. 750 volts is available for the intermediate stages. Tuned-choke filters, adjustable for 50- and 60-cycle power frequencies, are used on both the 1500-volt and the 750-volt outputs. Bias voltage (−300 volts) and low-voltage plate power (+300 volts) are obtained from two rectifiers each using 5U4G tubes and single-section filters.

4. Radio-Frequency Units

Three basic radio-frequency units cover the frequency ranges 200 to 540 kilocycles, 2 to 20



Two basic units are mounted in a single cabinet. The left-hand assembly contains a power supply at the bottom with its filter just above; then come the modulator, bias and low-voltage power supplies, and two radio-frequency units. Four additional radio-frequency units may be accommodated in the right-hand assembly. The subassembly panels have been removed to show the general arrangement of components. In operation, the full doors are closed, the meters being visible through the windows at the top.

megacycles, and 108 to 140 megacycles. They are interchangeable in the cabinet. The low- and very-high-frequency units have 72-ohm coaxial output transmission lines while the high-frequency unit is provided with a 600-ohm spaced parallel-wire line. The tube complements for the three types of radio-frequency units are listed in Table I.

TABLE I

VACUUM-TUBE COMPLEMENTS FOR RADIO-FREQUENCY UNITS

Function	200–540 Kilocycles	2–20 Megacycles	108–140 Megacycles
Oscillator	6AC7	6AC7	807
Buffer	6V6	—	—
1st Multiplier	—	6V6	807
2nd Multiplier	—	2E22	815
Intermediate Power Amplifier	2E22	2E22	815
Power Amplifier, Two Tubes	152TH	152TH	4–125A

Crystal-controlled oscillators are used in all radio-frequency units, although the circuits may be modified easily to permit frequency-shift keying or variable-frequency oscillator operation.

Blocked-grid keying is used in the low- and high-frequency units to permit keying speeds up to 500 words per minute. The very-high-frequency unit is not intended for radiotelegraph operation.

Frequency range adjustments are made with movable links and flexible leads that are soldered to coil taps. Original designs provided tap switches to permit bandsetting from the front panel, but most applications require so few frequency changes during the life of the equipment that the initial cost and the maintenance expense of band switches are not justified.

The low- and high-frequency units use a novel construction of intermediate stages. Each tube is mounted upside-down in an inverted U-shaped bracket so as to project through the shelf. Bypass capacitors, grid leak, and parasite suppressors are mounted on the top and sides of the bracket with very short leads to tube socket terminals and with considerably improved accessibility for test and replacement. The shelf and bracket is produced as a subassembly to facilitate manufacture.

5. *Modulator*

Approximately 350 watts of audio-frequency power is available for modulation of one radio-frequency unit. The input level for 100-percent modulation is 25 decibels below 6 milliwatts in the 500-ohm input circuit. The frequency response is designed for voice transmission. Low-pass input and output filters may be inserted to reduce higher-order sidebands. Distortion, without the low-pass filters, is less than 10 percent at 100-percent modulation.

All the audio-frequency amplifier stages are push-pull triodes. Resistance coupling is used between the 6SL7GT first amplifier, 6SN7GT second amplifier, and 6B4G driver tubes. The modulator stage uses two 805 tubes operating class *B*. The use of triodes in the driver stage makes inverse feedback unnecessary.

6. *Acknowledgment*

Development work on the equipment was directed by J. W. Butterworth who is now Chief Engineer of the Marine Division of Mackay Radio and Telegraph Company.

Modernization of International Telephone Service, and Its Reaction on National Telephone Systems*

By E. P. G. WRIGHT

Standard Telecommunication Laboratories, Limited, London, England

THE present state of the international telephone service is of interest because pre-war techniques are, in many cases, obsolete and because, in any event, dislocations caused by the war necessitate some revision.

This paper gives general information about foreign telephone systems, outlines continental networks and their interconnection, and describes future planning procedure and the progress already made.

Specialists in international telephony believe that the traffic will expand quickly with a rapid and efficient service. To provide higher speed, manual operation must be replaced by semi- or fully automatic methods. Consequently, existing national services need reviewing and, if necessary, replanning as integral parts of the international system. The modernization of some foreign national systems is discussed, with particular regard to quicker service.

There is striking evidence that harmony between nations is likely to be obtained only by mutual understanding, and that international communication should form one of the principal ties upon which the future of civilization will depend. It is probably unavoidable that national and international projects will be needing expenditure simultaneously, and it will be a tragedy if the expansion of the international service is stifled in order to allow a disproportionate effort on the national services.

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1. *Existing European International Service*

1.1 Description of the Service in 1939

During the 20 years between the two world wars, periodical reports were made of the introduction of new international circuits, permitting the public an ever-increasing range over which telephone calls could be established.

The service which has grown up has many admirable features, but a searching examination shows that progress has been hampered by the circumstances existing in the years during which the service was growing.

It would be expected that the international telephone service would benefit from its ability to adopt all the better practices that were in use by different administrations and to avoid any failures that were found to exist in national systems. In actual fact, these benefits have not accrued, and there has been a tendency for the international service to be less efficient than the best service provided nationally.

This result is perhaps not so unexpected when it is realized that the development, trial, acceptance, and introduction of a new practice require the agreement of two or more groups of technicians for an international service, whilst only one group is involved for a national service. Secondly, there is rather more scope for the introduction of improved methods in a national service, where replaced equipment can be retired for use on subsidiary circuits having less exacting requirements, but this convenience is seldom so easily planned in connection with international circuits. Thirdly, the difference in language and custom in different countries sometimes rules out the simplest solution and results in the continuance of a well-tried, but inefficient, practice. Finally, there is the important fact that an efficient international service cannot be established on the basis of several different practices, and many administrations would prefer to operate all their different international circuits on a uniform practice, even though this were not particularly efficient, rather than mix several methods, some good and some indifferent.

1.2 Function of the C.C.I.F.

In spite of the handicaps already mentioned, the interests of the international service are fostered by those who, by their foresight and wisdom, have created on a sturdy, yet resilient, basis the Comité Consultatif International Téléphonique, (C.C.I.F.), whose object is the coordination of the international service, and by

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* Presented, Institution of Electrical Engineers, London, England, March 20, 1947, and Norwegian Society of Electrical Engineers, Oslo, Norway, September 29, 1947. This paper received the Fahie Premium of the Institution of Electrical Engineers.

those who have helped to ensure the successful working of this association, believing that its objectives would benefit the different peoples of the world. It is the function of the Comité Consultatif International Téléphonique to think in terms of international service, disregarding purely national matters. In general terms, this work includes the investigation and study of new problems and specific instances of difficulty with existing practice, the establishment of standards for engineering design and procedure, operating and maintenance practice, documentation, statistics, and nomenclature.

All the large administrations in Europe are members of the Comité Consultatif International Téléphonique, and most of the larger administrations in North and South America are represented, in addition to several in other parts of the world.

This association, which has been established for more than 20 years, is not dissimilar to the United Nations Organisation in its wideness of outlook. It maintains a permanent Secretary-General, secretarial staff, and reference laboratory, and all expenses are met by fees paid by the co-operating members in proportion to their size. It is abundantly clear that the Comité Consultatif International Téléphonique provides the machinery for maintaining the international service in a healthy and vigorous condition.

1.3 Causes of Service Delays

Not unnaturally, the international service has grown stage by stage as technical knowledge and the prospect of a reasonable financial return have enabled more and more international circuits to be built. It is an undisputed fact that this manner of growing does not lead to the best result, as is demonstrated by the delay experienced by the public in obtaining international calls. Most of the principal cities of the world have automatic telephones, and local calls can usually be established in about 20 seconds; international calls, on the other hand, often require 1, 2, or even 4 hours, and there is no insoluble technical reason for such a serious disparity. The delays can be attributed to an inadequate provision of circuits, to the operating procedure, and to the primitive nature of signalling facilities. To a lesser extent the service may be restricted by the variable quality and instability of the circuits.

An examination of these causes shows clearly that replanning is essential. For example, in 1938, out of a possible 406 pairs of countries in Europe, 121 pairs were interconnected, with approximately 4 circuits per route; there were 316 subsidiary groups, with an average of less than 2 circuits per route; in three instances there were as many as 30 separate groups between adjacent countries, with less than 2 circuits per group.[1] In such circumstances, inefficiency is unavoidable—either the subscribers are made to wait so that the circuits are reasonably loaded or, if delay is to be eliminated, the circuits will not earn a satisfactory revenue. With large quantities of small groups, a satisfactory service is almost beyond the control of the administration. In most cases, the traffic increases and the delay grows simultaneously, until a point is reached when the delay is so great that it stifles further increase of traffic. It is evident that the continuance of these small groups does not easily admit the possibility of an adequate revenue without a hampering service delay.

The most obvious rearrangement to operate a smaller number of larger groups involves the introduction of additional switching points, and it has been judged in the past that the creation of switching points is a greater evil than the use of small groups. The limitations of manually operated switching centres are well known, and detailed operation procedures and carefully trained operators are necessary to obtain even a normal grade of service. The handling time of a call with manual transit-switching is several times that of a direct call; if two such switching points are involved, a further increase in time is introduced. Bitter experience has resulted in a widespread disinclination to adopt manually switched calls whenever a direct service is possible.

This state of affairs may be largely explained by the following facts:

A. The designation of the international call is longer than that of the national call and therefore more difficult to memorize.

B. The charge to the subscriber must be based on a ticket including all relevant information.

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[1] "Nomenclature des Circuits Téléphoniques Internationaux" (Bureau de l'Union Internationale des Télécommunications, Berne, February, 1939).

C. The passing of the call order from operator to operator and the further recording of the information is slow on account of the length of the designation and because at some point language translation may be necessary.

D. The prospect of finding three circuits in series—all available simultaneously—is poor, and when finally this is achieved there is an increased difficulty in finding the calling party because it is likely that he has moved from one place to another while waiting for the call to mature.

These difficulties are accentuated by the fact that it is the invariable practice to use ringers for signalling and that the facilities thus provided are equivalent to those on magneto switchboards, which have long been obsolescent on all the major national circuits.

2. Possibility of Improving Service

2.1 APPLICATION OF RECENT DEVELOPMENTS TO INTERNATIONAL SERVICE

The use of automatic switching and modern line construction and signalling methods in the national networks has shown that a quick and accurate service can be provided, and superficially there is no reason why a corresponding application of new methods to the international networks should not be equally effective.[2]

2.2 WHEN TO PLAN MODERNIZATION

Although such a proposal as that in Section 2.1 may be the logical conclusion, there remains the question whether this is the correct time to plan a general revision of the international service. Once a new plan is approved, it will take several years to introduce, and it is therefore desirable that it should remain in operation for at least 25 to 30 years.

During this time, technical progress will be made, and it is clearly inadvisable to exclude the benefits of any major developments which may become available. On this account, the system planner needs to scrutinize the situation and consider the possibility of:

A. New and cheaper line construction by means of wave-guides, coaxial cables, or radio.

B. The introduction of new switching means, such as the use of electronic methods.

C. Modifications to the subscriber's instrument to introduce higher-quality or auxiliary services, and the possibility in either case of consequential reaction on the frequency band transmitted.

D. Rearrangement of the basis of subscriber numbering or dialling.

E. Rearrangement of the basis of tariff or method of recording charges.

F. Introduction of associated services.

The outcome of such an examination can scarcely be precise, and, although few technicians would agree that research on these subjects will be unproductive during the next 30 years, there is little indication that anything is to be gained by delay from the research point of view. Recent development work with coaxial transmission systems has been carried to the point where it has been established that an efficient service is available for large groups of long lines at a reduced cost. Further new systems do not appear to offer the possibility of such striking improvements which would be universally applicable within the next decade.

The amount of switching equipment that will be needed at automatic transit centres is relatively small, and for this reason special arrangements can be made to provide desirable facilities, which could not be attempted if the scale were more extensive. Furthermore, the relatively small amount of equipment involved for switching means that it would not be a serious problem to replace it completely if new developments of sufficient importance became available. Any new international plan will be influenced by modern ideas on numbering, dialling, and charge recording.

The argument in favour of replanning in Europe is strengthened by the devastation caused by the war. Many of the international circuits passed through Germany, Berlin being the principal switching centre for eastern Europe. The rebuilding of the European international system is unavoidable, and there is everything to gain by replanning it on a world-wide basis.

2.3 NEED FOR INTERNATIONAL ENTERPRISE

Assuming that the technical merit of a new international switching plan could be proved to satisfy the critics, and that it could be established that the volume of traffic was likely to increase

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[2] H. T. Kohlhaas, "Milestones of Communication Progress," *Electrical Communication*, v. 20, n. 3, pp. 143–185; 1942.

in a predictable manner with improved service, there still remains a doubt whether there exists sufficient international enterprise to permit the logical expansion to take place. This aspect cannot be overlooked, because, although there is a potential demand for a better international service on the basis of existing charges, the new plant needs a capital expenditure requiring Government financial approval. In the process of seeking such approval, this international enterprise will be in competition with a number of national projects backed by the national Press or other eager supporters. If it is appreciated that the fundamental need for this enterprise lies in promoting international understanding—and, as such, it is a moral obligation comparable with the accepted principles of the United Nations Organisation—there is a reasonable chance of progress; but, if the case is judged on a narrower issue, who is there in a position to expostulate with the financial authority who sees but small advantage in promoting the project? If it is a fact that a general plan of co-operation between nations has a better chance of obtaining financial approval than a series of individual plans, this provides a considerable argument in favour of such a co-operative international plan, because of its greater prospect of surmounting adverse nationalistic or political interests.

3. *Telephone Service Throughout the World*

3.1 SURVEY OF INTERNATIONAL AND INTERCONTINENTAL SERVICES

In order to gain perspective, it is wise to study the question, What proportion of the international service is provided in Europe? The answer seems to be larger than might be expected.

The largest number of telephone calls originate in the United States of America, which has a network of international circuits to other parts of the world. A great volume of traffic passes between the United States and Canada, and probably more international calls pass over this frontier than over any other. The network of circuits to Mexico, the West Indies, and Central and South American states is probably roughly equal to the network of radio circuits to different capitals in Europe. Both Argentina and Brazil have direct radio connection to a number of capitals in Europe. Argentina has a network of land lines and radio circuits to other states in South America, but the quantities are all relatively small. Table I gives an approximate summary of intercontinental circuits in 1939.

TABLE I

INTERCONTINENTAL CIRCUITS IN 1939

	North America	Oceania	Central America and West Indies	South America	Asia	Africa
Europe	25	1	—	14	3	5
North America		9	6	9	—	—
Oceania			—	—	2	—

Note: These figures are based on reconnections recommended by the 13th Plenary Assembly of the Comité Consultatif International Téléphonique (London), 1945.

In Europe at present, there are some 28 independent states, and in 1939 there were some 600 different international routes, many of which passed through intermediate territories. The service between Great Britain and France was provided by a group of more than 100 circuits. It is probable that the negotiations necessary to operate the European traffic between so many administrations, involving as it must the complications of many languages and many currency systems, needs more effort than corresponding work in America, where a very large proportion has been negotiated between two administrations on the basis of one language and one currency.

It is evident that the technical problems in America are very great on account of the long distances involved, but to a large extent these difficulties have been overcome by the development work of the American Telephone and Telegraph Company and the Bell Telephone Laboratories to meet national requirements. Although an association like the Comité Consultatif International Téléphonique has immense technical resources, it is easier in some ways for a single organization, working independently, to develop and introduce a new service.

As regards international networks in other parts of the world, it is noteworthy that some of the long-distance national circuits between states in the Commonwealth of Australia (as shown in Table II) are of equivalent length to other international circuits, but, although the technical problems are equivalent to those encountered in

Europe, the technical administration for handling problems and deciding on policy is centralized as in the U.S.A.

South Africa and the Middle and Far East contain important telephone centres, but their international service constitutes a very small percentage of the whole. For example, India has a community of interest with Burma to the east and Ceylon to the south. The Soviet Union covers an immense territory, partly in the European network, but it seems unlikely that the eastern portions will have needs extending beyond adjacent countries, which will be determined directly by the administrations concerned. This applies also to China; and the international traffic from Japan is not extensive. The establishment of many of the standards for international service is likely to be settled independently in Europe and North America for land lines and switching, while for radio circuits initial standards are likely to be agreed mutually between these two continents.

TABLE II

AUSTRALIAN INTER-STATE TOLL CABLE ROUTES*

Circuit	Equipments × channels per equipment	Distance in kilometres
Adelaide–Darwin	1×3	3 050
Adelaide–Melbourne	1×12, 8×3, and 5×1	770
Adelaide–Perth	10×3 and 8×1	2 625
Brisbane–Sydney	2×12, 20×3, and 7×1	1 060
Melbourne–Sydney	4×12, 18×3, and 5×1	933

* Longest and largest routes in use or undergoing installation.

3.2 OUTLINE OF DEVELOPMENT PROPOSALS IN NORTH AMERICA

Proposals for the ultimate development of a continental switching plan for North America have been published. In general, it is hoped to treat the United States and Canada as a whole. As a means for improving the long-distance service, facilities will be provided for each long-distance operator to have access, at her own control, to every subscriber connected to a machine-switching office. This plan envisages the construction of a national numbering plan, whereby any subscriber may be called from any part of the country by using the appropriate long-distance number. In a sense, this is true at the present time, but the essential significance of the national numbering plan is that each number can be called by the operator on her dial or keyset. This procedure means that the name of the town and state must be replaced by a number or code letter, and obviously each telephone network must have a characteristic code. The plan provides not only for operator connection but also for subscriber dialling in the future; this does not necessarily anticipate the complete elimination of long-distance operators, but it is certain that subscribers will be given facilities for dialling over an increasing area, and it is very desirable that a national numbering plan should be applicable to both operator and subscriber dialling.

One of the difficulties involved in subscriber dialling is to find a long-distance dialling code that will identify the class of call as rapidly as possible and route the call independently of the local network. It is undesirable to arrange local registers to deal with long-distance traffic and nearly as wasteful to switch a call initially to a local register and then to a long-distance register.

In America, it is proposed to choose long-distance codes including the digits 1 and 0, which never appear in existing multi-office codes. Three-digit codes, of which the second digit is 1 or 0, will provide for some 75 zone centres which, it is thought, will suffice for the United States and Canada. It is intended that this code should be retransmitted from point to point until the terminal zone centre is reached, and by this means the choice of alternative routes will be greatly simplified. The crossbar system provides an excellent opportunity for rapid switching at the transit centres, and the large toll switching office at Philadelphia gives a practical example of the speed of service obtained with keyset sending, call announcing, and high-speed switching.[3–6]

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[3] F. J. Scudder, and J. N. Reynolds, "Crossbar Dial Telephone Switching System," *Bell System Technical Journal*, v. 18, p. 76; 1939.

[4] B. C. Bellowes, "Philadelphia Adopts Automatic Toll Switching," *Bell Laboratories Record*, v. 22, p. 101; 1943.

[5] L. G. Abraham, A. J. Busch, and F. F. Shipley, "Crossbar Toll Switching System," *Electrical Engineering*, v. 63, p. 302; 1944.

[6] F. F. Shipley, "Nation-wide Dialling," *Bell Laboratories Record*, v. 23, p. 368; 1945.

4. Perspective of European International Service as Seen from Great Britain and the Continent

The geographical position of Great Britain is unique, and the absence of a land frontier has not seriously obstructed modern methods of communication, because the largest group of international circuits lies between London and Paris, a distance of about 450 kilometres. This is the shortest international route from London, but it is much longer than the shortest route from many other countries. Fig. 1 illustrates this fact; it represents an outline of Great Britain laid over the north-west of Europe, with the south coast of England lying along the coasts of Belgium and The Netherlands. The important continental towns and countries have been shown in their relative positions within this outline in order to emphasize their proximity to one another. It will be observed that the area of England and Scotland contains portions of France, Belgium,

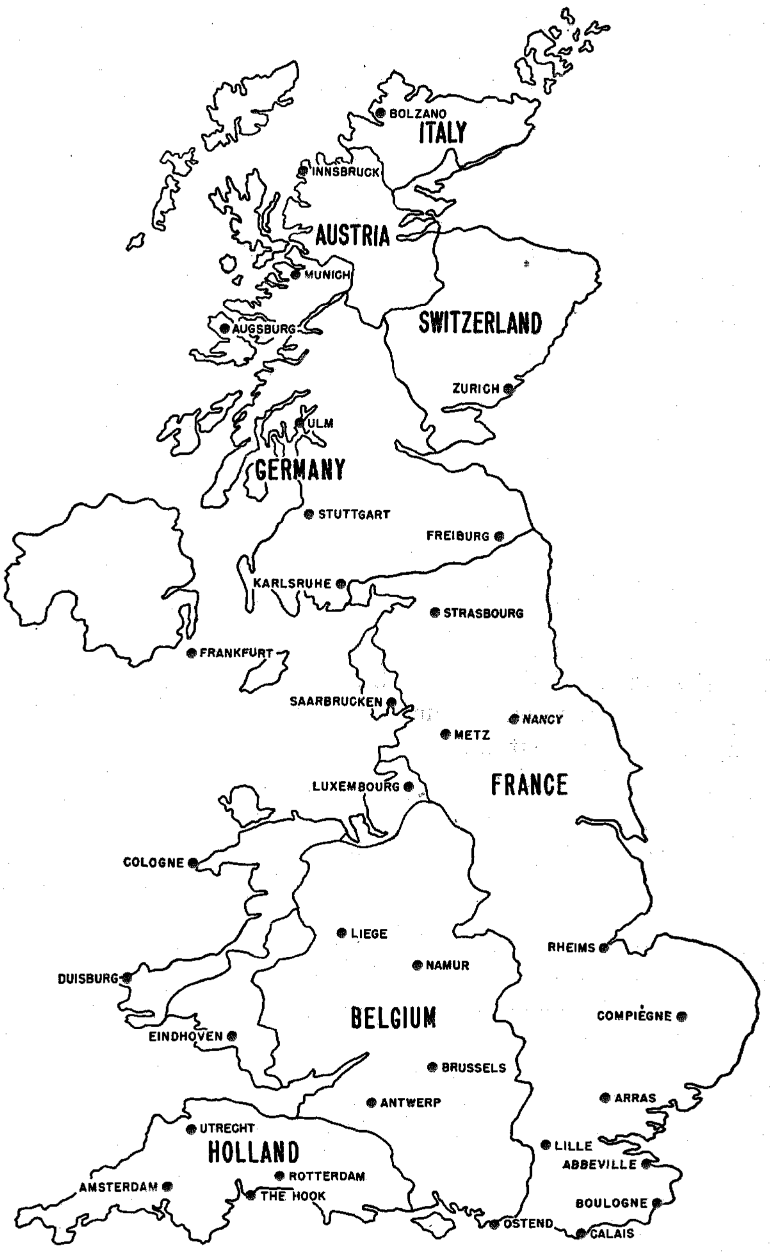

Fig. 1—Superimposition of map of British Isles on the continent of Europe, illustrating close association of European countries.

Holland, Luxembourg, Germany, Switzerland, Italy, and Austria.

The proximity of all the towns in this area to their nearest frontier is striking, and in many cases international calls involve a shorter distance than a trunk call in Great Britain. It must be realized that, consequently, the need for subscriber dialling on international calls is likely to arise sooner on the continent than in this country.[7]

5. Revised European Fundamental Switching Plan

5.1 National Numbering Plan

The fundamental switching plan in Europe is likely to follow principles similar to those outlined for North America, but elementary differences are likely to arise. In America, the continental switching plan is being designed to reach the wanted subscriber, whilst in Europe the international switching plan will be designed to take the call to the international *tête-de-ligne* centre of the country required, the completion of the call from this point being a national matter. In America, a single numbering plan will embrace the whole of the United States and Canada, but in Europe each country will wish to set up an independent national numbering plan. The designation of the called number must be transmitted over the international circuit, and it is of importance to know whether the number of digits is variable and, if letters are used as well as digits, the number of impulses to which the letters correspond.[8]

Table III shows the number-letter combinations in New York, London, Paris, Berlin, Sydney, and Copenhagen. The relationship between the dial numbers and the corresponding impulses also needs consideration. Fig. 2 compares the dial in most general use with that used in Sweden.

The letter-number variation is more difficult to allow for than the variation in the number ring for which compensation can be arranged in the national incoming register.

It seems very desirable that all the administrations in Europe should have national numbering plans. When this is achieved an international call can be passed into a country from any direction and then extended to the wanted subscriber over the national network without the intervention of an incoming operator. This fact is best illustrated by the case of Germany, where traffic from the surrounding countries would be presented to the international *tête-de-ligne* centres at Hamburg, Cologne, Frankfurt, Stuttgart, Münich, Berlin, and Stralsund. With a national numbering plan, each of these centres would have access to all the others for the national service, and the international traffic could be merged with the national traffic at these centres.[9] It might be thought that for countries having only a single international *tête-de-ligne* centre—such as Great Britain, Norway, or Portugal—the national numbering plan would be less essential, but such is not the case, because, if the service is to be based on the number being set up by the originating *tête-de-ligne* operator, it is necessary that she should be able to call this number on her dial or keyset, and this require-

TABLE III

NUMBER-LETTER COMBINATIONS

Numbers	Letters					
	New York	London	Paris	Berlin	Sydney	Copenhagen
1	—	—	—	A	A	Central
2	ABC	ABC	ABC	B	B	ABD
3	DEF	DEF	DEF	C	E	EFG
4	GHI	GHI	GHI	D	J	HIK
5	JKL	JKL	JKL	E	L	LMN
6	MNO	MN	MN	F	M	OPR
7	PRS	PRS	PRS	G	U	STU
8	TUV	TUV	TUV	H	W	WXY
9	WXY	WXY	WXY	J	X	AEØ
0	Z	O	OQ	K	Y	—

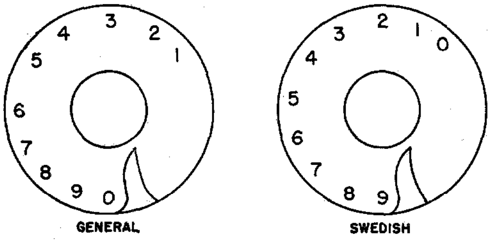

Fig. 2—Comparison of dial number plates.

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[7] "Long Distance Telephony," *The Post Office Green Papers*, no. 43; 1938.

[8] "Comité Consultatif International Téléphonique—Report of Sub-Commissions' Meeting in Paris, June and July, 1946," *Post Office Electrical Engineers' Journal*, v. 39, p. 117; 1946.

[9] E. Muller-Mees, "Automatic Methods of Trunk Operation used by the Reichspost," *Europaischer Fernsprech Dienst*, v. 60, p. 29; 1942.

ment necessitates a unique combination of digits for each subscriber.

It is possible to use a national numbering plan for setting up calls before full dialling facilities are actually available, because it enables the originating operator to set up the call without passing the designation verbally to an inward operator. No very formidable difficulties are expected in the layout of national numbering plans. Switzerland, Belgium, The Netherlands, and Denmark have national numbering plans wholly or partly in use at present; France, Italy, Rumania, and Hungary have plans prepared; and Sweden has a numbering plan covering the more populated part of the country. (See Section 8, Appendix.)

It is apparent that the use of national numbering plans will entail some further education of subscribers. More frequent access to directories is not necessary, but subscribers must be encouraged to include both their national and local telephone numbers in their correspondence letter-heads; usually the national number will be the local number with the addition of a 2- or 3-digit prefix. This prefix will serve to replace the name of the town, and subscribers must learn to ask for the country and the national number, and not the country, town, and local number. This seems to be a complication, but in Switzerland and Belgium it has not caused any serious trouble. It is obvious, of course, that the same national number should be applicable to both the international and the national services.

5.2 General Technical Requirements

Although the introduction of national numbering plans is possibly not essential for a semi-automatic international service, it is certainly very desirable, and it seems likely that most administrations will adjust their national service to enable the international service to benefit. There are, however, other subjects which need to be studied in connection with a semi-automatic international service, e.g., consideration of international traffic, fundamental cable plan, fundamental transmission plan, speed of connection, signalling system, service tones, language differences, tariff, alternative routes, and numbering plans. Each of these subjects deserves extensive examination.

5.2.1 Consideration of International Traffic

Attention has already been directed to the fact that a large number of small groups results in delay, and one of the chief benefits that should emerge from a new cable plan is a reduced number of groups, each containing a larger number of channels. The cable plan must be in harmony with the international traffic originated by each country. This traffic can conveniently be divided up into three parts, namely, traffic between neighbouring towns across a national frontier; traffic to adjacent countries; and traffic which must pass through intermediate countries.

The first part will probably be handled by frontier circuits limited to terminal traffic only. There will be cases where two towns relatively close together cannot be satisfied by frontier circuits limited to terminal traffic, and in such cases direct or transit international circuits will be needed. Copenhagen and Malmö are good examples. The second part contains about 60 per cent of the whole; the cables for this service can be fairly readily planned, as in most cases they will need to join the capital towns and the route will be indicated by existing cables. These routes will have a bearing on the third part, which can be treated in several ways, and the choice of the most suitable arrangement depends on many separate issues needing examination.

If the cable plan is to depend chiefly on the transit traffic, the anticipated requirements of this type of traffic should be set down as follows:

A. Switching should be rapid—if possible as rapid as that of the direct circuits.

B. Provision of alternative and emergency routes.

C. The operator should not need to know whether the service is direct or indirect.

D. Large traffic groups are preferable to small.

E. Merging with national traffic is undesirable on the transit circuits.

F. The use of the cable routes provided for the direct routes is advantageous.

5.2.2 Fundamental Cable Plan

Apart from the provision of frontier circuits and direct circuits as justified, there are three possible ways of preparing the cable plan:

A. The provision of one central transit centre with cables radiating in all directions to every country. The

TABLE IV

Forecast of European International Traffic for 1952

	Austria	Belgium	Bulgaria	Czechoslovakia	Denmark	Finland	France	Germany	Great Britain	Greece	Hungary	Italy	Luxembourg	Netherlands	Norway	Poland	Rumania	Spain and Portugal	Sweden	Switzerland	Turkey	U.S.S.R.	Yugoslavia
Austria	—																						
Belgium	3.8	—																					
Bulgaria	0.25	0.05	—																				
Czechoslovakia	(*a*) 22 (*b*) 30	2	0.5	—																			
Denmark	0.5	2	0.1	2	—																		
Finland	0.3	0.05	0.05	0.25	2	—																	
France	7.5	(*a*) 60 (*b*) 30	0.25	6	2	0.5	—																
Germany	20	20	0.25	20	10	1	(*a*) 15 (*b*) 15	—															
Great Britain	4	30	0.25	2	8	0.5	80	21	—														
Greece	0.25	0.05	4	0.25	0.1	0.05	0.2	0.25	0.25	—													
Hungary	40	2	0.33	16	0.25	0.25	3	8	4	0.33	—												
Italy	(*a*) 12 (*b*) 10	2	0.25	3	0.25	0.1	(*a*) 10 (*b*) 12	20	14	0.5	7	—											
Luxembourg	0.2	12	0.05	0.5	0.1	0.05	(*a*) 5 (*b*) 5	4	0.1	0.05	0.05	0.05	—										
Netherlands	4	(*a*) 25 (*b*) 25	0.05	6	6	0.1	36	(*a*) 32 (*b*) 30	29	0.05	3	4	2	—									
Norway	0.25	0.1	0.03	0.5	5	1	1	4	6	0.03	0.1	0.15	0.05	0.75	—								
Poland	6.4	2	0.1	6	4	1	4	(*a*) 15 (*b*) 7	4	0.03	4	4	0.15	2	2	—							
Rumania	5	0.1	2	10	0.1	0.05	4	3	2	1	(*a*) 7 (*b*) 7	1.5	0.05	0.05	0.05	2	—						
Spain and Portugal	0.5	0.1	0.05	0.1	0.1	0.05	(*a*) 5 (*b*) 5	2	0.2	0.05	0.1	0.2	0.1	0.1	0.05	0.1	0.05	—					
Sweden	0.5	2	0.05	2	24	15	4	16	0.5	0.05	1	0.2	0.2	4	(*a*) 15 (*b*) 8	2	0.1	0.1	—				
Switzerland	20	10	0.1	4	2	0.1	(*a*) 38 (*b*) 30	(*a*) 35 (*b*) 20	28	0.05	8	(*a*) 40 (*b*) 40	2	26	1	4	0.1	4	2	—			
Turkey	0.25	0.05	3	0.25	0.1	0.05	0.2	0.25	0.25	4	0.33	0.25	0.05	0.05	0.03	0.05	1	0.05	0.05	0.05	—		
U.S.S.R.	3	0.25	0.5	2	0.25	1	2	5	2	0.25	2	1	0.05	0.5	0.1	(*a*) 3 (*b*) 3	2	0.25	1.5	0.2	0.15	—	
Yugoslavia	11	0.1	4	3	0.1	0.05	2	3	1	5	12	8	0.05	0.1	0.03	2	5	0.05	0.05	4	1.5	2	—

All traffic expressed in erlangs (call-hours). The traffic in both directions is assumed to be equal. Letters (*a*) and (*b*) indicate main and frontier traffic, respectively.

TABLE V

SUMMARY OF CABLE PLAN STUDIES FOR 23 COUNTRIES IN EUROPE, SHOWING COMPARISON BETWEEN STAR AND RING FORMATIONS

	Austria	Belgium	Bulgaria	Czechoslovakia	Denmark	Finland	France	Germany
Star Formation								
Direct circuits	156 350	102 140	20 700	132 930	70 540	22 650	366 440	(323 265)
Overflow from transit centre	9 450	1 870	1 560	10 300	4 540	700	11 670	(See
Indirect circuits	6 430	13 210	6 640	11 620	10 880	6 420	8 680	Note
Circuits to transit centre	8 000	19 460	18 700	7 920	9 540	18 480	22 320	2)
	180 230	136 680	47 600	162 770	95 500	48 250	409 110	323 265
Ring Formation								
Direct circuits	65 670	32 620	17 800	54 440	19 140	13 590	89 310	270 820
Terminations from ring	75 200	12 080	3 760	39 400	9 590	2 090	52 360	39 130
Ring circuits	33 040	87 720	1 350	58 270	54 820	5 120	238 520	680
Circuits to ring	—	—	14 250	—	—	24 000	—	—
	173 910	132 420	37 160	152 110	83 550	44 800	380 190	310 630

	Gt. Britain	Greece	Hungary	Italy	Luxembourg	Netherlands	Norway	Poland
Star Formation								
Direct circuits	326 730	32 880	132 030	181 450	9 330	190 710	60 710	139 760
Overflow from transit centre	5 740	1 569	8 700	11 100	2 920	3 150	4 840	7 960
Indirect circuits	7 890	6 640	8 690	13 190	3 690	6 550	3 210	2 100
Circuits to transit centre	15 600	27 500	24 400	30 000	11 000	9 880	21 600	20 400
	355 960	68 580	173 820	235 740	26 940	210 290	90 360	170 220
Ring Formation								
Direct circuits	12 160	17 800	44 880	78 760	7 180	42 090	50 440	50 690
Terminations from ring	43 210	3 270	20 600	22 700	1 510	38 640	2 930	17 430
Ring circuits	120 080	1 350	52 170	39 600	3 470	100 180	12 330	94 720
Circuits to ring	136 250	26 920	35 270	39 960	10 150	—	21 210	—
	311 700	49 340	152 920	181 020	22 310	180 910	86 910	162 840

	Rumania	Spain and Portugal	Sweden	Switzerland	Turkey	U.S.S.R.	Yugoslavia	TOTALS
Star Formation								
Direct circuits	77 430	37 200	110 720	306 300	25 500	80 380	69 700	2 768 570
Overflow from transit centre	8 440	1 830	3 510	3 380	1 560	6 690	11 150	122 620
Indirect circuits	9 610	5 040	6 280	5 660	6 640	10 930	8 390	375 665
Circuits to transit centre	33 250	24 000	14 000	10 800	26 950	50 400	31 200	455 400
	128 730	68 070	134 510	326 140	60 650	148 400	120 440	3 722 255
Ring Formation								
Direct circuits	33 220	6 000	50 690	59 800	17 800	35 170	50 700	1 120 770
Terminations from ring	6 780	4 040	14 250	81 360	3 270	9 670	4 870	508 140
Ring circuits	17 070	6 200	60 040	165 200	1 350	26 390	5 490	1 185 160
Circuits to ring	50 450	39 950	—	—	20 400	45 500	43 000	507 310
	107 520	56 190	124 980	306 360	42 820	116 730	104 060	3 321 380

Note 1: All quantities represent circuit-kilometres.

Note 2: The star formation total for Germany includes 207 275 circuit-kilometres radiating from Berlin and 115 990 circuit-kilometres from other centres in Germany. The former group combines with the "overflow" and "indirect" groups from other countries and is included in the "indirect" total.

Note 3: The circuit-kilometre totals should not be given too great a significance. The annual charges will depend very largely on the number of circuits per route.

Note 4: The route-kilometres involved are as follows:

Star Formation		*Ring Formation*	
Direct	260 000	Direct	90 000
Overflow from transit centre Indirect through transit centre	27 000	Termination from ring	26 000
		Ring circuits 2 × 4 600 =	9 200
		To ring	26 000
To transit centre	27 000		

Note 5: Dividing the circuit-kilometre totals by the lengths given in Note 4, the following loading rates are obtained:

Star Formation

Direct $\frac{2\,769\,000}{260\,000} = 10.65$

Overflow and Indirect $\frac{498\,000}{27\,000} = 18.4$

To transit $\frac{455\,000}{27\,000} = 16.85$

Ring Formation

Direct $\frac{1\,121\,000}{90\,000} = 12.45$

Terminations $\frac{508\,000}{26\,000} = 19.5$

Ring $\frac{1\,185\,000}{9\,200} = 128.8$

To ring $\frac{507\,000}{26\,000} = 19.5$

cable plan in 1939 approximated to this arrangement, Berlin being the transit centre for eastern European traffic.

B. The provision of a number of primary transit centres, each having direct circuits to all other transit centres, and secondary transit centres having lines to at least one of the primary transit centres. Such an arrangement limits to two the international transit centres in any connection. The limit of one primary-to-primary circuit and two primary-to-secondary circuits provides a ready means of regulating the maximum permissible transmission loss between the originating and terminating *tête-de-ligne* centres. This type of plan seems likely to be adopted for the United States and Canada.

C. The provision of a transit ring arranged to connect a number of regional centres. Regional or national centres not lying on the ring would be given access to at least two of the regional centres on it. The length of the ring circumference and its position would be indicated both by the international telephone traffic and by the best natural cross-country routes. These factors have led to the establishment of the following routes in Europe: Paris–Brussels–Rotterdam–Copenhagen–Malmö; London–Rotterdam–Copenhagen–Malmö; London–Paris–Zürich–Milan; Paris–Zürich–Vienna; and Vienna–Brno–Cracow–Warsaw.

The international routes for traffic from Oslo, Amsterdam, Rotterdam, London, Brussels, Paris, Zürich, Madrid, Milan, and Rome to Berlin may not be reconstructed, because the traffic to Germany can be terminated at closer zone centres.

The choice of site for each transit centre should be influenced by the desirability of nominating not more than one town in each country (preferably one, such as Malmö, close to the frontier nearest to the centre of Europe).

For purposes of comparison between the single-transit-centre plan (Case A) and the transit-ring (Case C), a forecast of the international traffic for 1952, based on the somewhat abnormal traffic of 1938, is given in Table IV. The comparison is based on a central transit centre at Berlin and a ring connecting Malmö, Copenhagen, Rotterdam, Brussels, Paris, Zürich, Vienna, Brno, Cracow, and Danzig.

Table V gives a summary of the circuit-kilometre requirements of a number of countries. It is noteworthy that the direct circuits are often of use to only two countries, whereas terminations to or from the transit centres are common to a greater extent and the circuits around the transit ring are of general use. It is also to be noted that the transit ring shows a considerable circuit-kilometre saving for countries outside the ring. The total length of each class of circuit is also included, because this enables the size of the different groups to be calculated. The length of the transit ring is shown doubled in order to allow for two traffic rings, one used in each direction.

The transit ring appears to have several technical advantages, namely, economy of initial cost, less switching at each transit centre leading to faster switching, and facilities for extensions to be made more readily. From the point of view of stability of service, the ring provides a means of setting up emergency circuits, and with two routes to the ring there should be much less chance of interruption of service. The provision of a ring calls for the co-operation of a number of administrations, and it may be easier for many of the administrations to obtain funds to build a portion of an international transit ring than to obtain the same amount in order to reduce delay on existing international routes.

It is apparent that the single transit centre suffers because outlying national centres are too remote, and it follows that some saving can be introduced by adding more transit centres. Suppose, for example, that the capitals of the largest countries are chosen, i.e., London, Berlin, Paris, and Moscow. Paris can be useful as a distribution centre for the extreme west, but neither London nor Moscow can help to reach the north, the south or the south-east. A more promising rearrangement would be the choice of the most suitable geographical centres for these regions, e.g., a site in Sleswig for Scandinavia, a site in Switzerland to serve the south and south-west, a site in Czechoslovakia or eastern Austria to serve the surrounding region, and a site on the lower Danube basin to reach the Balkans. Such a modification transforms the single-transit-centre plan into the primary-transit-centre plan. However, the latter differs from the transit ring only by virtue of the fact that, with the ring, full interconnection between all the regional centres is not specified. In Europe, there is a tendency for the international telephone traffic to be inversely proportional to distance, so that it is found that in several cases over 95 per cent of the outgoing traffic does not need to pass over more than two ring transit centres. This condition is demonstrated in Table VI, which shows the volume of traffic from each of the named transit centres to each of the others. It is apparent that there is no case for interconnecting all the transit centres by direct circuits, and in fact it seems that in most cases direct circuits to two other centres (in each direction) are sufficient. The ring gives access to a number of zone centres in Germany, and it is assumed that any call to Germany may enter by any of these zone centres.

If the international traffic in Europe were more widely dispersed, there would be a greater appeal in the primary-transit-centre plan, but there would still remain the problem of whether a regional centre could be agreed to for serving several countries; if each country decided it needed a regional centre, then the pre-war situation would be re-established and the next stage would be to set up a central transit centre, which represents the first plan again.[10]

5.2.3 *Fundamental Transmission Plan*

Most of the problems in obtaining a high-grade transmission standard will be found in the national areas, particularly where existing plant is inadequate. Between most international *tête-de-ligne* centres there is little doubt that zero-loss, high-velocity carrier systems will be used. The stability of these circuits will be improved by 4-wire switching at the transit exchanges and at the *tête-de-ligne* centres. The conversion from 4-wire to 2-wire will take place inside the national networks. The recommendations of comprehensive studies of international transmission plans have already been published.[11]

It seems likely that most of the circuits required for a transit ring will be of the coaxial type. In this way cables with a large capacity for future growth can be made quickly and

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[10] M. Langer, "Studies on Telephone Engineering Problems," R. Oldenbourg, Munich, 1939.

[11] B. H. McCurdy, "Fundamental Transmission Planning of Telephone Networks," *Electrical Communication*, v. 18, pp. 25–32; July, 1939: v. 19, pp. 18–28; July, 1940.

TABLE VI

FORECAST FOR 1952 OF TRAFFIC BETWEEN TRANSIT CENTRES ARRANGED IN RING FORMATION

Clockwise	Brussels	Rotterdam	Copenhagen	Malmö	Danzig	Warsaw	Cracow	Brno	Vienna	Zürich	Paris
Paris	120	36	3	4.5	—	—	—	—	—	—	—
Brussels	—	44	6	3	2.5	—	—	—	—	—	—
Rotterdam	—	—	21	25	8.5	—	—	—	—	—	—
Copenhagen	—	—	—	26	4	—	—	2.3	1	—	—
Malmö	—	—	—	—	7	—	—	2.3	2.3	—	—
Danzig	—	—	—	—	—	—	—	1.1	0.1	0.2	—
Warsaw	—	—	—	—	—	—	—	11	8.5	4.5	6
Cracow	—	—	—	—	—	—	—	17	5	2	—
Brno	2.5	6	—	—	—	—	—	—	20	—	8
Vienna	6	7	—	—	—	—	—	—	—	34	16
Zürich	13	30	3	2	—	—	—	—	—	—	100

Anti-clockwise	Brussels	Paris	Zürich	Vienna	Brno	Cracow	Warsaw	Danzig	Malmö	Copenhagen	Rotterdam
Rotterdam	27	36	30	5	6	—	—	—	—	—	—
Brussels	—	60	14	4	2	—	—	—	—	—	—
Paris	—	—	114	30	8	4	—	—	—	—	—
Zürich	—	—	—	20	4.5	4	—	—	—	—	—
Vienna	—	—	—	—	32	13	—	0.05	1.2	1	—
Brno	—	—	—	—	—	12	—	0.25	2.75	2	—
Cracow	—	—	—	—	—	—	—	—	1.1	0.25	—
Warsaw	4.5	—	—	—	—	—	—	—	4	2.25	6.5
Danzig	—	—	—	—	—	—	—	—	8	2	—
Malmö	2.2	4.1	2.2	—	—	—	—	—	—	24	14
Copenhagen	3.3	3.2	1.2	—	—	—	—	—	—	—	20

Note 1: The traffic is expressed in erlangs (call-hours).

Note 2: The figures assume that traffic to Poland joins the national network at Danzig and Cracow.

Note 3: The traffic shown is not only that from the country in which the transit centre is situated. The clockwise traffic from Zürich includes that from Italy, Yugoslavia, and the Balkans. The anti-clockwise traffic from Zürich includes Spain, Portugal, and Luxembourg.

Note 4: A traffic group greater than 20 erlangs will have a good occupancy, and a group less than 10 erlangs will have a poor occupancy, unless an overflow to an alternative route is available.

installed at a cost which compares favourably with other means of transmission. Most of the circuits making connection with the ring will also be coaxial, especially where the number of channels required is considerable. In other cases, where the traffic needs only a small number of circuits and older types of cable construction exist, some time may elapse before multichannel transmission systems can be justified.

With coaxial and similar systems, it is convenient if each traffic group can be allocated with a group, or supergroup, of transmission circuits so that they may be switched conveniently without the circuits being terminated individually. If the circuits to a transit centre comprise, for example, 10 different traffic groups, it will be necessary to separate the groups sooner or later, and unnecessary expense can be eliminated if terminating equipment for converting the circuits to audio work is avoided. It is unlikely that the traffic groups will be of such a size that they will exactly fill a supergroup, and in fact it is doubtful whether it will be possible to avoid breaking into groups. Care will be necessary in the planning of the groups to avoid all unnecessary frequency-band transformations. Problems will arise concerning the desirability of combining traffic groups in order to take advantage of reduced quantities in larger bundles. The annual-charge plant expense can be evaluated by a simple formula comparing the length of line with the annual charges. If the line charges (including repeater stations) per kilometre-channel are expressed by C_L, and C_C represents the charges per circuit for switching equipment, C_T the charges for each terminating equipment (including signalling equipment), N_2 is the number of extra channels to be switched, and N_1 the number of physical channels saved, then the length of line must exceed

$$\frac{N_2 \times C_C + 2(N_2 - N_1)C_T}{N_1 \times C_L}.$$

To take an example, let $C_L = 1$, $C_C = 6$ and $C_T = 75$, and let 30 direct circuits be reduced by $6(N_1)$ to $24(N_2)$; then the distance must exceed

$$\frac{24 \times 6 + 2(24 - 6)75}{6 \times 1} = \frac{144 + 2\,700}{6}$$
$$= \frac{2\,844}{6} = 474 \text{ kilometres.}$$

With small groups, it may be found that N_1 exceeds N_2. If the direct-route distance (d_1) differs from the combined-route distance (d_2), then

$$(N_1 + N_2)d_1 \times C_L - N_2 \times d_2 \times C_L$$

must exceed the annual charges

$$N_2 \times C_C + 2(N_2 - N_1)C_T.$$

If the direct route is of different construction and type from the combined route, it may be necessary to use C_{L1} and C_{L2} to represent different charge rates. Similarly, it may be necessary to replace C_T by C_{T1}, C_{T2}, or C_{T3}, where C_{T1} represents the complete termination to audio, C_{T2} represents a simple group filter, and C_{T3} represents only signalling equipment for cases when the through circuit is converted to audio. The annual equipment charge expression would then be

$$N_2 \times C_C + 2N_2 \times T_2 \text{ (or } T_3) - 2N_1 \times T_1.$$

Each switching point added may involve a small additional insertion loss due to the connection of signalling equipment.

5.2.4 Speed of Connection

This subject merits serious consideration in the planning of a new long-distance switching system. An important reason for a revised fundamental plan is to speed up the service. In the past, much of the delay has been associated with the process of operating—not owing to any failing on the part of the operators but because arrangements did not easily permit a high-speed service to be provided. One contributory factor was the inadequacy of circuits, and it is clear that with more automatic switching this hindrance could be removed. It is the general experience that on long-distance circuits the provision of an increased number of working circuits reduces the delays only for a short period, because the better service encourages more traffic until an overload is re-established. It is a common condition to find such traffic growing by 25 per cent per annum in cases where additional channels are provided to maintain a speedy service. The saturation point has not been reached in any country, although the capacity of the subscribers to make long-distance calls must be finite. A second contributory factor has been the result of the many processes involved in establishing an international call, including the preparation of the demand ticket, the transfer to the line position, the request to the distant operator, the recall of the calling party, and the monitoring necessary to ensure that the two parties are making progress with the conversation. The alternative method with semi-automatic operation can be speeded up in the following way:

A. Combined line and recording (c.l.r.) operation.

B. Operator uses keyset, storing required number in local sender.

C. Called number indicated on the combined-line-and-recording operator's position, avoiding the use of pencil and ticket before starting to complete the connection.

D. Connection to register in terminating country established on high-speed basis.

E. Transfer of information, in high-speed code, from local sender to distant register.

F. Preparation of ticket automatically.

With a local sender normally available, the operator can set up the called number immediately it is requested. She can check back with the caller the number set up and therefore recorded on her indicator. It seems probable that the national numbers can be more easily visualized if a combination of figures and letters is used rather than all figures. Compare the following examples:

217	BO2	5 800	and	217	262	5 800
816	HOL	8 765	and	816	405	8 765
94	CE	2 674	and	94	23	2 674

It is assumed that the country required can be selected by the use of an individual key for each country.

The connection between the sender and the terminating register can be achieved very quickly and at little cost if there is a group of direct circuits. Not very much greater expense need be involved if the transit centres are designed to make selection in a few hundred milliseconds each. It seems unlikely that there can be adequate justification for transmitting digits in the ordinary sense for setting up the connection to the register in the country required. For example, a call from London to Milan might possibly use a direct circuit from London to Milan or it might use a circuit from London to Zürich and a new circuit from Zürich to Milan. As a further possibility, it might use London–Paris and Paris–Milan circuits or Paris–Zürich and Zürich–Milan circuits. All different combinations might lie in the same multi-channel transmission system over the complete distance. The sender in London would presumably cause the different circuits to be searched in a definite order to maintain a high occupancy for the direct circuits. Whichever type of circuit is chosen, a particular seizing signal could be transmitted, which would be sufficient to set up the connection to the incoming register at Milan, from which a proceed-to-send signal would be transmitted to permit the national number to be sent. Such a simple and rapid means of completing the connection would hardly be possible with a single centralized transit centre, which would need a numerical signal to choose the correct direction, as the number of possibilities would be much larger. The same difficulty would arise with a number of regional centres all interconnected. A brief seizing signal controlling selection is valuable, not only in time of connection, but also because it tends to reduce the signal time, which needs to be kept as low as possible to avoid overloading the line amplifiers.

The transfer of the call description need not exceed approximately 2 seconds. The selection of the required subscriber in the terminating country may take some seconds because, although the call can be expedited until the local exchange is reached, from this stage the time taken cannot easily be less than for a local call. It seems reasonable that on most international calls the required subscriber's line should be selected within 30 seconds of the request being made. With such a quick service the calling subscriber is likely to be awaiting the call or otherwise capable of being summoned quickly.

Within the next 25 years, it is to be expected that improved automatic systems will be available, permitting relatively high-speed switching of local connections and, unless the international service is designed for high speed, it is liable to be found inadequate and an obstacle to the provision of the highest quality service. The initial cost of the equipment and the maintenance expense are not likely to be altered very much, whether the transit service is handled by quick or by normal switching arrangements.

For these reasons, the initial planning of any new international network should study and strive to incorporate rapidity of operation.

5.2.5 Signalling System

The elementary requirement from the signalling system is simplicity of operation, but experience has shown clearly that this result is not obtained by the use of a simple signalling system which is achieved by limiting the number of different signals. As an example, the ringers used on international circuits represent the most primitive and simple form of signalling and result in difficulties in operating and a low-grade service to the subscribers. To improve and speed up the service, it is necessary to have a universal operating method, an adequate code of signals to allow the operators to work quickly and effectively, signals which are not subject to interference, and a signalling system which promotes, and does not impede, a high-speed service.

The amount of signalling equipment which appears on international circuits is appreciable, but it may very well be a false saving to cut down on this equipment if it means delay and difficulty in operating efficiently. The Comité Consultatif International Téléphonique have studied the question of signalling in some detail, and a number of decisions have been reached.

TABLE VII

SIGNALLING COMPARISON

Objection	Signalling by voice-frequency currents	Signalling over separate channels
Interference from speech	Needs safeguards	None
Interference between systems	Needs safeguards	None
Interference from echo suppressors	Needs safeguards	None
Overloading of amplifiers in multi-channel transmission systems	Needs safeguards	Reduced liability to trouble
Speed of signalling:— (*a*) Direct (*b*) Transit	 High High	 High Needs delay for repetition
Passage of signals through transit exchange	Automatic	All signals must be transferred to direct current and superimposed on talking circuit
Line expense	None	Variable. Probably minor if quantities are large
Possibility of establishing connection on channel on which speech circuit is out of order	Unlikely	Possible
Distortion due to repetitions	Very small	Increases with each repetition

As on most international circuits, it is to be expected that 4-wire high-velocity speech circuits will be used. The use of direct-current or 50-cycle-per-second alternating-current signalling systems is ruled out, because such signals will not be transmitted over the carrier circuits. The interruption of the transmitted carrier current for signalling purposes is undesirable on account of intermodulation effects. The two main alternatives which remain are the use of voice-frequency and separate signalling channels; opinion favours the former. The relative advantages and disadvantages of the two methods are shown in Table VII.

It can be seen that most of the objections to voice-frequency systems concern interference. Interference from speech currents is liable to occur, especially if the signalling power used is very small and the receivers are very sensitive. It has been found that guard circuits provide a satisfactory safeguard, especially if the signal length is not too short. Most of the circuits equipped with voice-frequency apparatus in Europe have used either 600 and 750 cycles per second or 2400 and 2500 cycles per second. It is claimed that greater speech immunity can be obtained by using 600 and 750 cycles per second simultaneously, and tests carried out indicate that for any fixed time-delay the interference with the single frequencies is several hundred times that with the frequencies used simultaneously. This fault liability can be balanced by increasing the signal length, 150 milliseconds of compound-frequency being roughly equivalent to 400 milliseconds of simple-frequency operation. It seems likely that other frequencies, such as 2000, 2400, 2700 and 3000 cycles per second, may also be superior to 600 and 750 cycles per second, and tests are being undertaken to establish to what extent this is true. The frequency/response efficiencies of the subscribers' transmitters and the cut-off frequencies of existing cables are other aspects that must be considered at the same time. The existing recommendation by the Comité Consultatif International Téléphonique is to use 600 and 750 cycles per second, and the frequency or frequencies eventually chosen will have little bearing on the speed of the service and not a great effect on the expense of the equipment involved.

TABLE VIII

COMPARISON OF SELECTIONS OF NATIONAL SERVICE-TONES

Tone	Austria	Denmark	France	Germany	Great Britain	Sweden	Switzerland
Dialling	Uninterrupted		Interrupted in cadence (66/33 c/s)	Uninterrupted	Uninterrupted (33 c/s)	Uninterrupted	Uninterrupted
Ringing	1-sec tone 5-sec silence	1-sec tone 3.5-sec silence	1.66-sec tone 3.33-sec silence	1-sec tone 5-sec silence	0.4-sec tone 0.2-sec silence 0.4-sec tone 2-sec silence	1-sec tone 9-sec silence	1-sec tone 5-sec silence
Busy (normal)	0.5-sec tone 0.2-sec silence 0.5-sec tone 1-sec silence	0.175-sec tone 0.47-sec silence	0.5-sec tone 0.5-sec silence	0.5-sec tone 0.2-sec silence 0.5-sec tone 1-sec silence	0.75-sec tone 0.75-sec silence	0.25-sec tone 0.25-sec silence	0.25–0.50-sec tone 0.25–0.50-sec silence
Busy (subscriber busy but "no call waiting" given to operators only)			0.5-sec tone 2-sec silence				
Number unobtainable					5-sec tone 1-sec silence	0.25-sec tone 0.25-sec silence 0.75-sec tone 0.25-sec silence	0.5-sec tone 0.1-sec silence 0.1-sec tone 0.1-sec silence 0.1-sec tone 0.1-sec silence
Information		0.06-sec tone 0.09-sec silence 0.06-sec tone 0.09-sec silence etc., every 10th silence of 0.6-sec duration					

The possibility of interference to and from other systems has not been sufficiently widely understood; the trouble is due to signals passing naturally from end to end of a connection, and, in consequence, signals transmitted by one system may cause an unwanted response in another system. This can result in an incorrect signal or irregular release and therefore needs safeguarding. The Comité Consultatif International Téléphonique have recommended the use of a prefix signal which opens the line, thereby isolating the essential portion of the signal and preventing it from passing out to cause interference in any other system. This safeguard may delay the answer signal by 50 milliseconds and cause a small increase in cost, but it involves no real technical difficulty.

Interference from echo suppressors is a more serious menace to voice-frequency systems. One method of avoiding such interference is to repeat the signals until they have been acknowledged. This process adds to the stability of the service but it increases the cost and complication of the equipment. As future international circuits will be almost entirely of the high-velocity-carrier type, the trouble is only temporary, and it may be found possible to eliminate the interference by arranging the position of the echo suppressors so that they are not connected between the voice-frequency receivers.

The overloading of the line amplifiers in multi-channel transmission systems is probably the most difficult problem. These amplifiers have sufficient capacity for normal speech currents, but a large proportion of the conversation contains no speech at all in one direction or the other, so that the mean energy level is low and the permissible power and signal duration should both be limited.[12–14]

5.2.6 *Service Tones*

The difference in national service-tones is a good example of the difficulty which can arise as a result of independent development by separate administrations. At the present time,

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[12] W. G. Radley, and E. P. G. Wright, "Voice Frequency Signalling and Dialling in Long-Distance Telephony," *Journal I.E.E.*, v. 89, Part III, p. 43; 1942.

[13] F. P. O'Grady, "Dialling on Long Distance Trunk Lines—A Review of Developments 1944," *Telecommunication Journal of Australia*, Feb., 1945.

[14] H. Jacot, "Développement de la signalisation et la sélection automatique à fréquences vocales dans le réseau téléphonique suisse," *Bulletin de l'Association Suisse des Electriciens*, No. 21, 1946.

there are so many different codes of service tones in operation that it would be a real difficulty for the originating international operator to recognize the significance of any particular tone.

The recommendation made by the Comité Consultatif International Téléphonique in 1938 is too general to assist recognition, and administrations have been reluctant to make widespread changes to achieve any single standard. Table VIII shows a comparison of some national service tones, and it is hoped that it may be possible to overcome the difficulty mentioned in the following way:—

Dialling Tone	This will be generated in the adjacent sender. The voice-frequency proceed-to-send signal can be standardized. It will indicate when transmission of the stored information can be commenced.
Ringing Tone	An effort will be made to identify this tone in the incoming *tête-de-ligne* centre. On recognition, the line will be opened and a standard voice-frequency signal transmitted which will enable the outgoing *tête-de-ligne* centre to connect the national ringing tone, which would be recognized by the caller. When the call is answered the line is reconnected.
Busy Tone (Transit Centre)	This can easily be turned into a voice-frequency signal and enable the outgoing *tête-de-ligne* centre to connect the national busy-tone and clear the international line.
Busy Tone (Subscriber)	An effort will be made to identify this tone in a similar way to the ringing tone, and to apply a standard voice-frequency signal which will enable the national busy-tone to be connected.
Miscellaneous, Number Unobtainable and Changed-Number Tones	It will be necessary for the outgoing operator to seek assistance from an incoming operator in the event of receiving an unrecognized tone. As most of the tones of this type need operator assistance this procedure may not cause any great delay.

It seems that this solution does not go far enough, and further study should be undertaken to investigate the possibility of reducing the present non-uniformity by agreeing upon a standard code which should become operative in, say, 20 years' time.

Uniformity of tones is valuable on national calls as well as international calls, because many travellers abroad are handicapped in making calls on account of the difficulty of recognizing the service tones.

5.2.7 Language Differences

International calls fall into two general classes, one in which the caller is intending to speak in the language of the country which he is calling, and the other in which the caller is intending to speak his own language and may not understand the language of the country which he is calling.

In the former case, especially if the traveller is speaking to his home country, most of the difficulty is experienced in obtaining access to the international operator and passing the details of the call. Once he is connected to his own country the only language difficulty will be that of the international operator in trying to determine whether the required connection has commenced. This should be indicated by the supervisory signals, and it is essential that the code of signals should make full provision for giving complete supervision.

If the caller is making a call to an individual in a foreign country and is unable to converse in that language, it is probable that he will originate a personal call to ensure reaching someone with whom he can talk. The line operator who is unable to speak the language concerned will presumably transfer such calls to an operator speaking the language or else call the inward *tête-de-ligne* operator to obtain her assistance.

TABLE IX

LANGUAGE DIVISIONS

Belgium	2 main languages, Flemish in north, and Walloon in south
Bulgaria	1 main language
Canada	2 main languages
China	Many variations
Czechoslovakia	2 main languages
Eire	2 main languages
Finland	1 main language, 2 fringe districts differ
Great Britain	1 main language, 1 variation
Greece	1 main language
Hungary	1 main language, 3 fringe variations
India	5 main languages, with variations
Poland	1 main language, 3 fringe variations
Rumania	2 main languages, 1 fringe variation
Switzerland	3 main languages
U.S.S.R.	Several variations
Yugoslavia	3 main languages, 4 fringe variations

Note: In addition to the variations in language, most countries have one or more dialects.

Language problems are likely to be complicated by the fact that several countries are bilingual while in other countries there is a considerable variation in dialect. Table IX emphasizes these variations.

It is abundantly clear that in spite of every precaution being taken at the originating *tête-de-ligne* centre there will be cases when an unexpected language is encountered in setting up an international call, and it is unreasonable that these calls should be broken down and re-set up through a second operator. Such a practice would tend to encourage the outward operator to set up all calls in this way in order to ease the operating. What is required is a signal that will enable the outward international operator to summon rapidly an inward international operator. It is believed that a forward transfer signal of this character could be incorporated without undue difficulty, and that such a signal is, in fact, used in some national voice-frequency signalling systems. One important rule which the language difficulty emphasizes is that the outward international *tête-de-ligne* operator should supervise the connection; any intermediate zone-centre operator who has helped to extend the call to the international centre should pass the control forward.

5.2.8 *Tariff*

Every administration has a settled scale for regular and special-attention calls, and the new toll plan does not need any fundamental change to the tariff structure. The record of the call can be made on a ticket prepared either by the international operator or by a machine; possibly in the former case, but certainly in the latter, the charge calculation will be automatic. In order that the operator may concentrate on setting up the connection, there are obvious advantages in allowing the ticket to be printed independently, and it is clearly desirable to employ some form of machine ticketing with any application involving subscriber dialling. In addition to the charge calculation, it is necessary to inform the caller, on request, of the charge to be made. It would be possible to provide "duration and charge advice" positions on which the details of the calling and called numbers, the duration of the call and the charge were all indicated; this information could then be passed to subscribers without delaying the long-distance operators.

It has recently been agreed that the international accounting shall be based on the tickets made at the originating centres. Such an arrangement is most appropriate for a high-speed semi-automatic or high-speed full-automatic service. If it is necessary for the transit or terminating countries to obtain information about the identity of the calling subscriber, more equipment is needed to record the information, and there is consequently a danger that the service may be retarded. The decision to use the ticket from the originating country need not deprive the transit and terminating switching centres of necessary information. There is a need for some method of checking traffic flow in order to determine the seasonal variations, the normal rate of growth and how much traffic is directed to different countries. It is clearly more difficult to collect information on these subjects on international than on national circuits. With the plan as outlined, it would be relatively simple to record at the transit centres the route on which a call occurs, the route to which it is switched, and the duration of the connection. These details could be recorded by ticket machines and the information analysed to ascertain the traffic characteristics.

5.2.9 *Alternative Routes*

Another subject associated with tariff rates, which is not easy to evaluate, is the best method of handling alternative routes. It should be made clear that emergency routes are subject to special-rate regulations which are not expected to cause serious complications in the future.

The alternative-route problem should be divided into two categories, since two quite separate principles are involved. In the first place it has been an operating practice for many years to have alternative means for a switchboard at A to reach a remote switchboard at B via a switchboard at either C or D, and sometimes through other routes as well. These diversions are based on the non-coincidence of busy-hour traffic. From the operating standpoint it is rather more elementary; if A finds that all the channels to C are in use she applies to D for a channel. If *D* has no direct channel to *B*, then *D* may ask *E* for a channel to *B*. The call is established, and it is difficult to know how much traffic is displaced by the diversions. If the groups are large, very little is gained; but if not, there is a serious tendency for traffic from a large group to swamp a small group. The arrangement as a whole is unsound, except as a temporary expedient.

The second application is based on a different principle. In this case certain direct routes are calculated on a high-loss rate, such as one call in four, and the overflow is arranged to be carried over transit groups which are materially larger than the direct groups. The transit groups are designed to cater for this overflow and are, in fact, less liable to variation in busy-hour traffic than individual groups. This arrangement has been used widely both in America and in Europe; it leads to considerable economy on direct circuits at very little cost on the transit circuits. On international circuits, the indirect route is liable to pass through an additional country. It is not difficult to record the traffic which uses the alternative route and to credit the third administration, so that the essential requirements can be met. There remains the fact that, on perhaps 5 per cent of the calls, a third administration draws a fee to the detriment of the administration in whose territory the call originated, and it is possible that the thought that something is being lost will be misunderstood and that an effort will be made to provide more direct circuits on an uneconomical basis, resulting in increased annual charges on the direct circuit. In this way, a greater loss would be incurred than by obtaining assistance from a third administration. In many cases reciprocal action would tend to balance the account, but it is certainly doubtful whether the basic economics of the arrangement will prevail where several administrations are concerned.

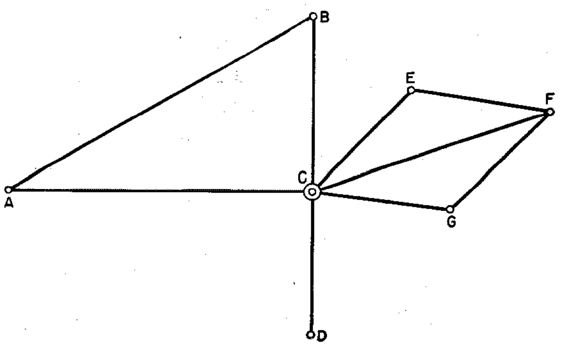

Fig. 3—Alternative routing with one principal transit centre.

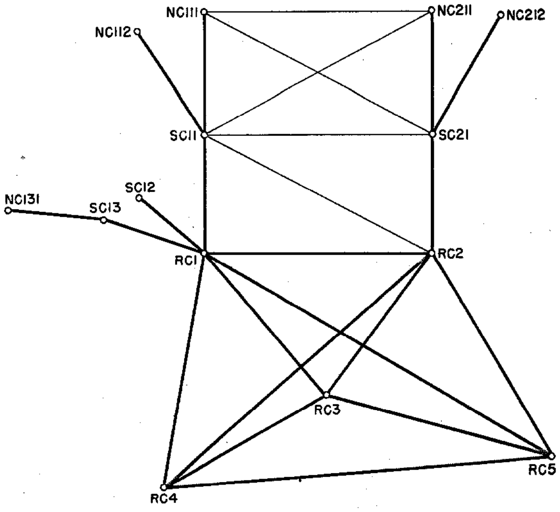

Fig. 4—Alternative routing with interconnected regional centres.

──── **High probability of loss circuits.**

━━━━ **Low probability of loss circuits.**

Figs. 3, 4, and 5 show the differences in the three cable layouts already considered as regards alternative routes.

In Fig. 3, *A* has a direct circuit to *B* and can overflow reasonably through *C*, unless *A–B* traffic is greater than *A–C* or *B–C* traffic. If *A* normally reaches *F* through *C*, and *A–C* is engaged, an attempt can be made through *B*, but this is satisfactory only if *A–B* and *B–C* are greater than *A–C*. Similarly, if *C–F* is engaged, overflow may be effected through *C–G* and *G–F* or *C–E* and *E–F*, provided these routes are greater than *C–F*, which is doubtful. It will be seen that the first case is opposed to the second. The efficiency of the plan depends very much on the length of *A–C*.

In Fig. 4, the basic plan presupposes that the regional centres *RC* are all fully interconnected and that each *RC* has a number of section centres *SC* to which, in turn, are connected national centres *NC*. All these circuits are high-grade, with a normal probability of loss. In addition, direct circuits with a high probability of loss may connect adjacent centres. In this case, a call from *NC*111 to *NC*211 might make a primary attempt at a direct connection, or, if this were not possible, an attempt through *SC*21, it being fundamental that *SC*21 has a normal-loss circuit to *NC*211. If this failed, *NC*111 would pass the call to *SC*11, which might try a high-loss circuit to *NC*211 and, if it had the direct circuits, via *SC*21 or *RC*2. Failing to make connection, the call is passed to *RC*1, and from there to *RC*2, *SC*21, and *NC*211 on normal-loss circuits throughout. In such an arrangement a high efficiency of direct circuits is obtained, which is very desirable if the geographical situation of *RC*1, *RC*2, etc., tends to involve heavy back-hauls. It should be noted that the concentration of the *RC* transit centres adjacent to one another is purely to simplify the sketch; in practice these transit centres would be well dispersed over the whole area.

In Fig. 5, *A* may have direct circuits to *K*, *L*, and *M* at a high probability of loss; also overflow access through the ring, which would provide normal-loss circuits. Alternative routes would also be available from *A* to *N* by means of *E* or *F*, which might be reached in the same direction of traffic or in opposite directions around the ring. If the international operator sets up the call requirements into adjacent senders it is probable that any alternative routing to be introduced will be independent of any special action of the operator. This is correct, as it is undesirable to burden the operator with considerations of which routes should be tried or, in fact, to allow any possibility of the operator setting up unauthorized routes.

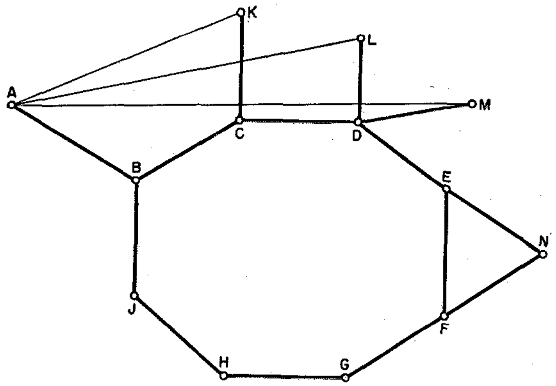

Fig. 5—Alternative routing with ring formation.

—— High probability of loss circuits.
━━ Low probability of loss circuits.

The quantities shown in Table V are based on the use of alternative routes, and it is noteworthy that a considerable economy of circuits is made possible. This can be appreciated from Table X, which compares the layouts (for Austria) with and without alternative routes for the star and ring formations corresponding to Figs. 3 and 5.

TABLE X

COMPARISON OF CABLE ROUTES EXPRESSED IN CIRCUIT-KILOMETRES WITH AND WITHOUT ALTERNATIVE ROUTES

Formation of cable plan	Type of circuit	*With* alternative routes	*Without* alternative routes
Star (Fig. 3)	Direct	156 350	167 120
	To centre	8 000	10 000
	Terminations (overflow)	9 450	—
	Terminations (indirect)	6 430	25 800
		180 230	202 920
Ring (Fig. 5)	Direct	65 670	66 330
	Ring	33 040	37 500
	Terminations	75 200	79 180
		173 910	183 010

The quantities in Table V demonstrate that the star arrangement is more dependent on alternative routing than is the ring arrangement, except in the case of Germany. With the ring formation, the direct and alternative routes are frequently passing through the same countries, but with the star formation it is usual for the direct and alternative routes to pass through different countries.

5.2.10 Numbering Plans

In comparison with North America, the switching in Europe, following different practices, is necessarily divided into more isolated groups. There is likely to be little desire for a continental numbering plan, and therefore the problem of numbering becomes a national affair, with a simple addition for the country selection. Both in America and in Europe, it is likely that there will be cases where a single numbering plan will spread over a national border, e.g., Buffalo, Detroit, etc.

5.3 SWITCHING EQUIPMENT

The desire to set up connections rapidly would appear at first sight to set some restrictions on the type of switching equipment used in the transit centres, but it is by no means certain that high speeds cannot be obtained with all the principal systems by some appropriate circuit rearrangement. There is also an argument that standardization is necessary to enable all transit centres to provide the necessary facilities. On investigation it seems that the argument is not as powerful in this case as in that of signalling. Non-standardization in signalling means repetition, involving additional equipment and delay. On the other hand, there is a good argument that the equipment to be used at the transit centres should be similar to that used normally for national service. Similarity in this way will ensure adequate maintenance knowledge and frequently mean that additional equipment can be obtained more quickly.

TABLE XI

Types of Switching System

Algiers, Morocco and Tunisia	R.6 and French step-by-step
Argentina	English step-by-step
Australia	English step-by-step
Austria	Austrian step-by-step
Belgium	Chiefly rotary, some step-by-step
Brazil	Rotary, American and English step-by-step
Bulgaria	German step-by-step
Canada	Canadian step-by-step
Chile	English step-by-step
China	English and German step-by-step
Cuba	American step-by-step
Czechoslovakia	Chiefly German step-by-step, some rotary
Denmark	Rotary, some Swedish Ericsson
Eire	English step-by-step
Finland	Chiefly German step-by-step, some Swedish Ericsson, and some crossbar
France	Paris and other big cities: rotary. Provinces: R.6
Germany	German step-by-step
Great Britain	5 cities: director type step-by-step. Provinces: normal step-by-step
Greece	German step-by-step
Hungary	Rotary
India	English and German step-by-step
Italy	German step-by-step, Swedish Ericsson and rotary
Japan	English, German, and Japanese step-by-step
Netherlands	Chiefly German step-by-step, some rotary, and Swedish Ericsson
New Zealand	English step-by-step, rotary
Norway	Rotary, some Swedish Ericsson
Poland	Chiefly Swedish Ericsson, some English step-by-step, and some rotary
Portugal	English step-by-step
Rumania	Rotary
Russia	Swedish Ericsson
South Africa	English and German step-by-step
Spain	Rotary
Sweden	Swedish Ericsson
Switzerland	Chiefly Haslar, some rotary, and some German step-by-step
Turkey	Swedish Ericsson and rotary
U.S.A.	Crosbar, panel, and American step-by-step
U.S.S.R.	Swedish Ericsson
Yugoslavia	Swedish Ericsson and rotary

The variations in the use of systems in different countries are very striking, as can be seen from Table XI.

It can be imagined that there might be some difficulty in deciding which system should be made a standard for transit centres.

5.4 Subscriber Dialling

This paper has referred chiefly to the provision of a semiautomatic service, but it is considered that as far as possible the plans should be suitable for subscriber dialling also. Objections are raised to the idea of subscriber dialling, but, from experience with long-distance dialling in national systems, many of the objections do not seem justified.

The case for international subscriber-dialling is as follows. Although many calls will continue to need assistance from an operator, there are many which the subscribers would prefer to make themselves. Some calls are little longer than local-area calls, e.g., Detroit and Windsor, Antwerp and Rotterdam, etc.

The objections to subscriber dialling are that most international calls are relatively expensive and that the subscriber has a right to the assistance of an operator, that servants and small boys will be free to set up long-distance and expensive calls, and that in the event of a wrong number the subscriber will be charged a considerable sum which he will resent.

The response to these arguments is that if the subscriber prefers to dial some calls direct, why should he be forced to have the assistance of the operator? Also, servants and small boys can ask for international calls through an operator; and it is not necessary to charge a 3-minute fee for a call which is released in under 15 seconds; it is found on long-distance national dialling that 10 seconds is sufficient to recognize a wrong number, and the charge for this period has been made the same as for a local call.

Two practical difficulties to subscriber dialling are the need for a dial number-plate carrying letters and figures, and the large expense involved in installing identification circuits for all subscribers where they do not exist already for the national service. It seems a possibility that an international dialling service may commence as a limited service, restricted to certain lines equipped for such a service. It seems very likely that if such a service were made available, a very high proportion of the requests would come from subscribers with private branch exchanges, operated by qualified operators.

6. National Telephone Systems

Appendix, Section 8, contains technical information of general interest concerning the national systems in Argentina, Belgium, Denmark, France, Hungary, The Netherlands, Portugal, Sweden, and Switzerland.

7. Acknowledgments

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8. Appendices

8.1 TELEPHONE INFORMATION ABOUT ARGENTINA

8.1.1 Toll Traffic

TABLE XII

NATIONAL AND INTERNATIONAL TOLL TRAFFIC IN MILLIONS OF CALLS

	As estimated in 1937	Actual
1930	—	7.5
1932	—	7.2
1934	—	7.8
1936	—	10.0
1938	13.0	12.7
1940	15.4	13.9
1942	17.7	16.0
1944	20.3	18.7
1945	21.8	20.0

8-year increase, 1930–1938 = 70 per cent.
7-year increase, 1938–1945 = 57.5 per cent.

8.1.2 International Traffic

The outgoing international calls in 1945 were 200 000, an increase of 87 per cent on 1938; the incoming international calls were 212 000 in 1945, an increase of 56 per cent on 1938.

Note: In Chile, the outgoing international calls increased between 1940 and 1945 by 233 per cent, and the incoming calls by 200 per cent. A large percentage of the international traffic from Chile passes into Argentina.

8.1.3 Primary Toll Centres

Buenos Aires
La Plata
Mar del Plata
Necochea
Tandil
T. Arroyos
Bahia Blanca
Pehuajo
Chililcoy
General Pico
Junin
Rufino
Rio IV
Pergamino
Va. Maria
Cordoba
Rosario
Santa Fé
Reconquista

Buenos Aires will have direct circuits to all other centres, except possibly Reconquista. The mean length of these circuits is approximately 400 kilometres, and the maximum about 800 kilometres. Buenos Aires will also have tributary circuits to another 45 centres with distances varying between 50 and 300 kilometres.

Other long-distance lines radiate to Mendoza, Tucuman, and Entre Rios. Land lines provide international circuits to Santiago and Montevideo.

There are radio connections to New York, a combined circuit for Madrid, Paris, London, and Berlin, and independent circuits to Rio de Janeiro, Bariloche, Lima, and La Paz.

8.1.4 Long-Distance Dialling

Thirty of the circuits between La Plata and Buenos Aires are arranged for operator key-sending, using a register at La Plata and a 50-cycle-per-second current for passing the information over the toll line.

The circuits between Mar del Plata and Buenos Aires operating over 384 kilometres employ direct-current dialling. These circuits are open wire and subject to low-resistance leakage to ground due to heavy mist and flying cobwebs.

The long-distance circuits will be converted from manual to semi-automatic working in accordance with the fundamental plan.

8.1.5 Transmission Systems

The country is suitable for carrier circuits, and it is expected that a considerable extension of 12-channel and 3-channel systems will be brought into service in the next 5 years.

8.1.6 National Numbering Scheme

No definite plan is yet established, but the ultimate nation-wide operator-dialling layout is likely to incorporate an overall numbering scheme.

8.1.7 Manual Toll-Traffic Switching

The present distribution of toll traffic by switching points is approximately:

Direct, 80 per cent
1 switch, 16.5 per cent
2 switches, 3 per cent
3 or more switches, 0.5 per cent,

the operating time-factor being:

Direct 1.0
1 switch, 3.14
2 switches, 5.28
3 switches 8.14

8.1.8 Relationship between Distance, Duration, and Percentage of Revenue for National Long-Distance Calls (See Table XII)

8.2 Telephone Information about Belgium

8.2.1 Transmission

Two-wire lines are used so long as no more than two toll-line sections are connected in tandem. These 2-wire lines are divided into separate groups for terminal and tandem traffic.

Four-wire lines are used for all except the first and last sections when the number of sections connected in tandem exceeds two.

8.2.2 Routing

If a secondary exchange is connected to two primary exchanges, both incoming and outgoing traffic has to be routed over the shorter route.

8.2.3 Tandem Routing

Up to 5 primary toll exchanges may be involved in one connection.

8.2.4 Numbering

The country is divided into 47 zones, each of which has a closed numbering system. Two or three zones have, or will have, 6-digit numbers and will be reached by a 2-digit toll prefix; the remaining zones have 5-digit local numbers and will be reached by a 3-digit toll prefix. The total number of digits to be dialled for calls outside the local zone is therefore 8 in every case.

8.2.5 Metering

All toll charges, with the exception of those for calls to neighbouring exchanges within the local call zone, will be registered by automatic ticket printers. The tariff is determined by the distances between the centres of each "sector," 23 of the so-called "larger zones" being divided into a number of "sectors," which may vary between two and six, whereas the remaining small zones have only one "sector." The fee may be determined by the prefix and the two or three first figures of the subscriber's number.

TABLE XIII

Distance kilometres	Mean duration minutes	Percentage of total calls	Percentage of total revenue
0–50	2.4	41.72	12.15
51–100	2.73	26.84	14.93
101–150	2.91	7.81	7.71
151–200	3.1	4.93	6.68
201–250	3.24	4.92	9.12
251–300	3.35	4.62	11.2
301–350	3.45	1.73	4.56
351–400	3.55	3.27	10.34
401–450	3.61	0.76	2.65
451–500	3.68	0.76	2.97
501–600	3.75	0.92	4.47
601–700	3.81	0.904	4.96
701–800	3.88	0.139	0.97
801–900	3.95	0.042	0.34
901–1 000	4.01	0.353	3.62
1 001–1 100	4.08	0.163	1.77
1 101–1 200	4.15	0.003	0.04
1 201–1 300	4.21	0.1	1.34
1 301–1 400	4.28	0.014	0.16
1 401–1 500	4.35	0.002	0.02

8.2.6 Time Limit

Forced release takes place after 99 minutes of conversation, but means are provided to release in case of emergency after 3 minutes.

8.2.7 Traffic Metering

A traffic meter is provided in each toll register in order to indicate the total number of connections set up to each zone. All remaining traffic information is obtained from the tickets prepared by the automatic ticket printers.

8.2.8 Registers

Arranged for a total of 8 figures (as explained in Section 8.2.4), code translation for 1 to 6 selections of 1 to 20 impulses, and for controlled action with "seize" impulse and "end-of-selection" impulse. Impulses are transmitted in the forward direction.

8.2.9 Outlet Searching

All toll lines work in ideal groups.

8.2.10 Selection Time

No maximum fixed.

8.2.11 Alternative Routing

Provision is made for alternative routing independent of register translation.

8.2.12 Impulsing

Primary toll centres are provided with impulse correctors.

8.2.13 Signalling

By 50-cycle-per-second current; also on 4-wire lines.

8.2.14 Signals

The same as for Switzerland (see Section 8.10), except for "offering" which is omitted.

8.2.15 Pulse Release

Subscriber dialling, 30–60 seconds.
Incoming register, 30–60 seconds.
Incoming tandem line, 5–10 seconds.
Busy or ringing condition—operator is called in after 30–60 seconds.

8.2.16 Insertion of Repeaters

Each incoming tandem line is provided with a repeater, cancelling the loss of the associated line.

8.2.17 4-Wire Switching

This is provided.

8.2.18 Prefix for International Traffic

004.

8.3 TELEPHONE INFORMATION ABOUT DENMARK

8.3.1 National Numbering Scheme (See Table XIV.)

TABLE XIV

*01	Copenhagen	†055	Nyköbing Fl.
		056	Maribo
021	Helsingör	057	Nakskov
022	Hilleröd	058	Vordingborg
023	Frederiksund	059	Stege
024	Roskilde		
025	Holboek	*061	Aarhus
026	Kalundborg	062	Randers
027	Köge	063	Grenaa
028	Ringsted	064	Rönde
029	Noestved	065	Odder
020	Rönne	066	Horsens
		067	Skanderborg
†031	Odense	068	Silkeborg
032	Svendborg	069	Hammel
033	Faaborg	060	Tranebjoerg
034	Assens		
035	Middelfart	†071	Herning
036	Bogense	072	Skjern
037	Kerteminde	073	Ringköbing
038	Nyborg	074	Holstebro
039	Rudköbing		
		†075	Skive
*041	Kolding	076	Viborg
042	Vejle	077	Lemvig
043	Fredericia	078	Hurup
044	Sönderborg	079	Thisted
045	Haderslev	070	Nyköbing M.
046	Aabenraa		
047	Tönder	†081	Aalborg
048	Gram	082	Skagen
049	Holsted By	083	Frederikshavn
		084	Hadsund
†051	Esbjerg	085	Hobro
052	Varde	086	Aars
053	Grindsted	087	Fjerritslev
054	Ribe	088	Brönderslev
		089	Hjörring

* International *tête-de-ligne* centre.
† National transit centre.
Remainder are group centres.

8.3.2 *Local Numbering Scheme*

Each of the 67 groups has an independent closed numbering scheme. For the smaller groups, it is a 4-digit system; some larger groups are on a 5-digit basis, while Copenhagen is 6-digit.

8.3.3 *Alternative Routing*

This is anticipated.

8.3.4 *Signalling Frequencies*

50 cycles per second and 2400/2500 cycles per second.

8.3.5 *Signals in Use or Proposed*

Seizing signal
Impulsing signal
Answer signal
Clear-back signal
Clear-forward signal
Proceed-to-send signal
Backward end-of-impulsing signal
Ring-forward signal
Trunk-offering signal
Breakdown signal
Release signal

8.3.6 *Number of Subscribers' Lines*

City subscribers	342 000
Rural subscribers	105 800
Official lines	2 700
	450 500

Instruments, including 2- and 4-party lines—558 150.

8.3.7 *Operating Administrations*

State	Möen South Jutland Toll network beyond 130 kilometres
Copenhagen Telephone Company	Zealand Isle of Bornholm
Lolland Falster Telephone Company	Lolland and Falster
Funen Telephone Company	Funen
Jutland Telephone Company	North Jutland
Saux Telephone Company	Saux

8.3.8

Copenhagen is semi-automatic; Odense has a call distribution system, and most of the other cities are manually operated.

8.3.9

Rural areas are 99 per cent manual. There is a total of about 1500 exchanges. Haderslev (2400) and Aabenraa (1800) are centres of rural automatic areas in South Jutland.

8.3.10

The new toll board at Aalborg will be of the cordless type.

8.3.11

A 12-channel carrier system is used between Copenhagen and Aalborg. There will probably be both 4-wire and 2-wire switching, the latter arranged for 4-wire switching, ultimately.

8.3.12 *Tariffs*

Local call	5 öre.
Less than 10 kilometres	10 öre.
Between 10 and 15 kilometres	15 öre.
Longer distances	Up to 40 öre.

8.4 TELEPHONE INFORMATION ABOUT FRANCE

8.4.1 *Transit Centres*

Principal: Paris, Lyons, Marseilles, Bordeaux, and Strasbourg; all interconnected by 4-wire circuits.

Ordinary: Rouen, Lille, Amiens, Nancy, etc. Each is connected to at least two of the principal transit centres.

8.4.2 *Distribution Centres*

Calais, Dunkerque, Dieppe, etc. Each is connected to one, or several, transit centres.

8.4.3 *Trunk and Toll Circuits*

These are divided into two groups and four categories:

Group 1.—Transit circuits.
Group 2.—Terminal circuits.

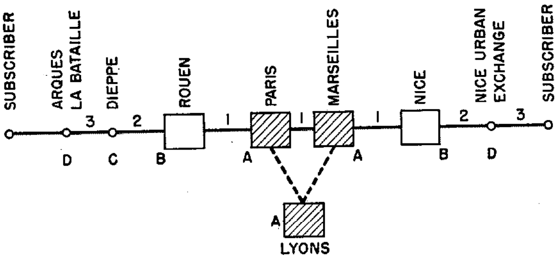
Category 1—4-wire circuits interconnecting transit centres.

Category 2—2-wire circuits interconnecting transit centre and distribution centre.

Category 3—Local junctions and subscriber circuits.

Category 4—Terminating circuits.

8.4.4 Example of Built-Up Connection



A—Principal transit centre.
B—Ordinary transit centre.
C—Distribution centre.
D—Group centre.

8.5 GENERAL PROPOSALS FOR FUTURE NATIONAL NUMBERING SCHEME

8.5.1 National Numbering

All national numbers will be on an 8-digit basis. Subscribers in Paris and the Seine Department will have a 7-digit regional number prefixed by a single digit to form the national number. All other subscribers will have a 6-digit regional number coupled with a 2-digit prefix.

The existing Paris numbers will be modified from a regional to a national basis by the addition of the prefix "1," e.g., VAU 2570 will become 1 VAU 2570.

Existing numbers in Lyons and Marseilles including a letter and four digits will add one letter to complete the regional number and two digits for the national prefix, e.g., (Lyons) F(ranklin) 4322 will become 53 FR 4322.

Existing 5-digit numbers will add a letter for the regional number and two digits for the national prefix, e.g., Rouen 25381 will become 27 R2 5381.

Existing 4-digit numbers will add a letter and a digit for the regional number and two digits for the national prefix, e.g., Elboeuf[15] 2345 will become 27 R8 2345.

8.5.2 Telephone Regions

The telephone region covers a district with a local community of interest, and the 6-digit regional number suffices for all connections in this district. Each region may contain a number of towns in different departments, e.g., the Lyons region would include St. Etienne, Grenoble, Annecy, Chambery, Bourg (i.e., the whole of the Department of the Rhône), and a part of the Ain, the Haute-Savoie, the Savoie, the Isère, and the Loire.

To simplify the dialling and the taxation, each department may be given a characteristic national prefix, in which case the telephone region required may be identifiable only with the first digit of the regional number.

8.5.3 Long-Distance Prefix

A subscriber obtains a long-distance call by the general prefix "16." Hence a Lille subscriber would call Paris, Lyons, and Rouen as follows:—

16	1	VAU 2570
16	53	FR 4322
16	27	R2 5381

8.5.4 Metering

A. Calls within canton	Single fee.
B. Calls within department or between adjacent departments	Determined by distance between chief places in cantons.
C. Calls between non-adjacent departments	Determined by distance between chief towns in departments.

8.5.5 Time Limit

Not determined.

8.5.6 Register and Translator Functions

The facilities provided with the R6 system for exchanges outside Paris, Marseilles, Nantes, and Arques would be as follows:—

A. Regional calls	Translation for three selections in addition to the 1st selector. Retransmission of part, or all, of the six digits received.

---

[15] Elboeuf is a satellite of Rouen.

B. National calls Receives "16" and eight digits without requiring a second dialling tone. Retransmits the eight digits to district national register.

The register sends a number of impulses not exceeding 18 to a meter-control circuit which operates the subscriber's message register every three minutes after the conversation has commenced.

8.5.7 Selection Time

2–5 seconds.

8.5.8 Alternative Routes

These will be provided.

8.5.9 Impulsing

Impulse correctors will be used.

8.5.10 Signalling

Tests are being carried out with 50 cycles per second, 600 and 750 cycles per second compound, and 2000 cycles per second.

8.5.11 Code of Signals

A. Terminal traffic:—

Seize, proceed-to-send, impulsing, answer, release.

B. Transit traffic:—

All terminal traffic signals, forward end-of-selection, backward end-of-selection, called-subscriber-free signal, clearback signal, backward-recall signal.

TABLE XV

Name	Number of Lines	Name	Number of Lines
Budapest	80 000	Balassagyarmat	1 250
Hatvan	1 250	Szolnok	2 500
Kecskemét	1 500	Székesfehérvár	1 750
Komárom	1 200	Miskolc	2 750
Eger	1 100	Sátoraljaujhely	600
Debrecen	2 800	Nyiregyháza	1 900
Mátészalka	600	Karcag	700
Békéscsaba	2 500	Széged	3 800
Szekszárd	1 200	Baja	1 600
Pécs	2 800	Kaposvár	1 300
Dombóvár	600	Barcs	500
Balatonföldvár	600	Keszthely	700
Nagykanizsa	1 100	Zalaegerszeg	700
Pápa	750	Veszprém	1 100
Szombathely	1 600	Györ	2 200
Sopron	1 200		

8.5.12

Insertion of repeaters. No switched repeaters will be used.

8.6 TELEPHONE INFORMATION ABOUT HUNGARY

8.6.1 Toll Centres (See Table XV.)

8.6.2 Long-Distance Dialling Circuits (See Table XVI.)

TABLE XVI

Dialling Frequency Cycles per Second	Number of Circuits
50	53
2900	16
600/750	11 } Installed, but no longer in service
150	8 }

8.6.3 Subscriber Long-Distance Dialling

This will require seven digits; local service inside Budapest, six digits; and country districts, four digits. Digits "0" and "9" are reserved for long-distance service.

8.6.4 Call Duration

Local—2 minutes.
Toll—3–4 minutes.

8.6.5 Long-Distance Setting-Up Time

12–18 seconds.

8.6.6 Multi-Metering

It is planned that calls within an area will be multi-metered. Long-distance calls will be recorded automatically by ticket printers.

8.7 TELEPHONE INFORMATION ABOUT THE NETHERLANDS

8.7.1 National Numbering Scheme (See Table XVII.)

TABLE XVII

Toll Regional Centres	Code	Toll Terminal Areas
Goes	011	Middelburg, Zierikzee, Kruiningen, Oostburg, Ijzendijke, Terneuzen, Hulst
Breda	016	Tholen, Bergen op Zoom, Steenbergen, Roosendaal, Oudenbosch, Oosterhout.
's Gravenhage	017	Leiden, Alphen, Delft, Naaldwijk

TABLE XVII *Continued*

Toll Regional Centres	Code	Toll Terminal Areas
Rotterdam	018	Gouda, Gorinchem, Sliedrecht, Dordrecht, Oude Beijerland, Spijkenisse, Middelharnis
Alkmaar	022	Den Burg, Den Helder, Middenmeer, Schagen, Noord Scharwoude, Hoorn, Enkhuizen
Haarlem	025	Beverwijk, Zandvoort, Hillegom, Hoofddorp, Lisse
Amsterdam	029	Purmerend, Zaandam, Weesp, Amstelveem, Hilversum, Loenen
Utrecht	034	Harderwijk, Amersfoort, Tiel, Barneveld, Woerden, Culemborg, Leerdam, Jutphaas, Vreeswijk, Doorn
's Hertogenbosch	041	Tilburg, Waalwijk, Heusden, Zaltbommel, Oss, Veghel
Maastricht	044	Gulpen, Heerlen, Sittard
Venlo	047	Roermond, Helden-Panningen, Horst, Venray
Eindhoven	049	Eersel, Valkenswaard, Helmond, Deurne, Weert
Leeuwarden	051	Franeker, St. Anna Parochie, Dokkum, Veenwouden, Drachten, Sneek, Akkrum, Workum, Lemmer
Zwolle	052	Heerenveen, Wolvega, Steenwijk, Vollenhove, Meppel, Kampen, Ommen, Dedems Vaart, Elburg
Hengelo	054	Rijsen, Almelo, Goor, Neede, Oldenzaal, Groenlo, Enschede, Winterswijk
Beilen	055	Assen, Smilde, Borger, Emmen, Terapel, Hoogeveen, Oosterwolde, Coevorden
Groningen	059	De Leek, Winsum, Veendam, Middelstum, Appingedam, Hoogezand, Zuidhorn, Winschoten, Onstwedde
Deventer	067	Raalte, Epe, Apeldoorn, Zutphen, Vorden, Lochem, Uddel
Arnhem	083	Wageningen, Zevenaar, Dieren, Terborg, Doetinchem
Nijmegen	088	Cuijk, Grave, Druten, Zetten

8.7.2 Transmission

Although on some routes 2-wire lines are still in existence, 4-wire lines will be used throughout in the ultimate transmission plan.

Long-distance circuits are 12- or 24-channel carrier systems, which in future, and when necessary, may be extended to 36- or 48-channel systems. All these lines work on zero overall loss, and are used indiscriminately for terminating and tandem traffic, whenever the two types exist in one direction.

8.7.3 Routing

The latest plan foresees that, with a few exceptions, all district centres (total of 20 for the country) are directly interconnected by circuits with a high probability of loss. The overflow is alternatively routed through a common transit centre. The latter is directly connected to all district centres by circuits with a low probability of loss. The number of direct circuits is so calculated that some 5 or 10 per cent of the traffic overflow to a group of overflow circuits (common for all directions) which leads to one of the 20 district centres acting as an overflow tandem-exchange. This latter exchange is directly connected to all other district centres by a sufficient number of circuits (incoming and outgoing) to cater for the handling of all overflow traffic directed to it.

8.7.4 Numbering

The toll prefix comprises five figures, i.e., first figure "0" indicates toll call; combination of second and third figures indicates one of 20 districts into which the country is divided; the fourth figure indicates one of 10 sectors into which each district is divided; and the fifth figure indicates one of the 10 local exchange areas of each sector.

The local subscriber numbers may be 3, 4, 5, or 6 figures. The toll prefix is omitted for local calls exclusively.

8.7.5 Metering

The present metering is determined by the airline distances between the local-exchange areas and by time.

The fee may be determined completely by the prefix.

A new metering system is now under consideration, which is as follows. There will be only 3 zones, namely:

Zone *A*, comprising the home sector;

Zone *B*, applying to all adjacent sectors, in both the home and other districts; and

Zone *C*, comprising any other sectors. In this way the tariff may be determined by the first three figures of the prefix.

The numbering of metering impulses is as follows:

(*a*) At the moment of answering—1 impulse.

(*b*) After 5–10 seconds—a number of metering impulses which may be varied between 0 and 15, and which may be determined differently for each tariff, e.g., for Zone *A*—0 impulses, for Zone *B*—3 impulses, and for Zone *C*—10 impulses.

After this first period of 5–10 seconds has expired, one metering impulse is given for each time unit which is different in length for the 3 zones, e.g., for Zone *A*—1 impulse every 60 seconds, for Zone *B*—1 impulse every 30 seconds, and for Zone *C*—1 impulse every 10 seconds.

8.7.6 Time Limit

With the present tariff, the possibility is provided of breaking down the connection after 9 minutes; with the future tariff this breakdown may be done after a time varying between 6 and 12 minutes.

8.7.7 Traffic Metering

Each time- and zone-metering circuit is provided with two meters, which record, respectively, the total number of metering impulses and the total number of effective connections per circuit.

In order to obtain an indication of the ratio between the different kinds of calls, one time- and zone-metering circuit is provided with two totalling meters—one registering the total number of metering impulses for each tariff class, and the other meter registering the total number of effective calls per tariff class.

8.7.8 Register

Registers are at present provided only in rotary exchanges, but in future they will be introduced in all other exchanges and will take all figures dialled for toll calls and transmit them in tone-frequency code-impulse form.

8.7.9 Outlet Searching

In rotary exchanges all toll lines are in ideal groups; this condition is approached in step-by-step exchanges by the use of mixing selectors.

8.7.10 Selection Time

No limit fixed

8.7.11 Alternative Routing

See Section 8.7.3.

The possibility may be provided for routing calls through a second overflow exchange if all circuits to the first overflow exchange are occupied

8.7.12 Impulsing

Except on some 50-cycle-per-second junctions, all connections are set up via voice-frequency lines with end-to-end signalling, so that no impulse correctors are needed.

8.7.13 Signalling

On some existing circuits 50-cycle-per-second signalling is used, but on most circuits single voice-frequency signalling is applied, at 2400 cycles per second in one direction and 2500 cycles per second in the opposite direction.

8.7.14 Signals

First dialling tone(150 cycles per second).
Selection impulses.
Answering.
Forward clearing.
Release.
Offering:—

(*a*) Initiate offering.
(*b*) Acknowledgment of signal (*a*).
(*c*) Proceed offering } may be repeated.
(*d*) Undo offering }

Toll breakdown.
Re-ringing.
Clear back.
Second dialling tone after prefix (450 cycles per second).
Ringing control (450 cycles per second).
Busy tone (450 cycles per second).

8.7.15 Pulse Release

Incoming line—1–2 minutes.

8.7.16 Insertion of Repeaters

Terminal repeaters are used with pad switching.

8.7.17 4-Wire Switching

This is provided only on tandem connections.

8.7.18 Prefix for International Traffic

Not provided.

8.8 TELEPHONE INFORMATION ABOUT PORTUGAL

8.8.1 Transmission

Toll links are divided into separate groups for terminal and tandem traffic.

8.8.2 Tandem Routing

This has not yet been determined.

8.8.3 Metering

This will be determined by the code prefix.

8.8.4 Time Limit

Not yet decided.

8.8.5 Alternative Routing

This will probably be provided.

8.8.6 Prefix for International Traffic

Net yet determined.

8.8.7 Insertion of Repeaters

Probably terminating repeaters, and therefore no intermediate repeaters required.

8.8.8 Signalling

Trial installation with 600 and 750 cycles per second with compound prefix.

8.9 TELEPHONE INFORMATION ABOUT SWEDEN

8.9.1 Division of Country

300 areas which will require independent 3- or 4-figure codes commencing with the digit "0."

8.9.2 Local Numbering

Five or six figures, the first two figures indicating the identity of the exchange.

8.9.3 Open Numbering

Within each area only the local number is used; to reach other areas the code and local number must be dialled.

8.9.4 Tariff

The average price of a call is 4 öre. Experiments have been tried where the length of the call determines the fee. The basic length of conversation charge might be a period of 6 minutes.

8.9.5 Cordless Switchboard

Planned for Stockholm, and would probably be used elsewhere.

8.9.6 Toll Operator

Will set up long-distance connections with key set.

8.9.7 Priority

Is given to toll calls over local calls.

8.9.8 Repeaters

These will be automatically inserted.

8.9.9 4-Wire Switching

This will be adopted.

8.9.10 Transmission

The overall attenuation for a toll connection should not exceed 1.0 neper.

8.9.11 Toll Impulsing

Condenser discharge—50-cycle-per-second and 150-cycle-per-second alternating current; impulse—40 milliseconds, interruption—60 milliseconds.

8.9.12 National Numbering Scheme

An automatic national numbering scheme is planned.

8.10 TELEPHONE INFORMATION ABOUT SWITZERLAND

8.10.1 *National Numbering Scheme* (See Table XVIII.)

TABLE XVIII

Toll Regional Centres	Code	Toll Terminal Areas	Code
Lausanne	021	Genève (Primary)	022
		—	023
		Yverdon	024
		Aigle	025
		Martigny	026
		Sion	027
		Brig	028
		Bulle	029
Bern	031	Biel	032
		Thun	033
		B'dorf	034
		Langnau	035
		Interlaken	036
		Fribourg	037
		Neuchâtel	038
		La Chaux de Fonds	039
		Zweisimmen	030
Lucerne	041	Zug	042
		Schwyz	043
		Altdorf	044
		Sursee	045
Zürich	051	Winterthur	052
		Schaffhausen	053
		Frauenfeld	054
		Rapperswil	055
		Baden	056
		Wohlen	057
		Glarus	058
Basel	061	Olten (Primary)	062
		Langenthal	063
		Aarau	064
		S'thurn	065
		Delemont	066
St. Gallen	071	Weinfelden	072
		Wil	073
		Wattwil	074
Chur	081	St. Moritz	082
		Davos	083
		Schuis	084
		Sargano	085
		Ilanz	086
Lugano	091	Bellenzona	092
		Locarno	093
		Faido	094

8.10.2 *Transmission*

Primary, and in some cases secondary, toll links are divided into separate groups for terminal and tandem traffic.

8.10.3 *Routing*

If a secondary exchange is connected to two primary exchanges, both incoming and outgoing traffic has to be routed over the shorter route.

8.10.4 *Tandem Routing*

No connection should involve more than three primary toll exchanges.

8.10.5 *Numbering*

Four-, 5-, and 6-digit, and when necessary 3-digit, toll prefix.

8.10.6 *Metering*

Determined by distance between primary toll exchanges and therefore controlled by prefix. Facilities are included for determining fee by prefix and the two (or three) first figures of subscriber's number.

8.10.7 *Time Limit*

Facilities for applying forced release after 12 minutes on connection between primary toll centres; on secondary toll exchanges forced release may be applied after 6 minutes.

8.10.8 *Traffic Metering*

Totalling meter for day and night local connections.

Totalling meter for seizures per direction for outgoing terminal and tandem traffic in primary and secondary toll exchanges.

Overflow meters per direction in primary and secondary toll exchanges.

Totalling meters for seizures of originating outgoing primary and secondary toll traffic.

Totalling meters for transit connections in primary toll exchanges.

8.10.9 *Registers*

Arranged for 2 code and 6 numerical digits, code translation for 1–3 selections of 1–20 impulses and for controlled action with "seize" impulse and "end-of-selection" impulse. Impulses are transmitted in the forward direction.

8.10.10 *Outlet Searching*

Primary links are in ideal groups.

8.10.11 *Selection Time*

Maximum of 4 seconds.

8.10.12 Alternative Routing

Provided via alternative primary centre independent of register translation.

8.10.13 Impulsing

Primary toll centres are provided with impulse correctors.

8.10.14 Signalling

By 3000-cycle-per-second and 50-cycle-per-second alternating current.

8.10.15 Signals

Seize
Proceed-to-send.
Selection impulses.
End-of-selection impulses.
Answering.
Release.
Ringing control (400 cycles per second).
Busy tone (400 cycles per second).
Offering.
Re-ringing.
Clear-back.
Test busy.

8.10.16 Pulse Releases

Subscriber dialling	20–30 seconds
Incoming register	30 seconds
Incoming tandem line	5–10 seconds
Busy or ringing-tone condition	3 minutes

8.10.17 Insertion of Repeaters

By automatic means when connection exceeds 1.2 nepers.

8.10.18 4-Wire Switching

This is provided.

8.10.19 Prefix for International Traffic

014.

Shunted-Amplifier Input Admittance

By A. SHADOWITZ

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A METHOD is presented for finding the input admittance of an amplifier having a shunt impedance between its output and input. The shunted-amplifier gain is the sum of two terms that have simple physical significance. Several cases are worked out for a shunting capacitance, and include a single stage with both resistive and tuned loads and a double stage with resistance-capacitance coupling. Application to a frequency modulator is given in detail. Extensions are indicated to the theory of the tuned-plate tuned-grid oscillator, multivibrator, and negative-impedance devices.

• • •

When a passive network, such as a capacitance, is connected from input to output directly across a vacuum-tube amplifier, many of the amplifier characteristics may be radically altered. The input admittance, in particular, becomes a function of several variables; general and specific examples of its behavior have been discussed previously.[1–4]

The dependence of the input admittance of several closely related types of shunted amplifiers on input frequency, load impedance, and value of input-output shunting capacitance is examined. The ratio of the load impedance to the shunting impedance is introduced as the independent variable.

The circuit under study has been described[5] as a shunt-type feed-back amplifier. The action, so far as gain is concerned, is similar to that of the ordinary series-type feed-back circuit, in which a voltage from the output is applied in series with the input signal voltage. In the present case, if the generator is assumed to have zero internal impedance, the signal voltage from grid to cathode is specified by the assumed generator voltage. It cannot, therefore, be assumed to fluctuate with variation of the circuit parameters.

Two alternative assumptions are possible with respect to the effect of the shunt path:

A. Current which is dependent on the input voltage is introduced into the output.

B. Current which is dependent on the output voltage is introduced into the input.

The first assumption provides a logical basis for gain calculations; the second for calculation of input admittance.

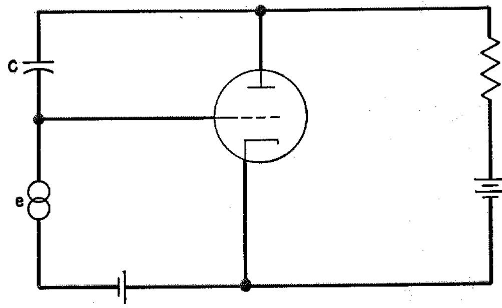

Fig. 1—Single-stage class-*A* amplifier with resistive plate load and with a capacitance *C* connected between input and output circuits.

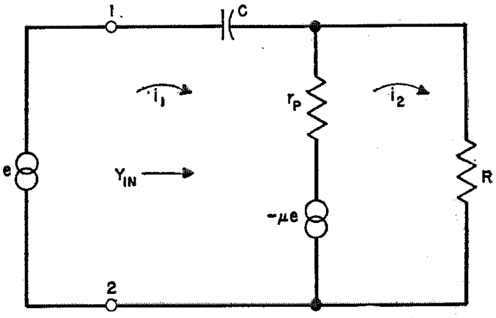

Fig. 2—Equivalent circuit of Fig. 1.

The theory developed below is applied to explain the operation of a particular frequency modulator[6] and is extended to the treatment of several circuits as negative-resistance oscillators.

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[1] J. M. Miller, "The Input Impedance of a Vacuum Tube," Bulletin No. 351, U. S. Bureau of Standards; 1919.

[2] S. Ballantine, "On the Input Impedance of the Thermionic Amplifier," *Physical Review*, v. 15, pp. 409–420; 1920.

[3] K. McIlwain and J. G. Brainerd, "High Frequency Alternating Currents," 2nd edition, John Wiley and Sons, New York, New York, 1939; p. 134.

[4] F. E. Terman, "Radio Engineering," 2nd edition, McGraw-Hill Book Company, New York, New York, 1937; p. 231.

[5] H. W. Bode, "Network Analysis and Feedback Amplifier Design," 1st edition, D. Van Nostrand and Company, New York, New York, 1945; p. 36.

[6] E. M. Ostlund, A. R. Vallarino, and M. Silver, "Center-Frequency-Stabilized Frequency-Modulation System," *Proceedings of the I.R.E.*, v. 35, pp. 1144–1149; October, 1947.

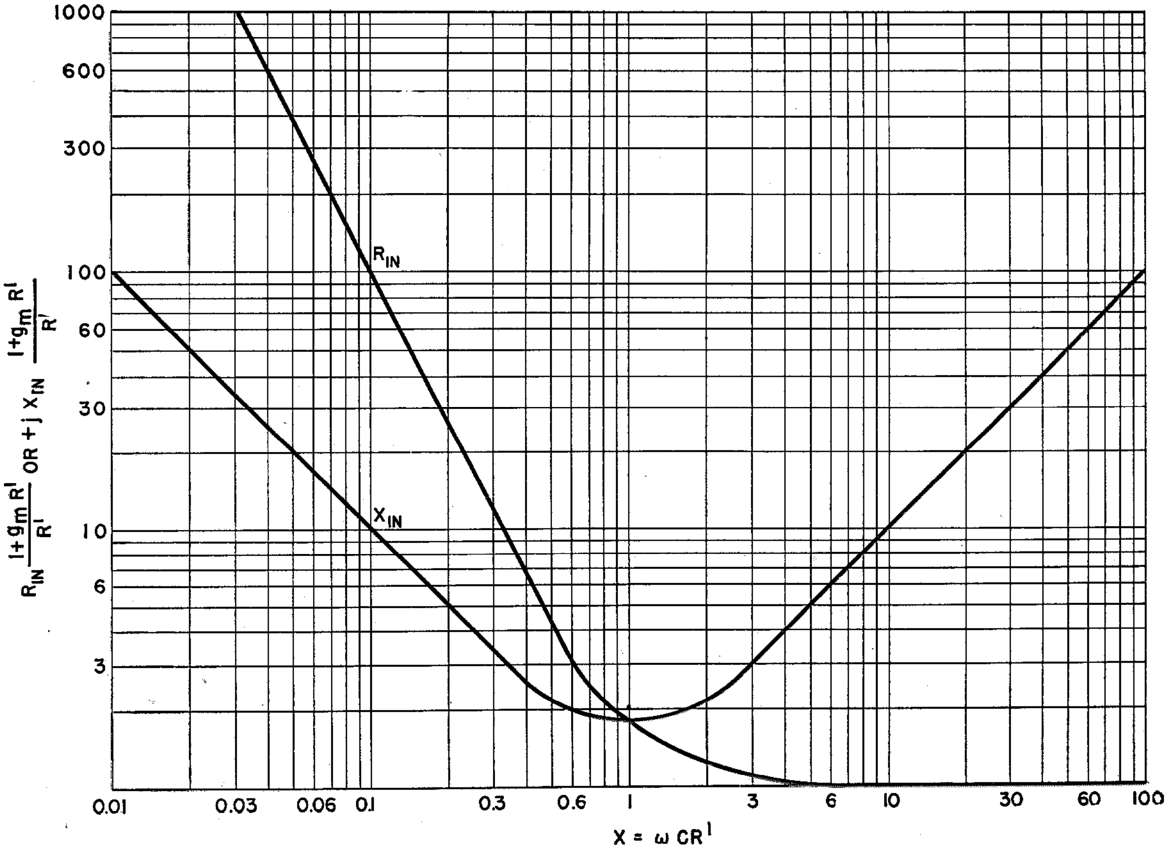

Fig. 3—Input resistance and reactance of circuit of Fig. 1.

1. *Single Stage With Resistive Load*

Fig. 1 is a schematic of a single-stage class-A amplifier with a capacitance connected between input and output, the amplifier being driven by a zero-impedance generator. The equivalent alternating-current circuit for frequencies below the transit-time domain is shown in Fig. 2. The grid-plate capacitance is included in C. The grid-cathode and plate-cathode capacitances may be neglected: the former constitutes a constant term additive to the input admittance, while the only effect of the latter is to limit (in practice but not in principle) the range of value of the load resistance. The input admittance Y_{in} between points 1 and 2 is

$$Y_{in}=\frac{i_1}{e}=\frac{r_p+R(1+\mu)}{r_pR+(r_p+R)\dfrac{-j}{\omega C}}. \qquad (1)$$

Setting

$$R'=\frac{r_pR}{r_p+R},$$

$$x=\omega CR',$$

and

$$g_m=\frac{\mu}{r_p},$$

(1) may be written

$$Y_{in}=\frac{1+g_mR'}{R'}\times\frac{x^2+jx}{1+x^2}. \qquad (2)$$

If Y_{in} is represented as a resistance R_{in} in parallel with a reactance X_{in}, then

$$R_{in}=\frac{R'}{1+g_mR'}\times\frac{1+x^2}{x^2}, \qquad (3)$$

$$X_{in}=-j\frac{R'}{1+g_mR'}\times\frac{1+x^2}{x}. \qquad (4)$$

Graphs of (3) and (4) are given in Fig. 3.

The gain A of the stage is given by Ri_2/e.

So,

$$A=\frac{\dfrac{Rr_p}{R+r_p}}{\dfrac{Rr_p}{R+r_p}+\dfrac{-j}{\omega C}}+\frac{-\mu\dfrac{R\frac{-j}{\omega C}}{R+\frac{-j}{\omega C}}}{r_p+\dfrac{R\frac{-j}{\omega C}}{R+\frac{-j}{\omega C}}}.$$

The gain is therefore the sum of two terms:

A. *Voltage division;* the parallel combination of load impedance and plate resistance, in series with the grid-plate impedance, divides the voltage impressed on this circuit.

B. *Tube amplification;* the tube load consists of the load impedance in parallel with the grid-plate impedance.

This is a general method of computing gain, and is used later in the paper. Thus,

$$A=\left(\frac{x^2+g_m^2R'^2}{x^2+1}\right)^{\frac{1}{2}}\tan^{-1}\left[\frac{(1+g_mR')x}{x^2-g_mR'}\right]. \quad (5)$$

It is evident that if

$$x\ll 1, \quad \text{then} \quad A\rightarrow -g_mR',$$

while when

$$x\gg 1, \quad A\rightarrow +1.$$

It should be noted that the above analysis is not strictly valid at frequencies considerably above approximately 20 megacycles per second for the following reasons:

A. The reactances of the various lead inductances become appreciable, so that Fig. 2 does not remain a true representation of Fig. 1. The main effect of these inductances is that the input admittance acquires a conductance component proportional to $\omega^2 g_m$.

B. Transit-time effects become important. Here also the input admittance acquires a conductance component proportional to $\omega^2 g_m$.

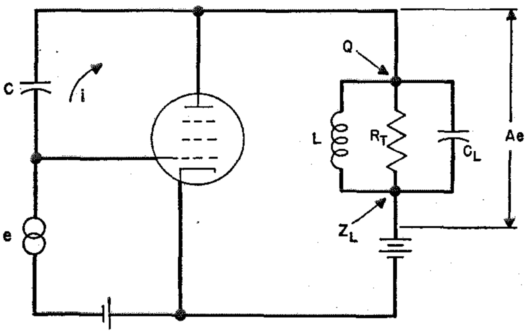

Fig. 4—Single-stage class-*A* amplifier with a tuned-circuit plate load and with a capacitance *C* connected between input and output circuits.

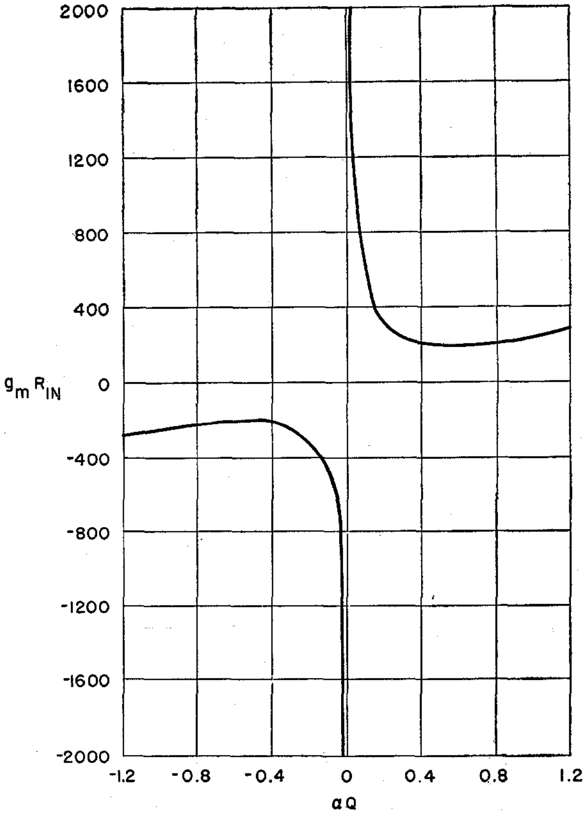

Fig. 5—Input resistance of circuit of Fig. 4 for $g_mR_T=20$ and $x=0.01$.

1.1 SMALL COUPLING

Consider first the case where C is very small, i.e.,

$$\frac{1}{\omega C}\gg\frac{r_pR}{r_p+R}, \quad \text{or} \quad x\ll 1.$$

The shunt input resistance R_{in} is then much larger than the shunt input reactance X_{in}, so the former may be neglected. It is interesting to note from (3), however, that when $g_mR'\gg 1$ and $x\ll 1$,

$$R_{in}=\frac{1}{g_m}\times\frac{1}{\omega^2C^2R'^2},$$

or that the input conductance varies as $\omega^2 g_m$. This is similar to the result of the lead-inductance and transit-time effects noted above. From (4), with $x\ll 1$,

$$X_{in}=-j\frac{R'}{1+g_mR'}\times\frac{1}{\omega CR'}$$

$$=\frac{-j}{\omega C(1+g_mR')}, \quad (6)$$

which is the reactance of a capacitor of value $C_{in}=C(1+g_mR')$. But it was shown above that if $x\ll 1$, then $A=-g_mR'$. Thus,

$$C_{in}=C(1-A). \tag{7}$$

This is the well-known formula for the Miller effect, which this case actually represents, the input admittance being a capacitance, the value of which depends on A. By placing the input terminals across the tank of an inductance-capacitance oscillator, or across a capacitance in a resistance-capacitance oscillator, the instantaneous effective capacitance may be varied, and the oscillator thereby becomes frequency modulated.

1.2 LARGE COUPLING

When C is very large,

$$\frac{1}{\omega C}\ll\frac{r_pR}{r_p+R}, \quad \text{or} \quad x\gg 1.$$

From Fig. 3 it is evident that the shunt input reactance X_{in} is now much larger than the shunt input resistance R_{in}, so the former may be neglected. Then, from (3),

$$R_{in}=\frac{R'}{1+g_mR'}. \tag{8}$$

If $g_mR'\gg 1$,

$$R_{in}=\frac{1}{g_m}. \tag{9}$$

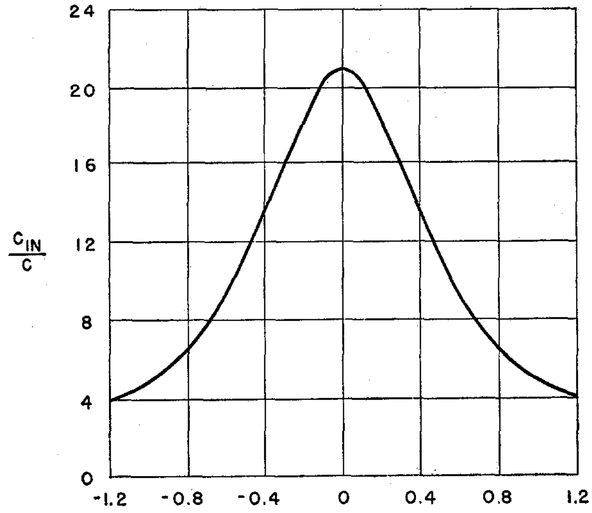

Fig. 6—Input capacitance of circuit of Fig. 4 for $g_mR_T=20$ and $x=0.01$.

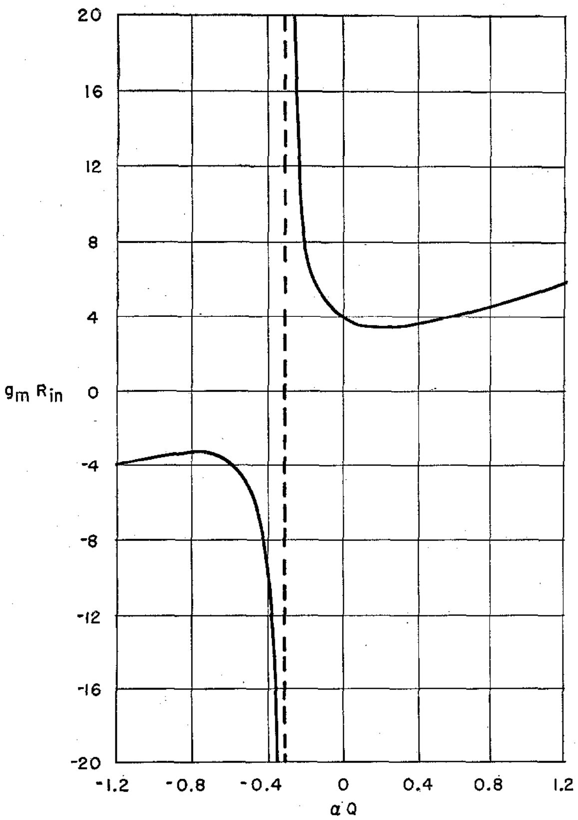

Fig. 7—Input resistance of circuit of Fig. 4 for $g_mR_T=20$ and $x=0.6$.

The input admittance in this case is a conductance, the value of which is proportional to g_m. This is for $x\gg 1$, which is the case not only when the coupling capacitor is made large, but also when the frequency is high. The behavior of this conductance with g_m is similar to that noted above for lead inductance and transit time; here, however, there is no longer a variation with frequency.

Since the input admittance is a resistance, the value of which depends on g_m, it is possible to vary the instantaneous frequency of a resistance-capacitance oscillator by placing the input terminals of the amplifier across the grid resistor of the oscillator. By using a pentode instead of a triode for the amplifier and varying g_m by applying audio-frequency voltage to the screen rather than to the control grid, the variable resistance may be obtained without accompanying voltage variations which could amplitude modulate the oscillator. This occurs because the amplifier gain, when $x\gg 1$, is unity regardless of the

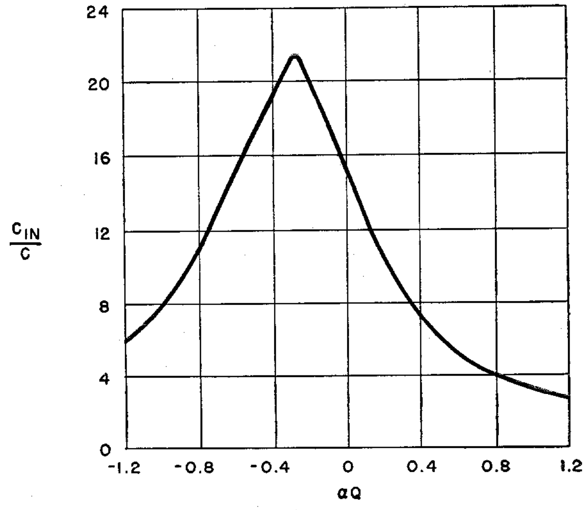

Fig. 8—Input capacitance of circuit of Fig. 4 for $g_mR_T=20$ and $x=0.6$.

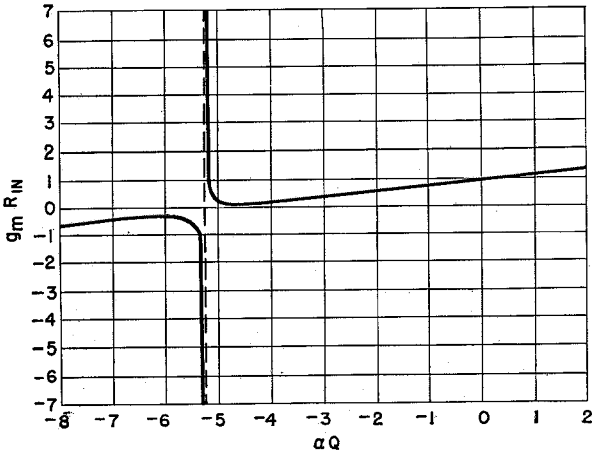

Fig. 9—Input resistance of circuit of Fig. 4 for $g_mR_T=20$ and $x=10$.

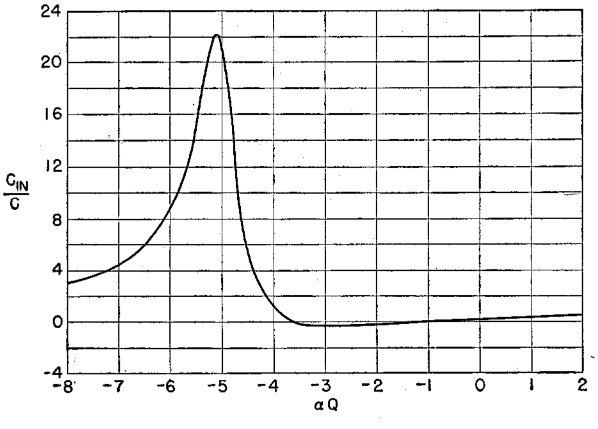

Fig. 10—Input capacitance of circuit of Fig. 4 for $g_mR_T=20$ and $x=10$.

value of g_m; no audio-frequency voltage can then appear at either grid or plate. To minimize amplitude modulation caused by the changing resistance itself, the input of the modulator may be placed across both resistors of the resistance-capacitance oscillator, rather than across the grid resistor alone.

2. *Single Stage Having Tuned-Circuit Load*

Fig. 1 may be modified by the substitution of a tuned circuit for the plate load resistor, as shown in Fig. 4. Fig. 3 is then no longer a valid representation of this new circuit, although it still holds true for any given frequency at which the tuned circuit is resonant.

From Fig. 4,

$$Y_{in}=\frac{i}{e}=\frac{1}{e}\left[\frac{e-Ae}{\dfrac{-j}{\omega C}}\right]=j\omega C(1-A). \qquad (10)$$

To calculate the gain A for an arbitrary C, the method derived above is used. If $\alpha=(\omega-\omega_0)/\omega_0$, where ω_0 represents the resonant angular velocity of the load alone, and discussion is restricted to the region near resonance, the load impedance is

$$Z_L=\frac{R_T}{1+j2\alpha Q}, \qquad (11)$$

where

$$Q=R_t/\omega_0L.$$

If we set $x=\omega CR_T$ (assuming r_p large compared to R_T), we obtain

$$A=\frac{x^2+x2\alpha Q-g_mR_T+jx(g_mR_T+1)+2\alpha Qg_mR_T}{x^2+x4\alpha Q+4\alpha^2Q^2+1}. \qquad (12)$$

Substituting in (10),

$$R_{in}=\frac{x^2+x4\alpha Q+4\alpha^2Q^2+1}{x^2\left(g_m+\dfrac{1}{R_T}\right)+x2\alpha Qg_m}, \qquad (13)$$

and

$$C_{in}=C\frac{x2\alpha Q+4\alpha^2Q^2+1+g_mR_T}{x^2+x4\alpha Q+4\alpha^2Q^2+1}. \qquad (14)$$

These equations do not lend themselves to simple analysis as they stand. By limiting the range of applicability, however, it is possible to simplify them greatly.

2.1 $\alpha=0$

If the response at the resonant frequency of the tuned circuit is considered for various values of coupling, i.e., $\alpha=0$, the results are the same as those given in (3) and (4) for a resistive load. Fig. 3, therefore, applies in this case.

2.2 $x \ll 1$

Next is considered a region near the resonant frequency of the tuned circuit for small coupling between grid and plate, i.e., $x \ll 1$. This case is shown in Figs. 5 and 6, where $x=0.01$, $g_mR_T=20$. In order that ω shall not enter into both the abscissas and ordinates, these graphs imply that the plate tuning is changed from one point on the abscissa to another.

2.3 $x=0.6$

Next, let the coupling have some median value, such as $x=0.6$, and assume that $g_mR_T \gg 1$. This case is shown in Figs. 7 and 8 for the value $g_mR_T=20$.

2.4 $x \gg 1$

Finally, when the coupling is very large, and $g_mR_T \gg 1$, the case is shown in Figs. 9 and 10 for the values $x=10$ and $g_mR_T=20$.

3. Frequency Modulator

Fig. 11 is a schematic diagram of a frequency-modulated oscillator,[6] based on the principles just outlined. Several details of the modulator action may be of interest.

The frequency of operation, 4 megacycles, the grid-plate capacitance of 10 micromicrofarads, and the modulator load resistor of 2500 ohms are such that the value of x is approximately 0.6. Tuning of the modulator plate circuit is accomplished by inserting an audio-frequency voltage at the grid of the modulator. Correct tuning is then characterized by minimum audio-frequency voltage across the radio-frequency-bypassed part of the oscillator grid-leak resistor.

A pentode modulator with external capacitance is used, rather than a triode with its internal grid-plate capacitance, because of the greater linearity of the pentode g_m-e_g curve. The operating point is at the center of the most linear portion of the curve. The radio-frequency and audio-frequency voltages must never be so large that their sum extends beyond this linear region. For this reason, the modulator grid is tapped down on the oscillator coil (this connection is also used to minimize the effect of the modulator input-resistance variations).

Since operation is in the linear portion of the g_m-e_g curve, it is also in the curved portion of the i_p-e_g curve. The 6AB7 therefore acts as a van der Bijl grid modulator, and its plate-circuit

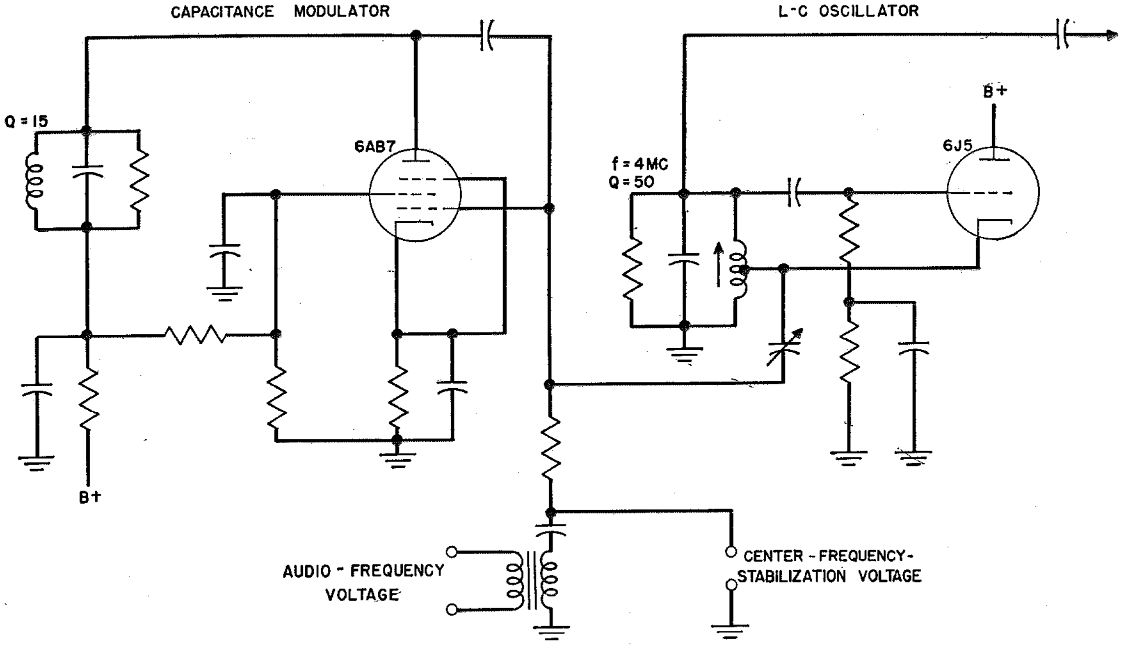

Fig. 11—Frequency-modulated oscillator.

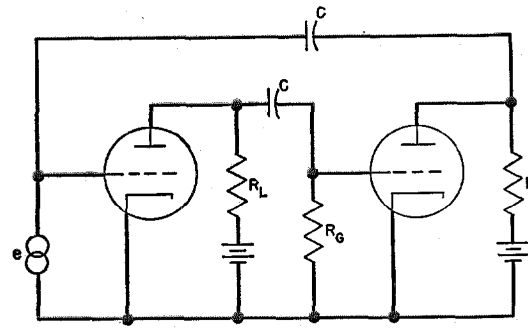

Fig. 12—Two-stage resistance-capacitance coupled class-*A* amplifier with a capacitance *C* connected between input and output circuits.

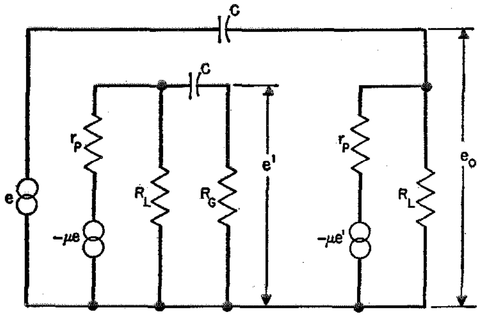

Fig. 13—Equivalent circuit of Fig. 12.

voltage will show the radio-frequency carrier plus the audio-frequency sidebands. The plate voltage is amplitude modulated as well as frequency modulated. Furthermore, amplitude components at twice the audio frequency are introduced into the plate voltage by the slight variation of the tuned-circuit impedance as the frequency varies on both sides of resonance. If, in addition, the plate-dropping, cathode, and screen bypass capacitors have a low reactance at radio frequency, but not at audio frequency, additional audio-frequency components are introduced into the plate voltage.

All these audio-frequency addition terms in the modulator plate voltage do not affect the voltage at the modulator grid since the grid-plate capacitor is very small. The audio-frequency modulated terms (radio-frequency sidebands), however, do feed back. The oscillator, therefore, is slightly amplitude modulated by the presence of the amplitude-modulated terms in the plate voltage of the modulator. Another way of saying the same thing is that both the modulator C_{in} and

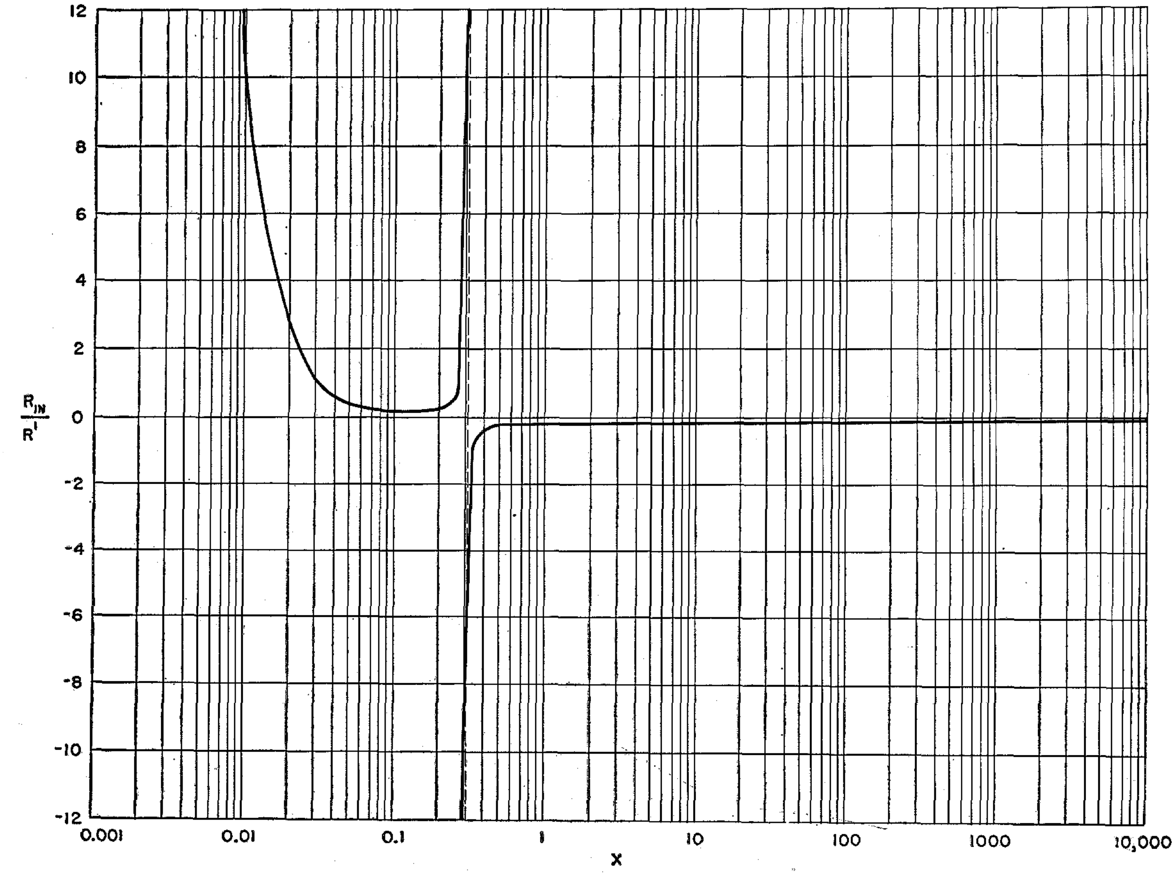

Fig. 14—Input resistance of circuit of Fig. 12 for $a=10$, $b=1000$, and $d=0$.

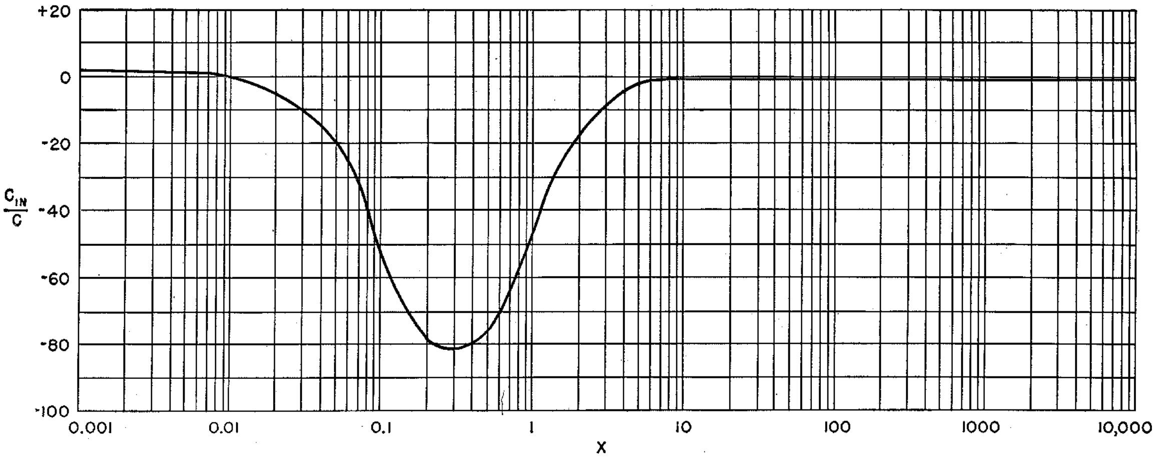

Fig. 15—Input capacitance of circuit of Fig. 12 for $a=10$, $b=1000$, and $d=0$.

R_{in} appear across the oscillator tuned circuit, the Q of which is varied with fluctuations of these terms, thus amplitude modulating the output. The effect is a small one, of the order of -50 decibels, if the modulator plate circuit has a low Q ($Q\approx 15$).

There are numerous ways in which the frequency deviation may be controlled: the audio-frequency level, the capacitance from 6AB7 grid to 6J5 cathode, the feed-back capacitance of the 6AB7, the Q of 6AB7 tuned circuit, etc.

The modulator distortion at 100-percent modulation, during a test of such a modulator, was approximately -46 decibels. The noise was -75 decibels.

4. *Tuned-Plate Tuned-Grid Oscillator*

The equations for input admittance developed in the previous sections may be applied for a rough explanation of the behavior of a tuned-plate tuned-grid oscillator, assuming that operation is class A rather than class C.

Equations (18) and (19), which give the input resistance and capacitance of the circuit of Fig. 4, may be written, if we let $\gamma=\omega/\omega_0$ and $K=\omega_0 CR_T$,

$$g_m R_{in}=1+\frac{Q}{K}-\frac{\frac{Q}{K}}{\gamma^2}+\frac{\frac{1+\frac{1}{g_m R_T}}{QK}}{\gamma^2\left(1+\frac{K}{Q}\right)-1}, \qquad (15)$$

$$\frac{C_{in}}{C}=\frac{Q(\gamma^2-1)}{\gamma^2(K+Q)-Q+\frac{\gamma^2}{\gamma^2(K+Q)-Q}}+\frac{\gamma^2 g_m R_T}{[\gamma^2(K+Q)-Q]^2+\gamma^2}. \qquad (16)$$

If a parallel resonant circuit is now placed across the input of this circuit, then for stable oscillations to occur the following condition must be satisfied

$$R=-R_{in}. \qquad (17)$$

From this condition, it is possible to show that three conditions may occur: no oscillation, one possible frequency of oscillation, and two possible oscillation frequencies. The values of these frequencies, as well as the value of inductance required in the grid, may also be calculated.

5. *Two-Stage Amplifier with Resistance-Capacitance Coupling*

A circuit of this type is shown in Fig. 12. The equivalent circuit, shown in Fig. 13, is based on assumptions of negligible capacitance between grid and plate of each stage, negligible capacitance from each grid and plate to ground, and class-A operation for each stage. Let the reactance of C be X_c. If

$$\left.\begin{aligned} R'&=\frac{R_L r_p}{R_L+r_p},\\ x&=\omega CR',\\ a&=\frac{R_G}{R'},\\ b&=g_m^2 R_G R', \end{aligned}\right\} \qquad (18)$$

then Figs. 14 and 15 show the variation of input resistance and capacitance for the values $a=10$, $b=1000$, corresponding, e.g., to $R_G=10^5$, $R'=10^4$, and $g_m=10^{-3}$.

If it is desired to determine the frequency of stable oscillation when a resistance R_G is

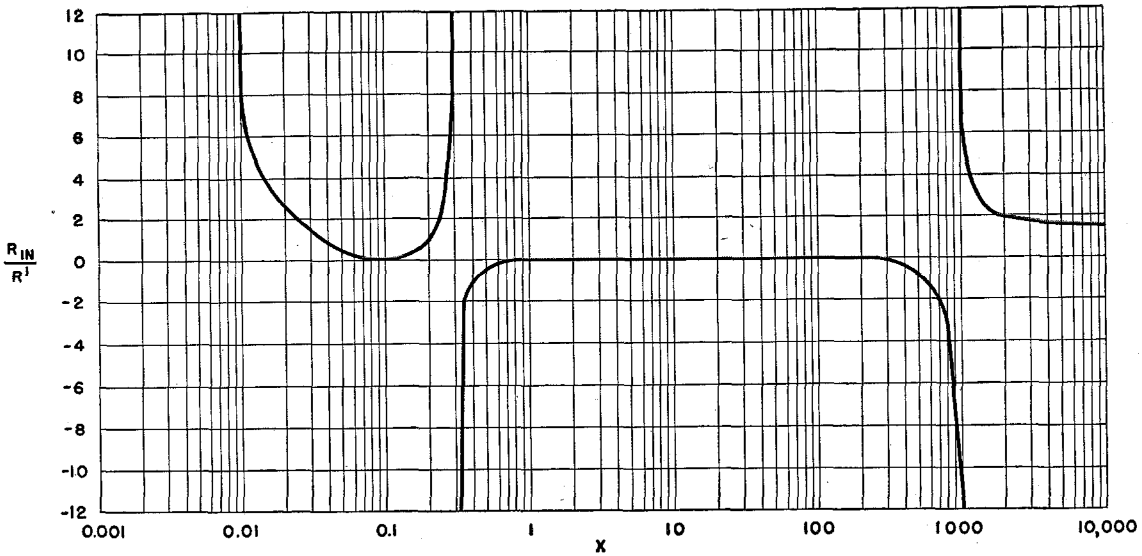

Fig. 16—Input resistance of circuit of Fig. 12 with capacitance C_1 between second grid and ground for $a=10$, $b=1000$, and $d=0.01$.

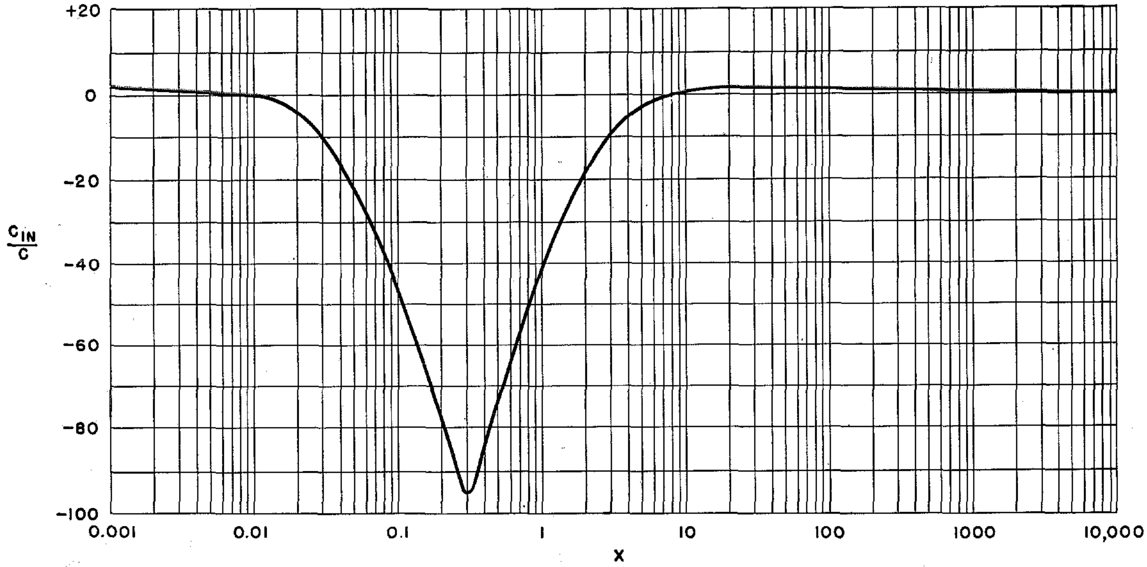

Fig. 17—Input capacitance of circuit of Fig. 12 with capacitance C_1 between second grid and ground for $a=10$, $b=1000$, and $d=0.01$.

substituted for the generator in Fig. 12, the conditions to be met are

$$\frac{C_{in}}{C}=0, \qquad (19)$$

$$\frac{R_{in}}{R'}=-a. \qquad (20)$$

These two conditions are incompatible in the present case: R_{in}/R' is positive when $C_{in}/C=0$. Thus no oscillations will occur. It is shown below, however, that it is only necessary to make a simple assumption to permit oscillation.

If, for example, a capacitance C_1 of reactance X_1 is assumed to exist between the second grid and ground (in parallel with R_G in Fig. 12), and we let $d=X_c/X_1$, with R', x, a, and b having the same meaning as before, then the variation of input resistance and capacitance is shown in

Figs. 16 and 17 with $d=0.01$. It is seen that the conditions (19) and (20) for oscillation can now be fulfilled.

Previous analysis,[7] assuming a small inductance in series with one of the leads, has yielded a similar result. It was shown that if this inductance were zero, oscillations could not occur. Actually, it is immaterial whether series inductance or shunt capacitance is assumed; both are physically present.

6. *Conclusion*

The method outlined above for obtaining the input admittance of a shunted amplifier can be extended to many cases of practical importance; for example, oscillations in multistage intermediate-frequency amplifiers, the analysis of negative-resistance circuits such as are used to give a high variable Q in some wave analyzers, Eccles-Jordan center circuits, etc. The gain of the shunted amplifier is also obtained in simple fashion by this method.

In all these cases, by presupposing a generator of zero internal impedance between grid and cathode, it is possible to make the mathematical analysis without resorting to the concept of feedback.

7. *Acknowledgment*

The writer wishes to acknowledge the help of Messrs. E. M. Ostlund, M. Silver, and A. R. Vallarino on various phases of this work.

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[7] B. van der Pol, "Relaxation Oscillations," *Philosophical Magazine*, 7th series, v. 2, pp. 978–992; November, 1926.

Physical Properties of Crystals and Their Symmetry

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PREDICTION of certain possible properties of crystalline materials is based on the development of moduli linking the cause and effect of physical phenomena. The use of tensor and matrix calculus permits a survey of crystalline substances for such properties as pyroelectricity, dielectric susceptance, piezoelectricity, elasticity, and piezomagnetism. The physical properties of bodies crystallizing in various systems are outlined.

• • •

Pierre Curie, at the age of 21, published a short note[1] in which he announced the discovery of piezoelectricity.

The origin of this discovery must be sought in the associations of ideas caused by the simultaneous study of physics and crystallography.

"These two sciences were equally familiar to him and complemented each other in his mind.

"For him, the symmetry of phenomena was an intuitive notion. Few physicists had as great a knowledge as he of the crystallographic forms and of the symmetry groups."[2]

These main ideas, which led him to the discovery of the new phenomenon, were published later in the following form:

When certain causes produce certain effects, the symmetry elements of the causes must be found again in the effects.

When certain effects show a certain nonsymmetry, this nonsymmetry must be found again in the causes which produced them.

The reverse of these two propositions is not true, at least practically, i.e., the effects produced may be more symmetrical than their causes.

The discovery of piezoelectricity was brought about by the following reasoning. Pyroelectricity, a phenomenon that has been known for a long time, is characterized, in tourmaline, by the appearance, when heated, of a dielectric polarization.

This polarization can be represented by a vectorial quantity having the symmetry of a polar vector, although it is caused by a rise in temperature, which is a scalar phenomenon.

The nonsymmetry peculiar to the electric field (direction and space orientation) can appear only in a medium having this kind of nonsymmetry, i.e., with no center.

Consequently, other causes than a temperature variation, for instance, elastic deformations caused in the crystalline medium if the symmetry of this medium is favorable, will also be able to cause the appearance of a dielectric polarization.

Such was the origin of Curie's discovery, which was brilliantly confirmed by experiment. Curie found piezoelectric samples in the 21 classes of crystals having no symmetry centers. A detailed study of piezoelectricity was limited to quartz.

A few years later, Voigt, the German physicist who succeeded Neumann in this field, made a general investigation of analytical methods of representing the symmetry of the physical phenomena taking place in crystallized matter. These methods led Voigt to the notion of tensors. The very great physical and mathematical importance of tensorial calculus, this new mode of expressing the relation between quantities, is well known.

Voigt was able to foresee the existence of all piezoelectric moduli of crystals having no centers. He applied his analytical methods to all physical phenomena in crystals including elasticity, dielectric constants, piezomagnetism, etc. Present requirements and conditions suggest consideration of the use of crystals other than quartz. To know how a given piezoelectric crystal can be used, it is necessary to know the existence and the meaning of the characteristic moduli of its crystal class. These data are given very completely and in great detail by Voigt.

However, when trying to retrace the calculations on which Voigt based his conclusions, one is hampered by the notations he used, these notations being ill adapted to the problem.

Since 1910, when Voigt's work was done, mathematicians have made available to physicists greatly improved methods of calculation by means of which all the fundamental results of

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[1] Pierre Curie, "Développement, par pression, de l'électricité polaire dans les cristaux hémièdres à faces inclinées" (Development by Pressure of Polar Electricity in Hemihedral Crystals With Inclined Faces), *Comptes Rendus de l'Académie des Sciences;* August 2, 1880.

[2] Mme Curie in the preface to Pierre Curie's works.

crystal physics can easily be found. For instance, by using appropriate notations, all theorems commonly applied in the study of symmetry can be obtained readily by the elementary rules of matrix multiplication. It is also possible to predict how a crystal, identified only by its main symmetry characteristics, must behave with respect to any physical property. Attention should first be directed to the theories that are indispensable to these calculations, which reduce to systematic use of linear forms and transformations.

The theory of tensors is obtained by associating the theories of linear transformations and of invariants. A tensor is a quantity characterized by the existence of several *numbers* (*components*) having particular values in a given reference system. However, any quantity defined by a system of components is not necessarily a tensor.

The main criterion of a tensor is the following:

With the components of a tensor, and in a manner which shall be defined more precisely later, it is possible to form an invariant.

If the reference system is changed, the tensor components also change, but the quantity represented by these components remains the same. It must be possible to calculate the new values of the components if the laws of transformation from the first system of reference to the second are known.

With regard to equality between tensors:

A. If two tensors are equal in one reference system, they are equal in all systems.

B. If a tensor is zero in one system, it is also zero in all others.

The relation between tensors is independent of the reference system. All the relations between physical quantities have this absolute characteristic of being independent of the reference system. This is commonly expressed by the notion of dimensions of physical quantities and of homogeneity of equations.

With crystalline media, the method of tensorial calculus becomes a necessity; the properties of symmetry will be characterized by the identity of the tensorial forms representing the quantities studied at two symmetrical points on the crystal.

1. *Linear Transformations*

Assume a group of variables of the same kind, $a_1, a_2, a_3, \ldots, a_i$ they will be subjected to a linear transformation by calculating a new group of variables $\bar{a}_1, \bar{a}_2, \bar{a}_3, \ldots, \bar{a}_n$ by the following rule:

To obtain $\bar{a}_1$, for instance, a_1 is multiplied by a numerical coefficient α, then a_2 is multiplied by a second coefficient, and so on. The sum of all the products thus formed will equal $\bar{a}_1$.

The α coefficient figuring in the expression for $\bar{a}_1$ shall be provided with two indices, a lower index equal to 1 and an upper index equal to the index of the variable that this coefficient multiplies.

$$\bar{a}_1 = \alpha_1^1 a_1 + \alpha_1^2 a_2 + \ldots + \alpha_1^n a_n. \qquad (1)$$

The transformation will include n equations similar to (1). They may be written in an abridged form as

$$\bar{a}_1 = \sum_j \alpha_1^j a_j. \qquad (2)$$

1.1 Mute Indices, Einstein's Convention

If, in one member of an equation, the same index appears in the two terms of one product, as an upper index in one term and as a lower index in the other, we agree to effect the summation with respect to this index and it is unnecessary to retain the $\sum$ sign, and therefore, any value may be given to the index with respect to which the summation is effected. Such an index is called a mute index; (1) will be written

$$\bar{a}_i = \alpha_i^j a_j. \qquad (3)$$

1.2 Reverse Transformation

To subject a new group of variables $\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_n$, to the reverse linear transformation, coefficients β_m^n are selected to produce the initial variables $a_1, a_2, \ldots, a_n$.

This transformation is written:

$$a_m = \beta_m^n \bar{a}_n. \qquad (4)$$

The β coefficient may be calculated from the α coefficient.

Let us consider the system of (3). Supposing a_j unknown and $\bar{a}_m$ known, we solve this system with respect to the a_j variables and, applying

Cramer's rule for relations of the type

$$a_1 = \frac{\begin{vmatrix} \bar{a}_1 & \alpha_1^2 & \alpha_1^3 & \cdots\cdot & \alpha_1^n \\ \bar{a}_2 & \alpha_2^2 & \alpha_2^3 & \cdots\cdot & \alpha_2^n \\ \cdots\cdot & \cdots\cdot & \cdots\cdot & \cdots\cdot & \\ \bar{a}_n & \alpha_n^2 & \alpha_n^3 & \cdots\cdot & \alpha_n^n \end{vmatrix}}{\begin{vmatrix} \alpha_1^1 & \alpha_1^2 & \alpha_1^3 & \cdots\cdot & \alpha_1^n \\ \alpha_2^1 & \alpha_2^2 & \alpha_2^3 & \cdots\cdot & \alpha_2^n \\ \cdots\cdot & \cdots\cdot & \cdots\cdot & \cdots\cdot & \\ \alpha_n^1 & \alpha_n^2 & \alpha_n^3 & \cdots\cdot & \alpha_n^n \end{vmatrix}} \qquad (5)$$

and developing,

$$a_1 = \frac{|M_1^1|\bar{a}_1}{|\alpha|} - \frac{|M_1^2|\bar{a}_2}{|\alpha|} \cdots\cdot \pm \frac{|M_1^n|\bar{a}_n}{|\alpha|}. \qquad (6)$$

By identifying with the first part of (4),

$$\beta_1^j = (-1)^{1+j}\frac{M_j^1}{|\alpha|} \qquad (7)$$

and, in general,

$$\beta_i^j = (-1)^{i+j}\frac{|M|_j^i}{|\alpha|}, \qquad (8)$$

$|M|_j^i$ being the minor obtained from the α determinant from which the jth line and the first column have been deleted.

Thus, the β coefficients are known when the α coefficients are known, provided, however, that (4) has a solution, that is, that the array of the α's be square and that determinant $|\alpha|$ be other than zero.

1.3 VARIANCE

Let us consider other variables $b_1, b_2, \ldots, b_n$ which, by virtue of their definitions, must undergo at the same time as variables a_i a linear transformation with the same coefficients α_i^j as variables a_j, i.e., such that

$$\bar{b}_i = \alpha_i^j b_j. \qquad (9)$$

Variables b_j are said to be *covariant.*

Suppose, now, that there exist other variables c^i which, at the same time as the a_i's become $\bar{a}_j$'s, are transformed into $\bar{c}^k$ in accordance with the relation

$$\bar{c}^i = \beta_k^i c^k, \qquad (10)$$

that is, by using for their direct transformation the coefficients of the reverse transformation of the covariant quantities. The variables c will be called *contravariant.*

These variables use α_i^j as a coefficient for their reverse transformation.

$$c^k = \alpha_l^k \bar{c}^l. \qquad (11)$$

Contravariant variables will be distinguished from covariant variables by placing their indices at the top.

1.4 INVARIANT

Consider now the result of multiplying each variable a_j taken in the initial group by a quantity A and adding the terms.

$$S = A^j a_j. \qquad (12)$$

A linear form is obtained.

Let us assume that when variables a_j are subjected to the initial linear transformation, the sum S remains constant.

$$S = \bar{A}^k \bar{a}_k. \qquad (13)$$

This implies that the quantities A_j are functions of such a nature that

$$A^j a_j \equiv \bar{A}^k \bar{a}_k \equiv \bar{A}^k \alpha_k^j a_j. \qquad (14)$$

This relation gives n equations

$$A^j = \alpha_k^j \bar{A}^k, \qquad (15)$$

which implies that the quantities A^j are contravariant.

The quantities A are said to be the components of a tensor of the first order, once contravariant. Such quantities are familiar and are vectors.

1.5 NOTE

If, in (4), which defines the reverse transformation, the quantities $\bar{a}$ are expressed as functions derived from the fundamental linear transformation (3), the following relation is obtained

$$a_m \equiv \beta_m^n \alpha_n^p a_p, \qquad (16)$$

which must be verified for all values of index p, the latter varying from 1 to n. Thus, n distinct equations are obtained between the α and the β coefficients.

If, for instance, $p = m$

$$a_m = \beta_m^n \alpha_n^m a_m \qquad (17)$$

or

$$\beta_m^n \alpha_n^m = 1. \qquad (18)$$

If p differs from m, there will be $n-1$ equations.

$$0 = \beta_m^n \alpha_n^p. \qquad (19)$$

Equations (18) and (19) can be grouped into a single form

$$\beta^n_m \alpha^p_n = \delta^p_m, \tag{20}$$

where δ^p_m (Kronecker's index) has a value of 0 for $m \neq p$ and of 1 for $m = p$.

Equation (8) is the solution of equation (19).

1.6 INVARIANT MULTILINEAR FORMS

Suppose that there must be considered a p series of covariant variables

$$\begin{matrix} a_1, a_2, \ldots, a_n \\ b_1, b_2, \ldots, b_n \end{matrix}$$

and a q series of contravariant variables

$$\begin{matrix} e^1, e^2, \ldots, e^n \\ f^1, f^2, \ldots, f^n. \end{matrix}$$

We shall form the products $a_i b_j \ldots e^k f^l$ in all possible manners giving indices i, j, k, and l, all possible values from 1 to n, and then multiply each product by a quantity A designated by the indices of the terms of the product arranged in opposite positions, adding all the terms obtained.

$$S = A^{ij}_{mn} a_i b_j \ldots e^m f^n \ldots \tag{21}$$

If the quantities A are such that the sum S remains invariant when the linear transformation is effected, that is, if they are transformed into a quantity A such that

$$S = \bar{A}^{pq}_{rs}{}^{\vdots\vdots}_{\vdots\vdots} \bar{a}_p \bar{b}_q \ldots \bar{e}^r \bar{f}^s \ldots, \tag{22}$$

these quantities A are said to be the components of a tensor of the $(p+q)$th order, p times contravariant and q times covariant.

If, in (22), the quantities $\bar{a}$, $\bar{b}$, ..., $\bar{e}$, $\bar{f}$ are replaced by their expressions in terms of a, b, c, ..., e, f, and if the coefficients of the same products are identified termwise, n relations are found, allowing the calculation of quantities $\bar{A}$ in terms of the A's.

$$\bar{A}^{pq}_{rs}{}^{\vdots\vdots}_{\vdots\vdots} = \beta^p_i \beta^q_j \alpha^m_r \alpha^n_s \ldots A^{ij}_{mn}{}^{\vdots\vdots}_{\vdots\vdots}. \tag{23}$$

These equations follow:

$$\bar{A}^{pq}_{rs} = \beta^p_i \beta^q_j \alpha^m_r \alpha^n_s \ldots A^{ij}_{mn}.$$

It will be seen that this condensed writing brings a considerable simplification, for (23) represents n^{p+q} equations each having n^{p+q} terms in their right-hand members.

Thus, in elasticity, use is made of tensors having 4 indices, representing in 3-dimensional space, $3^4 = 81$ components.

2. *Tensorial Algebra*

2.1 ADDITION

If two tensors have the same variance, they can be added termwise.

$$t^{ik}_m = r^{ik}_m + S^{ik}_m. \tag{24}$$

To demonstrate that the components t^{ik}_m have a tensorial nature, it will be sufficient to show that by collecting them, an invariant can be produced.

As r^{ik}_m and s^{ik}_m are the components of a tensor, the invariants are:

$$R = r^{ik}_m a_i b_k c^m, \tag{25}$$

$$S = s^{ik}_m a_i b_k c^m, \tag{26}$$

and

$$R + S = T = (r^{ik}_m + s^{ik}_m) a_i b_k c^m = t^{ik}_m a_i b_k c^m. \tag{27}$$

2.2 EXTERIOR MULTIPLICATION

Consider two tensors whose components are A^r_{pq} and B^{lm}_k; each of the components A can be multiplied successively by all the B components, thus obtaining a tensor C whose components are

$$C^{rlm}_{pqk} = A^r_{pq} \times B^{lm}_k. \tag{28}$$

By a reasoning identical with the above, it could be shown that the components C^{rlm}_{pqk} have a tensorial character.

2.3 CONTRACTION

By giving particular values to indices p, q and l, m, for instance, terms for which the lower index k of the B's is equal to the upper index p of the A's and form the sum, we may obtain components having only 4 indices.

$$D^{lm}_{pq} = \sum_r A^r_{pq} B^{lm}_r. \tag{29}$$

To show that these components D are tensorial in character, it is sufficient to verify that they are transformed in accordance with the law

$$\bar{D}^{zt}_{xy} = \alpha^p_x \alpha^q_y \beta^z_l \beta^t_m D^{lm}_{pq}. \tag{30}$$

As

$$A^u_{xy} = \alpha^p_x \alpha^q_y \beta^u_r A^r_{pq} \tag{31}$$

and

$$B^{zt}_s = \alpha^k_s \beta^z_l \beta^t_m B^{lm}_k, \tag{32}$$

the product will have new components, also tensorial in character.

$$C^{ztu}_{xys}=\alpha^p_x\alpha^q_y\beta^u_r\alpha^k_s\beta^z_l\beta^t_m A^r_{pq}B^{lm}_k. \tag{33}$$

If all the terms for which $r=k$ are added with respect to r,

$$C^{ztu}_{xys}=\sum_r \alpha^p_x\alpha^q_y(\beta^u_r\alpha^r_s)\beta^z_l\beta^t_m A^r_{pq}B^{lm}_r \tag{34}$$

or

$$\sum_r \beta^u_r\alpha^r_s=S^u_s,$$

Kronecker's index, which is equal to 1 when $u=s$ and to 0 when $u\neq s$, and as

$$\sum_r A^r_{pq}B^{lm}_r=D^{lm}_{pq},$$

there remains

$$\bar{D}^{zt}_{xy}=\bar{C}^{ztu}_{xyu}=\alpha^p_x\alpha^q_y\beta^z_l\beta^t_m D^{lm}_{pq}. \tag{35}$$

Again, in the present example, we can contract once more by selecting from all the terms D^{lm}_{pq} those for which $p=l$ and by adding with respect to l obtain the components of a tensor only once covariant and only once contravariant.

$$E^m_p=\sum_l D^{lm}_{pl}. \tag{36}$$

Finally, we can again add all the terms such as E^m_q in which $m=q$ with respect to m and obtain an invariant

$$J=\sum_m E^m_m. \tag{37}$$

Finally

$$J=A^r_{pq}B^{rq}_r. \tag{38}$$

3. *Matrixes*

The α^j_i coefficients of linear transformation (3) can be arranged in a table, the line number corresponding to the lower index and the column number to the upper index; this table is called the matrix of the linear transformation.

$$\left\|\begin{array}{ccccc}\alpha^1_1 & \alpha^2_1 & \alpha^3_1 & \dots & \alpha^n_1\\ \alpha^1_2 & \alpha^2_2 & \alpha^3_2 & \dots & \alpha^n_2\\ \dots & \dots & \dots & \dots & \dots\\ \alpha^1_n & \alpha^2_n & \alpha^3_n & \dots & \alpha^n_n\end{array}\right\| \tag{39}$$

This table must be placed between double bars to avoid confusion with the determinant of the coefficients, whose typographical arrangement is the same.

The matrix is only a symbol representing a series of operations, which are represented in another way by (3). The matrix is represented in an abridged form by

$$\|\alpha\| \quad \text{or} \quad (\alpha). \tag{40}$$

In the linear transformation for obtaining p quantities $\bar{a}$ from n quantities a, p may differ from n, and the matrix is very often rectangular.

The *diagonal* terms are the α^i_i; they form the main diagonal of the table of the α^i_j.

The matrix is *symmetrical* when the terms that are symmetrical with respect to the main diagonal are equal. For instance,

$$\begin{array}{|c|c|c|}\hline a & d & e\\ \hline d & b & f\\ \hline e & f & c\\ \hline\end{array}\ . \tag{41}$$

It is antisymmetrical when the terms that are symmetrical with respect to the main diagonal are equal but of opposite signs; then $\alpha_{ij}=-\alpha_{ji}$ and, consequently, $\alpha_{ii}=0$. For instance,

$$\begin{array}{|c|c|c|}\hline & d & e\\ \hline -d & & f\\ \hline -e & -f & \\ \hline\end{array}\ . \tag{42}$$

A diagonal matrix is a matrix in which all terms are zero except the diagonal terms.

$$\begin{array}{|c|c|c|}\hline a & & \\ \hline & b & \\ \hline & & c\\ \hline\end{array}\ . \tag{43}$$

A scalar matrix is a diagonal matrix in which all terms are equal.

$$\begin{array}{|c|c|c|}\hline a & & \\ \hline & a & \\ \hline & & a\\ \hline\end{array}\ . \tag{44}$$

The unit matrix is the scalar matrix in which $a=1$.

$$(1)=\begin{array}{|c|c|c|}\hline 1 & & \\ \hline & 1 & \\ \hline & & 1 \\ \hline\end{array}\,. \qquad (45)$$

4. *Matrix Algebra*

4.1 EQUALITY

Two matrixes are equal when their components having the same indices are equal.

$$\alpha_j^i=\gamma_j^i$$

or

$$(\alpha)=(\gamma).$$

4.2 SUM

If we consider two linear transformations of the same nature on the same variables $a_1 \dots a_n$, let

$$\left.\begin{aligned} &\text{let} && b_p=\alpha_p^q a_q \\ &\text{and} && C_p=\gamma_p^q a_p, \end{aligned}\right\} \qquad (46)$$

we can add the equalities having the same indices and place the corresponding terms as factors, obtaining

$$d_p=b_p+c_p=(\alpha_p^q+\gamma_p^q)a_p, \qquad (47)$$

which corresponds to the linear transformation

$$d_p=S_p^q a_p.$$

The matrix S_p^q is the sum of the matrixes α_p^q and γ_p^q.

$$(S)=(\alpha)+(\gamma). \qquad (48)$$

4.3 PRODUCT OF TWO MATRIXES

If, following a first linear transformation which acting on variables a transformed them into $\bar{a}$, we consider a second one transforming the $\bar{a}$'s into $\bar{\bar{a}}$'s,

let
$$\bar{a}_i=\alpha_i^j a_j, \qquad (49)$$

$$\bar{\bar{a}}_k=\gamma_k^i \bar{a}_i, \qquad (50)$$

we can note that the $\bar{\bar{a}}_k$ terms are linear functions of the a variables.

$$\bar{\bar{a}}_k=\delta_k^j a_j. \qquad (51)$$

Matrix (δ) is said to be the product of matrix (α) by matrix (γ).

$$(\delta)=(\alpha)(\gamma). \qquad (52)$$

Replacing, in (50), the a_i's by their values from (49) and identifying termwise with (51), we see that

$$\delta_k^j=\sum_i \gamma_k^i \alpha_i^j. \qquad (53)$$

This leads to the following rule for the multiplication of matrixes.

The two matrixes to be multiplied are placed side by side with the multiplier on the left of the multiplicand.

$$\overset{\longrightarrow}{\begin{array}{|c|c|c|}\hline \gamma_1^1 & \gamma_1^2 & \gamma_1^3 \\ \hline \gamma_2^1 & \gamma_2^2 & \gamma_2^3 \\ \hline \gamma_3^1 & \gamma_3^2 & \gamma_3^3 \\ \hline\end{array}} \;\overset{\downarrow}{\times}\; \begin{array}{|c|c|c|}\hline \alpha_1^1 & \alpha_1^2 & \alpha_1^3 \\ \hline \alpha_2^1 & \alpha_2^2 & \alpha_2^3 \\ \hline \alpha_3^1 & \alpha_3^2 & \alpha_3^3 \\ \hline\end{array} = \begin{array}{|c|c|c|}\hline \gamma_1^1\alpha_1^1+\gamma_1^2\alpha_2^1+\gamma_1^3\alpha_3^1 & \gamma_1^1\alpha_1^2+\gamma_1^2\alpha_2^2+\gamma_1^3\alpha_3^2 & \\ \hline & & \\ \hline & & \\ \hline\end{array}\,. \qquad (54)$$

To obtain the term δ_i^j, which is placed at the ith line and jth column of the product, we multiply successively each term of the ith line of the multiplier by the terms of the same number in the jth column of the multiplicand and add the products. The numbers are taken from left to right in the multiplier and from top to bottom in the multiplicand.

The result of this operation is not usually independent of the order of the factors. In general

$$(\alpha)(\gamma)\neq(\gamma)(\alpha). \qquad (55)$$

4.3.1 Note 1

This practical rule applies to the linear transformation of (3). When it is desired to calculate the terms $\bar{a}$ as functions of the α's and of the a's, it is sufficient to arrange the a terms in one vertical column as shown in (56) and to operate in accordance with the rule for the multiplication of matrixes.

$$\begin{array}{|c|c|c|}\hline \alpha_1^1 & \alpha_1^2 & \alpha_1^3 \\ \hline \alpha_2^1 & \alpha_2^2 & \alpha_2^3 \\ \hline \alpha_3^1 & \alpha_3^2 & \alpha_3^3 \\ \hline \end{array} \times \begin{array}{|c|}\hline a_1 \\ \hline a_2 \\ \hline a_3 \\ \hline \end{array} = \begin{array}{|c|}\hline \alpha_1^1 a_1+\alpha_1^2 a_2+\alpha_1^3 a_3 \\ \hline \alpha_2^1 a_1+\alpha_2^2 a_2+\alpha_2^3 a_3 \\ \hline \alpha_3^1 a_1+\alpha_3^2 a_2+\alpha_3^3 a_3 \\ \hline \end{array} = \begin{array}{|c|}\hline \bar{a}_1 \\ \hline \bar{a}_2 \\ \hline \bar{a}_3 \\ \hline \end{array}. \qquad (56)$$

4.3.2 *Note 2*

The variables a_k, by means of which the basic transformation was defined, are of a tensorial character since by associating them with the contravariant variables b^k an invariant can be formed.

$$J = a_k b^k. \qquad (57)$$

The components a can be considered as being the components of a tensor of the first order, once covariant, i.e., a vector.

This tensor may be symbolized by an abridged symbol similar to that used for the matrix, and the basic transformation (3) will be written

$$(\bar{a}) = (\alpha)(a). \qquad (58)$$

5. *Matrix Calculus Applied to Operations of Symmetry*

Operations of symmetry can easily be represented by linear transformation matrixes:

5.1 OPERATIONS OF THE FIRST KIND

In operations of the *first kind*, F passes to and can be made to coincide with $\bar{F}$. To bring about the coincidence of two geometrical figures, a translation and a rotation are combined.

In studying the symmetry of crystals, only rotation is considered. As the crystal is homogeneous, its properties depend only on orientation in space; directions alone need be considered.

The most general operation of the first kind is a rotation. If in Fig. 1 we pass from the coordinates of point M

$$M \left| \begin{array}{l} x^1 \\ x^2 \\ x^3 \end{array} \right.$$

to those of point $\bar{M}$, which is the new position of M when a rotation of angle θ is effected clockwise about axis ox^3, we find

$$\left| \begin{array}{l} \bar{x}^1 = \rho \cos(\alpha - \theta) \\ \bar{x}^2 = \rho \sin(\alpha - \theta) \\ \bar{x}^3 = x^3 \end{array} \right. \qquad (59)$$

and, as $x^1 = \rho \cos \alpha$, $x^2 = \rho \sin \alpha$,

$$\left| \begin{array}{l} \bar{x}^1 = x^1 \cos\theta + x^2 \sin\theta \\ \bar{x}^2 = -x^1 \sin\theta + x^2 \cos\theta \\ \bar{x}^3 = x^3. \end{array} \right. \qquad (60)$$

By analogy with the transformation relation of the contravariant quantities

$$\bar{x}^l = \beta_k^l x^k,$$

we see that transformation (60) can be represented by a matrix

$$B_k^i \rightarrow \begin{array}{|c|c|c|}\hline \cos\theta & \sin\theta & \\ \hline -\sin\theta & \cos\theta & \\ \hline & & 1 \\ \hline \end{array}. \qquad (61)$$

In what follows, we shall always consider the coordinates of a point as being contravariant

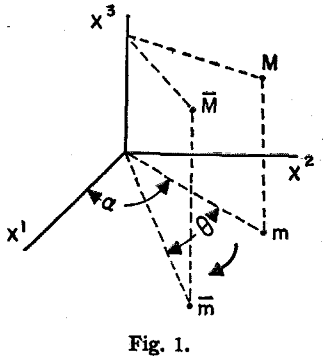


Fig. 1.

quantities. This follows the practice in vectorial geometry of using three unit vectors $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$ to represent any vector that defines a reference system. The vector to be measured $\vec{F}$ is the resultant of three oriented vectors along the directions of the unit vectors, and each of these components is measured with the corresponding vector. Thus, we have an invariant sum,

$$\vec{F}=\vec{a}_i x^i.$$

The x^i's are called the coordinates of the extremity of vector θF and are contravariant. As the basic transformations consist in changing the unit vectors, the latter are naturally considered as being covariant.

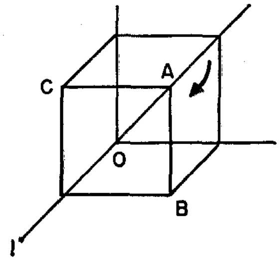


Fig. 2.

In the calculations which follow, we shall not use the methods of vectorial geometry, but those of metric geometry, the unit vectors common to all directions and to all systems shall be tacitly implied, but we shall continue to consider the coordinates of a point as contravariant quantities.

Furthermore, as we use rectangular axes, the matrix α of direct transformation of the covariant quantities is

$$\begin{array}{r|c|c|c|}\hline \alpha_i^j\rightarrow & c & -s & \\ \hline (\alpha)_3^\theta= & s & c & \\ \hline & & & 1 \\ \hline\end{array}\,, \qquad (62)$$

$$(\alpha)_1^\theta=\begin{array}{|c|c|c|}\hline 1 & & \\ \hline & c & -s \\ \hline & s & c \\ \hline\end{array}\,. \qquad (63)$$

It differs from matrix β only by a permutation of the lines and columns. For a simpler writing, we shall always write c and s instead of $\cos\theta$ and $\sin\theta$.

We shall designate the matrix by a lower index for the axis about which rotation takes place and an upper index indicating the angle of rotation.

A symmetry axis of order n is characterized by the value of $\theta=2\pi/n$.

If axis ox_3 is a binary axis, $\cos\pi=1$, $\sin\pi=0$,

$$(\alpha)_3^\pi=\begin{array}{|c|c|c|}\hline -1 & & \\ \hline & -1 & \\ \hline & & 1 \\ \hline\end{array}\,, \qquad (64)$$

and

$$(\alpha)_1^\pi=\begin{array}{|c|c|c|}\hline 1 & & \\ \hline & -1 & \\ \hline & & -1 \\ \hline\end{array}\,. \qquad (65)$$

In the study of crystals, we must obtain consider symmetry with respect to a ternary axis, Fig. 2, such as OA, oriented along the diagonal of a cube whose three sides coincide with the axes of coordinates.

The rotation, which changes point C to point B, corresponds to a circular permutation of the coordinates; the representative matrix is

$$(\alpha)^{2\pi/3}_{\text{diagonal}}=\begin{array}{|c|c|c|}\hline & 1 & \\ \hline & & 1 \\ \hline 1 & & \\ \hline\end{array}\,. \qquad (66)$$

5.2 OPERATIONS OF THE SECOND KIND

In operations of symmetry of the *second kind*, F and $\bar{F}$ are in the same relation as an object and its mirror image. The figures are said to be enantiomorphous.

5.3 SYMMETRY WITH RESPECT TO A PLANE

We pass, in Fig. 3, from a point M to a point $\bar{M}$, symmetrical with respect to the normal plane

ox^3, by changing the sign of its coordinate x^3,

$$(\beta)_3^p=(\alpha)_3^p=\begin{array}{|c|c|c|}\hline 1 & & \\ \hline & 1 & \\ \hline & & -1 \\ \hline\end{array}\,. \tag{67}$$

Similarly, a symmetry with respect to the plane normal to direction 1 is represented by the matrix

$$(\beta)_1^p=(\alpha)_1^p=\begin{array}{|c|c|c|}\hline -1 & & \\ \hline & 1 & \\ \hline & & 1 \\ \hline\end{array}\,. \tag{68}$$

5.4 Symmetry With Respect to a Center

To pass from a point M to its symmetrical point $\bar{M}$ with respect to the center of coordinates, simply change the signs of its coordinates.

$$(\beta)_c=(\alpha)_c=\begin{array}{|c|c|c|}\hline -1 & & \\ \hline & -1 & \\ \hline & & -1 \\ \hline\end{array}=(-1), \tag{69}$$

i.e., the matrix (-1).

5.5 Combinations of Operations

Combinations of operations can be represented by a matrix equal to the product of the two operations. For example, if a point is rotated successively through an angle α and then through an angle β about an axis 3, the final operation is represented by the matrix corresponding to the rotation $(\alpha+\beta)$. This can easily be verified by the rule for the multiplication of matrixes.

$$(\alpha)_3^{\alpha+\beta}=(\alpha)_3^{\beta}\times(\alpha)_3^{\alpha}. \tag{70}$$

$$\begin{array}{|c|c|c|}\hline \cos\beta & -\sin\beta & \\ \hline \sin\beta & \cos\beta & \\ \hline & & 1 \\ \hline\end{array}\times\begin{array}{|c|c|c|}\hline \cos\alpha & -\sin\alpha & \\ \hline \sin\alpha & \cos\alpha & \\ \hline & & 1 \\ \hline\end{array}=\begin{array}{|c|c|c|}\hline \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \\ \hline \sin(\alpha+\beta) & \cos(\alpha+\beta) & \\ \hline & & 1 \\ \hline\end{array}\,. \tag{71}$$

5.6 Plane of Alternate Symmetry of Order n

This is a plane direction, such that the crystal is restored to a position identical with its initial position by rotating it through π/n normal to the plane, and then taking its symmetrical point with respect to the plane. This involves two successive transformations, which are expressed by a matrix multiplication. For instance, for a plane of alternate symmetry of the second order,

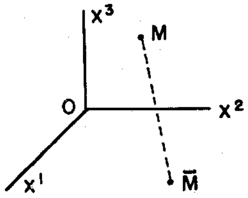


Fig. 3.

$$(\alpha)_3^{\bar{\omega}_2}=(\alpha)_2^p\times(\alpha)_3^{\pi/2}, \tag{72}$$

$$(\alpha)_3^{\bar{\omega}_2}=\begin{array}{|c|c|c|}\hline 1 & & \\ \hline & 1 & \\ \hline & & -1 \\ \hline\end{array}\times\begin{array}{|c|c|c|}\hline & -1 & \\ \hline 1 & & \\ \hline & & 1 \\ \hline\end{array}$$

$$=\begin{array}{|c|c|c|}\hline & -1 & \\ \hline 1 & & \\ \hline & & -1 \\ \hline\end{array}\,.$$

5.7 THEOREMS CONCERNING SYMMETRY

All theorems concerning symmetry may be proved by matrix multiplication.

5.7.1 Theorem 1

Any figure having an even-order axis L^{2n} passing through a symmetry center has a symmetry plane passing through the center and normal to the axis. Thus

$$(\alpha)^c \times (\alpha)_3^{\pi} = (\alpha)_3^{p}, \tag{73}$$

$$\underset{\text{Center}}{\begin{array}{|c|c|c|}\hline -1 & & \\ \hline & -1 & \\ \hline & & -1 \\ \hline \end{array}} \times \underset{\text{Axis}}{\begin{array}{|c|c|c|}\hline -1 & & \\ \hline & -1 & \\ \hline & & 1 \\ \hline \end{array}} = \underset{\text{Plane}}{\begin{array}{|c|c|c|}\hline 1 & & \\ \hline & 1 & \\ \hline & & -1 \\ \hline \end{array}}.$$

5.7.2 Theorem 2

Any figure having a plane of symmetry P passing through a center has an even-order axis L^{2n} normal to the plane and passing through the center.

$$\underset{\text{Center}}{\begin{array}{|c|c|c|}\hline -1 & & \\ \hline & -1 & \\ \hline & & 1 \\ \hline \end{array}} \times \underset{\text{Plane}}{\begin{array}{|c|c|c|}\hline 1 & & \\ \hline & 1 & \\ \hline & & -1 \\ \hline \end{array}} = \underset{\text{Axis}}{\begin{array}{|c|c|c|}\hline -1 & & \\ \hline & -1 & \\ \hline & & 1 \\ \hline \end{array}}. \tag{74}$$

5.7.3 Theorem 3

Any figure having a symmetry plane P and an even-order symmetry axis L^{2n} normal to the plane has a symmetry center P.

$$\underset{\text{Axis}}{\begin{array}{|c|c|c|}\hline -1 & & \\ \hline & -1 & \\ \hline & & 1 \\ \hline \end{array}} \times \underset{\text{Plane}}{\begin{array}{|c|c|c|}\hline 1 & & \\ \hline & 1 & \\ \hline & & -1 \\ \hline \end{array}} = \underset{\text{Center}}{\begin{array}{|c|c|c|}\hline -1 & & \\ \hline & -1 & \\ \hline & & -1 \\ \hline \end{array}}. \tag{75}$$

5.7.4 Theorem 4

The reverse occurs for odd-order axes.

$$\begin{array}{|c|c|c|}\hline \cos\dfrac{2\pi}{2n+1} & -\sin\dfrac{2\pi}{2n+1} & \\ \hline \sin\dfrac{2\pi}{2n+1} & \cos\dfrac{2\pi}{2n+1} & \\ \hline & & 1 \\ \hline \end{array} \times \begin{array}{|c|c|c|}\hline -1 & & \\ \hline & -1 & \\ \hline & & -1 \\ \hline \end{array} \neq \underset{\text{Plane}}{\begin{array}{|c|c|c|}\hline 1 & & \\ \hline & 1 & \\ \hline & & -1 \\ \hline \end{array}}. \tag{76}$$

5.7.5 Theorem 5

Any figure having an odd-order symmetry axis L_{2n+1} passing through a symmetry center C has a plane of alternate symmetry of order L_{2n+1}. As an example for $n=1$,

$$\begin{array}{|c|c|c|}\hline -1 & & \\ \hline & -1 & \\ \hline & & -1 \\ \hline\end{array} \times \begin{array}{|c|c|c|}\hline \cos\frac{2\pi}{3} & -\sin\frac{2\pi}{3} & \\ \hline \sin\frac{2\pi}{3} & \cos\frac{2\pi}{3} & \\ \hline & & \\ \hline\end{array} = \begin{array}{|c|c|c|}\hline \cos\frac{\pi}{3} & -\sin\frac{\pi}{3} & \\ \hline \sin\frac{\pi}{3} & \cos\frac{\pi}{3} & \\ \hline & & -1 \\ \hline\end{array} = (\alpha)^{\bar{\omega}_3}. \qquad (77)$$

5.7.6 Theorem 6

Any figure having an alternate symmetry plane of odd order $\bar{\omega}_{2n+1}$ and, consequently, perpendicular axis of order $(2n+1)$ has a symmetry center.

$$(\alpha)^{\pi/(2n+1)} \times (\alpha)^{\bar{\omega}_3} = (-1). \qquad (78)$$

$$\begin{array}{|c|c|c|}\hline \cos\frac{2\pi n}{2n+1} & -\sin\frac{2\pi n}{2n+1} & \\ \hline \sin\frac{2\pi n}{2n+1} & \cos\frac{2\pi n}{2n+1} & \\ \hline & & 1 \\ \hline\end{array} \times \begin{array}{|c|c|c|}\hline \cos\frac{\pi}{2n+1} & -\sin\frac{\pi}{2n+1} & \\ \hline \sin\frac{\pi}{2n+1} & \cos\frac{\pi}{2n+1} & \\ \hline & & -1 \\ \hline\end{array} = \begin{array}{|c|c|c|}\hline -1 & & \\ \hline & -1 & \\ \hline & & -1 \\ \hline\end{array}. \qquad (79)$$

5.7.7 Theorem 7

Any figure having a plane of alternate symmetry of an even order $\bar{\omega}_{2n}$ and, consequently, an even axis L_{2n} can have no center.

$$\begin{array}{|c|c|c|}\hline \cos\frac{\pi}{2n} & -\sin\frac{\pi}{2n} & \\ \hline \sin\frac{\pi}{2n} & \cos\frac{\pi}{2n} & \\ \hline & & -1 \\ \hline\end{array} \times \begin{array}{|c|c|c|}\hline \cos\frac{\pi p}{2n} & \sin\frac{\pi p}{2n} & \\ \hline \sin\frac{\pi p}{2n} & \cos\frac{\pi p}{2n} & \\ \hline & & 1 \\ \hline\end{array} = \begin{array}{|c|c|c|}\hline \cos\frac{\pi(p+1)}{2n} & \sin\frac{\pi(p+1)}{2n} & \\ \hline \sin\frac{\pi(p+1)}{2n} & \cos\frac{\pi(p+1)}{2n} & \\ \hline & & -1 \\ \hline\end{array} \neq (-1), \qquad (80)$$

as $\pi\frac{p+1}{2n} \neq \pi$.

6. Symmetry in Crystals

At two symmetrical points of a crystal, all physical properties are identical. By physical symmetry is meant a condition whereby if at point M certain causes produce certain physical effects, at a point $\bar{M}$, symmetrical with respect to the first point, the same causes produce the same effects. These relations between causes and effects are characterized by moduli, whose numerical values depend on the material.

In the analytical expression between causes and effects, space coordinates are involved. We are attempting to specify, therefore, in tensorial form, the dependence between causes, effects, and position coordinates of a point. As a result, the moduli are themselves tensors and when all moduli are known at point M, their new values at a different point $\bar{M}$ may be calculated.

If M and $\bar{M}$ are symmetrical and if the calculated moduli must be identical to the corresponding moduli as measured at M, as many relations will be obtained as there are moduli.

The whole group of relations thus established permit the assertion that certain moduli can or cannot exist within the limits of symmetry characteristic of the material studied. In some cases, it is found that certain moduli are equal to certain others with or without a change in signs.

In all cases, the equations between moduli are homogeneous equations of the type $ax+by+\ldots=0$; no constant in the right-hand member ever indicates the order of magnitude of these moduli. This method of calculation allows only a prediction of the possible existence of the moduli and their mutual relations.

To predict the properties of all crystalline systems, it is necessary to examine only a few particular relations of symmetry; the combination of relations found permits the inherent characteristics of each crystalline species to be determined.

7. *Pyroelectricity, Tensorial Moduli With One Index*

This phenomenon corresponds in a dielectric medium to the appearance of an electrostatic-induction vector when the temperature of the body is varied.

In the general case of a dielectric placed in an electric field, the induction b has the value

$$\vec{b}=Ko\vec{h}+4\pi\vec{P}. \tag{81}$$

We can assume, here, that the external electric field is zero, and therefore

$$\vec{b}=4\pi\vec{P}. \tag{82}$$

Vector $\vec{P}$ is homogeneous to vector $\vec{b}$ and will have the same variance.

The work of an electric charge e moved over a distance $d\vec{x}$ in a medium in which the electric field is $\vec{h}$ is

$$dw=e\vec{h}d\vec{x}, \tag{83}$$

or, using the tensorial notation,

$$dw=eh_idx^i,$$

where dw, as well as e, are invariants. We must thus consider h as a tensor of the first order, once covariant. The same holds for b in rectangular coordinates. The energy stored in volume V of a dielectric medium is

$$W=\frac{\vec{b}\vec{H}}{8\pi}dv,$$

$$8\pi W=b_ih_idx^id\sigma^i.$$

As b is proportional to the temperature variation θ in a pyroelectric body, this property must be characterized by moduli π that must satisfy

$$b_i=\pi_ix\theta. \tag{84}$$

As the two members of this equation must be tensors of the same order, it is evident that as θ is a scalar, i.e., a tensor of zero order, π must be a tensor of the first order, covariant.

Then, if we pass from one point M to another point $\bar{M}$, the moduli $\bar{\pi}$ at point $\bar{M}$ can be calculated in terms of the moduli π at point M by means of the linear relation

$$\bar{\pi}_k=\alpha_k^l\pi_l \tag{85}$$

and, moreover, if point $\bar{M}$ is symmetrical with respect to M, all physical properties at $\bar{M}$ are identical to those measured at M and, therefore,

$$\bar{\pi}_k=\pi_k. \tag{86}$$

By associating (85) and (86), we finally obtain the relation of the particular symmetry represented by the α coefficients,

$$\pi_k=\alpha_k^l\pi_l. \tag{87}$$

The linear transformation can be made very rapidly by matrix multiplication and (87) becomes

$$(\pi)=(\alpha)(\pi). \tag{88}$$

If, for instance, the system has a center of symmetry, the equation is written

$$\begin{array}{|c|}\hline \pi_1\\ \hline \pi_2\\ \hline \pi_3\\ \hline\end{array} = \begin{array}{|c|c|c|}\hline -1 & & \\ \hline & -1 & \\ \hline & & -1\\ \hline\end{array} \times \begin{array}{|c|}\hline \pi_1\\ \hline \pi_2\\ \hline \pi_3\\ \hline\end{array} = \begin{array}{|c|}\hline -\pi_1\\ \hline -\pi_2\\ \hline -\pi_3\\ \hline\end{array}. \tag{89}$$

Therefore:

$$\pi_1=-\pi_1=\pi_2=-\pi_2=\pi_3=-\pi_3=0. \qquad (90)$$

Pyroelectricity cannot exist in centered systems; all the moduli are zero.

The relations for symmetry about a nonbinary axis ox_3 will be written

$$(\pi)=(\alpha)_3^\theta(\pi) \qquad (91)$$

or

$$\begin{bmatrix}\pi_1\\ \pi_2\\ \pi_3\end{bmatrix}=\begin{bmatrix}c & -s & \\ s & c & \\ & & 1\end{bmatrix}\times\begin{bmatrix}\pi_1\\ \pi_2\\ \pi_3\end{bmatrix}=\begin{bmatrix}c\pi_1-s\pi_2\\ s\pi_1+c\pi_2\\ \pi_3\end{bmatrix}, \qquad (92)$$

which corresponds to equations

$$\left.\begin{aligned}\pi_1(c-1)&=s\pi_2\\ \pi_1 s&=-(c-1)\pi_2\\ \pi_3&=\pi_3\\ \text{or}\qquad (\pi_1)^2&=-(\pi_2)^2\\ \pi_3&=\pi_3.\end{aligned}\right\} \qquad (93)$$

The two first equations are verified only if moduli π_1 and π_2 are zero; the third one implies that π_3 *can* exist. The existence of modulus π_3 is not sure, but it is not in contradiction with the type of symmetry considered.

For symmetry about axis L_3, a diagonal of the cube, we have

$$\begin{bmatrix}\pi_1\\ \pi_2\\ \pi_3\end{bmatrix}=\begin{bmatrix} & 1 & \\ & & 1\\ 1 & & \end{bmatrix}\times\begin{bmatrix}\pi_1\\ \pi_2\\ \pi_3\end{bmatrix}=\begin{bmatrix}\pi_2\\ \pi_3\\ \pi_1\end{bmatrix}$$

or

$$\pi_1=\pi_2=\pi_3, \qquad (94)$$

three equal possible moduli of the same absolute value. In a crystal having only this symmetry, a rise in temperature would give three induction components, equal in the three directions ox_1, ox_2, ox_3, i.e., a resultant precisely in the direction of the diagonal of the cube.

7.1 ALTERNATE PLANE $\bar{\omega}_2$, PERPENDICULAR TO AXIS ox_3

The relations are:

$$\begin{bmatrix}\pi_1\\ \pi_2\\ \pi_3\end{bmatrix}=\begin{bmatrix} & -1 & \\ 1 & & \\ & & -1\end{bmatrix}\times\begin{bmatrix}\pi_1\\ \pi_2\\ \pi_3\end{bmatrix}=\begin{bmatrix}-\pi_2\\ \pi_1\\ \pi_3\end{bmatrix}. \qquad (95)$$

Hence, $\pi_1=\pi^2=-\pi^2=0$. π_3 alone can exist.

7.2 BINARY AXES OR PLANES OF SYMMETRY

If there exist still other axes or planes, these symmetries can reduce the number of moduli. The general relation for symmetry being

$$\pi_i=\alpha_i^k\pi_k, \qquad (96)$$

where the symmetrical elements are planes or axes, the transformation matrix is always diagonal, involves only coefficients of the type α_i^i, and is equal to $+1$ or to -1. Relation (96) can thus be written

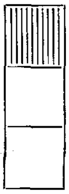
$$\pi_i=\pm\pi_i. \qquad (97)$$

Where the coefficient α_i^i is negative, the corresponding modulus cannot exist; where it is positive, however, pyroelectricity can appear in the corresponding direction.

Rectangular tables can be prepared in which + or − signs represent the result of matrix multiplication on a unit vector. The squares containing + signs may be cut out and the grid thus obtained superposed over tables of moduli. For example, the binary ox_1-axis case follows.

$$\begin{bmatrix}+1\\ +1\\ +1\end{bmatrix}\neq\begin{bmatrix}+1 & & \\ & -1 & \\ & & -1\end{bmatrix}\times\begin{bmatrix}+1\\ +1\\ +1\end{bmatrix}=\begin{bmatrix}+1\\ -1\\ -1\end{bmatrix}. \qquad (98)$$

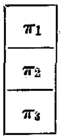
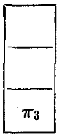
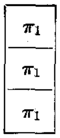
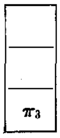
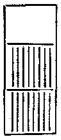
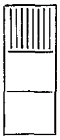
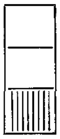
The top box containing +1 is cut out to produce the grid below.



The ox_1 axis already being transformed by the symmetries of the nonbinary axes ox_3 or, as the case may be, by the symmetries about the ternary axis or diagonal of the cube, the possible moduli for all crystalline systems will appear in the holes of the grid as they are the only ones consistent with all the symmetries of the system. (See Tables I and II.)

TABLE I

Grids for Use with Moduli Tables

No Symmetry	Nonbinary ox_3 Axis	Cube Diagonal	Alternate Plane
			
			
Plane Perpendicular to ox^1	Plane Perpendicular to ox^3	Binary Axis to ox	Binary Axis to ox^3

From the results for all crystalline systems, which will be given later, it is evident that pyroelectricity appears only in a few systems.

8. *Dielectric Susceptance, Tensorial Moduli With Two Indices*

If an isotropic dielectric is placed inside an electric field, the electric induction is equal to

$$\vec{b} = K_0\vec{h} + 4\pi\vec{P} = K_1\vec{h} = K_0\vec{h} + 4\pi x\vec{h}, \tag{99}$$

where K_1 is the specific inductive power and x the dielectric susceptance.

In an anisotropic dielectric medium, the direction of the induction is not necessarily that of the field; the above relation will be written:

$$b_i = K_0 h_i + 4\pi x_i^k h_k, \tag{100}$$

$$\frac{b_i - K_0 h_i}{4\pi} = x_i^k h_k. \tag{101}$$

The quantities $(b_i - K_0 h_i)/4\pi$ and h_k are once covariant; hence all the x_i^k are components of a tensor of the same order, once covariant and once contravariant.

The tensorial transformation for calculating the susceptances x at a point $\bar{M}$, when they are known at point M, are

$$\bar{x}_i^j = \alpha_i^k \beta_l^j x_k^l, \tag{102}$$

TABLE II

Moduli of 10 Crystalline Forms

System	Form	Moduli	Name	Symbol
Triclinic	Hemihedrism	π_1, π_2, π_3	Potassium Dichromate	$K_2Cr_2O_7$
Clinorhombic	Hemihedrism Antihemidrism	π_3 π_1, π_2		
Orthorhombic	Antihemidrism	π_3	Lithium Sulfate	Li_2SO_4
Rhombohedric	Antihemidrism Tetartohedrism	π_3	Scalclesite	—
Quadratic	Antihemidrism Tetartohedrism	π_3	Wulfenite	$PbMoO_4$
Hexagonal	Antihemidrism Tetartohedrism	π_3	Tourmaline Potassium Bromate Sodium Periodate	— $KBrO_3$ $NaIO_4$

and, if point $\bar{M}$ is symmetrical with respect to M, we must have the identities

$$\bar{x}_i^j = x_i^j, \tag{103}$$

hence the relation between the coefficients involved in a type of symmetry characterized by a particular matrix α is

$$\bar{x}_i^j = \alpha_i^k \beta_l^j x_k^l \tag{104}$$

and, as we are using rectangular axes and $\alpha_j^i = \beta_i^j$, these equations can be written

$$x_i^j = \sum_{kl} \alpha_i^k \alpha_j^l x_k^l. \tag{105}$$

It can easily be shown that a transformation of the type

$$a_{ij} = \alpha_i^k \alpha_j^l a_k^l,$$

to which (105) is similar, can be effected by matrix multiplication and

$$(a) = (\alpha) \times (a)(\alpha)^T. \tag{106}$$

Matrix $(\alpha)^T$ represents matrix α transposed, i.e., the lines and columns have been interchanged.

From the definition of a matrix product, obtaining the product $(\alpha)(a)$ involves finding terms such that

$$I_i^l = \alpha_i^k a_{kl}. \tag{107}$$

Then, obtaining the product $(I)(\alpha)^T$ involves finding terms such that

$$a_{ij} = I_i^l \alpha_l^j = \alpha_l^j \alpha_i^k a_k^l. \tag{108}$$

But α_l^j of the transposed matrix is equal to the term of the direct matrix. Transformation (105) can then be obtained by matrix multiplication (106),

$$(x) = (\alpha)(x)(\alpha)^T. \tag{109}$$

As in the case of one-index tensors, we have first found the relation imposed by symmetry about axis ox_3 for an angle different from π. We must next solve the system of equations.

$$\begin{array}{c|c|c|c|}
 & 1 & 2 & 3 \\ \cline{2-4}
1 & x_1^1 & x_1^2 & x_1^3 \\ \cline{2-4}
2 & x_2^1 & x_2^2 & x_2^3 \\ \cline{2-4}
3 & x_3^1 & x_3^2 & x_3^3 \\ \cline{2-4}
\end{array}
\equiv
\begin{array}{|c|c|c|}
\hline
c & -s & \\ \hline
s & c & \\ \hline
 & & 1 \\ \hline
\end{array}
\times
\begin{array}{|c|c|c|}
\hline
x_1^1 & x_1^2 & x_1^3 \\ \hline
x_2^1 & x_2^2 & x_2^3 \\ \hline
x_3^1 & x_3^2 & x_3^3 \\ \hline
\end{array}
\times
\begin{array}{|c|c|c|}
\hline
c & s & \\ \hline
-s & c & \\ \hline
 & & 1 \\ \hline
\end{array}, \tag{110}$$

$$\begin{array}{|c|c|c|}
\hline
c & -s & \\ \hline
s & c & \\ \hline
 & & 1 \\ \hline
\end{array}
\times
\begin{array}{|c|c|c|}
\hline
x_1^1 & x_1^2 & x_1^3 \\ \hline
x_2^1 & x_2^2 & x_2^3 \\ \hline
x_3^1 & x_3^2 & x_3^3 \\ \hline
\end{array}
=
\begin{array}{|c|c|c|}
\hline
x_1^1c - x_2^1s & x_1^2c - x_2^2s & x_1^3c - x_2^3s \\ \hline
x_1^1s + x_2^1c & x_1^2s + x_2^2c & x_1^3s + x_2^3c \\ \hline
x_3^1 & x_3^2 & x_3^3 \\ \hline
\end{array},$$

and

$$\begin{array}{|c|c|c|}
\hline
x_1^1c - x_2^1s & x_1^2c - x_2^2s & x_1^3c - x_2^3s \\ \hline
x_1^1s + x_2^1c & x_1^2s + x_2^2c & x_1^3s + x_2^3c \\ \hline
x_3^1 & x_3^2 & x_3^3 \\ \hline
\end{array}
\times
\begin{array}{|c|c|c|}
\hline
c & s & \\ \hline
-s & c & \\ \hline
 & & 1 \\ \hline
\end{array}
=
\begin{array}{|c|c|c|}
\hline
\begin{matrix} x_1^1c^2 - x_2^1cs \\ + x_2^2s^2 - x_1^2cs \end{matrix} & \begin{matrix} x_1^1cs - x_2^1s^2 \\ - x_2^2cs + x_1^2c^2 \end{matrix} & x_1^3c - x_2^3s \\ \hline
\begin{matrix} x_1^1cs + x_2^1c^2 \\ - x_2^2cs - x_1^2s^2 \end{matrix} & \begin{matrix} x_1^1s^2 + x_2^1cs \\ + x_2^2c^2 + x_1^2cs \end{matrix} & x_1^3s + x_2^3c \\ \hline
x_3^1c - x_3^2s & x_3^1s + x_3^2c & x_3^3 \\ \hline
\end{array}.$$

Equating the terms of the initial tensor and of the right-hand member, we have the equations

$$\left.\begin{array}{ll}
(11) & (x_2^2-x_1^1)s=(x_1^2+x_2^1)c \\
(12) & -(x_2^2-x_1^1)c=(x_1^2+x_2^1)s \\
(13) & x_1^3(c-1)=x_2^3 s \\
(23) & x_1^3 s=-x_2^3(c-1) \\
(31) & x_3^1(c-1)=x_3^2 s \\
(32) & x_3^1 s=-x_3^2(c-1),
\end{array}\right\} \quad (111)$$

whence $x_1^1=x_2^2$, $x_1^2=-x_2^1$, $x_1^3=x_2^3=x_3^1=x_3^2=0$, and $x_3^3=x_3^3$. The symmetry conditions in the case of a two-index tensor for a nonbinary axis ox^3 correspond to the tensor

$$\begin{array}{|c|c|c|}\hline x_1^1 & x_1^2 & \\ \hline -x_1^2 & x_1^1 & \\ \hline & & x_3^3 \\ \hline \end{array}\ .$$

There are now only three distinct susceptances instead of nine.

8.1 Symmetry About the Ternary Axis, Diagonal to the Cube

The tensorial transformation

$$x_i^j=\sum_{kl} \alpha_i^k \alpha_j^l x_k^l$$

reduces to an interchange of indices in the right-hand member, for in the matrix

$$(\alpha)_{\text{diagonal}}^{2\pi/3}=\begin{array}{|c|c|c|}\hline & 1 & \\ \hline & & 1 \\ \hline 1 & & \\ \hline \end{array}\ , \qquad (66)$$

all the coefficients are equal to unity but the numbers of their lines are greater by 1 than the numbers of their columns. The symmetry conditions are

$$\begin{gathered}
x_i^j=x_{i+1}^{j+1} \\
x_1^1=x_2^2=x_3^3 \\
x_1^2=x_2^3=x_3^1 \\
x_1^3=x_2^1=x_3^2.
\end{gathered}$$

Then this tensor must be

$$\begin{array}{|c|c|c|}\hline x_1^1 & x_1^2 & x_1^3 \\ \hline x_1^3 & x_1^1 & x_1^2 \\ \hline x_1^2 & x_1^3 & x_1^1 \\ \hline \end{array}\ .$$

8.2 Alternate Plane

We must write the equations

$$\begin{array}{|c|c|c|}\hline x_1^1 & x_1^2 & x_1^3 \\ \hline x_2^1 & x_2^2 & x_2^3 \\ \hline x_3^1 & x_3^2 & x_3^3 \\ \hline \end{array} \equiv \begin{array}{|c|c|c|}\hline & -1 & \\ \hline 1 & & \\ \hline & & -1 \\ \hline \end{array} \times \begin{array}{|c|c|c|}\hline x_1^1 & x_1^2 & x_1^3 \\ \hline x_2^1 & x_2^2 & x_2^3 \\ \hline x_3^1 & x_3^2 & x_3^3 \\ \hline \end{array} \times \begin{array}{|c|c|c|}\hline & 1 & \\ \hline -1 & & \\ \hline & & -1 \\ \hline \end{array}\ .$$

The tensor readily reduces to

$$\begin{array}{|c|c|c|}\hline x_1^1 & x_1^2 & \\ \hline -x_1^2 & x_1^1 & \\ \hline & & x_3^3 \\ \hline \end{array}\ .$$

8.3 Symmetry About Binary Axes and With Respect to Planes Perpendicular to the Axes of Coordinates

The reasoning is the same as in the case of tensors having two indices. Grids will be prepared in the same way with the boxes cut away (shaded in the text) for the terms +1 which correspond to possible moduli. The boxes are not shaded for factors −1, thus indicating that the corresponding susceptances are impossible. By superposing one or several of such grids on the four fundamental tables, it is possible to show which crystalline systems have susceptances that are not zero.

TABLES

No Symmetry

x_1^1	x_1^2	x_1^3
x_2^1	x_2^2	x_2^3
x_3^1	x_3^2	x_3^3

Nonbinary ox_3 Axis or Alternate Plane

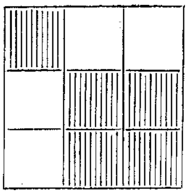
x_1^1	x_1^2	
$-x_1^2$	x_1^1	
		x_3^3

Cube Diagonal

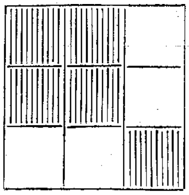
x_1^1	x_1^2	x_1^3
x_1^3	x_1^1	x_1^2
x_1^2	x_1^3	x_1^1

GRIDS

Axis L_1^2 and Plane P_1



Axis L_3^2 and Plane P_3



9. *Piezoelectricity, Tensorial Moduli With Three Indices*

Piezoelectricity is a phenomenon of interdependence between the elastic properties of a crystalline medium and electric induction in this medium.

This phenomenon has two inverse aspects. The first one, discovered by Curie, is that if a crystalline medium having no symmetry center is subjected to stresses, a dielectric polarization appears in the medium. The second one, which Lippmann had predicted from thermodynamic considerations as a consequence of the first, is the deformation of the medium when subjected to an electric field.

Before defining the various moduli possible in a material whose symmetry characteristics are known, we must define what we mean by deformations and by stresses in a solid body, and bring out the tensorial character of these quantities.

We shall consider first a plane deformation. In Fig. 4, let M and P be two neighboring points in a plane having the coordinates

$$M\left|\begin{array}{l} x^1 \\ x^2 \end{array}\right. \qquad P\left|\begin{array}{l} x^1+dx^1 \\ x^2+dx^2 \end{array}\right. \tag{112}$$

in the undeformed matrix. If the matrix is deformed, points M and P are displaced and move to M', P' having the coordinates

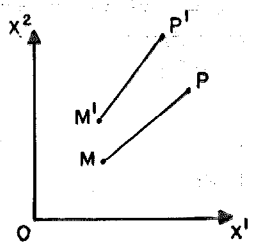


Fig. 4.

$$\left.\begin{array}{ll} M' & \left| \begin{array}{l} x^1+u^1 \\ x^2+u^2 \end{array}\right. \\ P' & \left| \begin{array}{l} x^1+u^1+dx^1+\dfrac{\partial u^1}{\partial x^1}dx^1+\dfrac{\partial u^1}{\partial x^2}dx^2, \\ x^2+u^2+dx^2+\dfrac{\partial u^2}{\partial x^1}dx^1+\dfrac{\partial u^2}{\partial x^2}dx^2. \end{array}\right. \end{array}\right\} \tag{113}$$

We see that vector MP has changed length as well as direction. The lengths $M'P'$ and MP are independent of the reference system as well as of any function depending on these lengths and, in particular, of the quantity:

$$\frac{\overline{M'P'}^2-\overline{MP}^2}{2}=J.$$

As a solid is not easily deformed, u^1 and u^2 are infinitely small. Also, the differential quotients $\partial u^1/\partial x^1$, $\partial u^1/\partial x^2$, $\partial u^2/\partial x^1$, $\partial u^2/\partial x^2$ are infinitely small quantities of the first order and, therefore, their squares or their products can be neglected. Consequently, we have

$$J=\frac{\partial u^1}{\partial x^1}(dx^1)^2+\frac{\partial u^2}{\partial x^2}(dx^2)^2+\left(\frac{\partial u^1}{\partial x^2}+\frac{\partial u^2}{\partial x^1}\right)dx^1dx^2. \tag{114}$$

$$S_{ij}=\frac{1}{2}\left(\frac{\partial u^i}{\partial x^j}+\frac{\partial u^j}{\partial x^i}\right)=S_{ji}. \tag{115}$$

The invariant J then appears as

$$J = S_{ij}\, dx^i\, dx^j. \tag{116}$$

We see that when $i=j=1$, for instance, $S_{11} = \dfrac{\partial u^1}{\partial x^1}$. S_{11} represents the relative lengthening in the direction 1. It can easily be verified by considering two small vectors of lengths du^1 and du^2 parallel to the axes before deformation, that the cosine of their angle after deformation is equal to

$$\cos\theta = \frac{\partial u^1}{\partial x^2} + \frac{\partial u^2}{\partial x^1}, \tag{117}$$

therefore,

$$S_{ij} = \tfrac{1}{2}\cos\theta_{ij}.$$

If points M and P had been taken outside the plane x^1ox^2, we should still have (116) but, in the summation we should have to make i, j successively equal to 1, 2, 3.

From the point of view of tensorial transformations, dx^1 and dx^j must be considered as contravariant variables, the nine quantities S_{ij} (of which only six are distinct since $S_{ij}=S_{ji}$) are the components of a second-order tensor twice covariant, and, knowing S_{ij} in one system, we can calculate S_{ij} for any system by means of the transformation relation

$$\bar{S}_{kl} = \alpha^i_k \alpha^j_l S_{ij}. \tag{118}$$

We can now conceive of the most general piezoelectric phenomenon as being the appearance of a deformation S_{ij} in a crystalline body placed in a field with a component h_k.

If this deformation occurs, assuming it to be linear, the coefficient of proportionality between S_{ij} and h_k may be designated by δ^k_{ij}; it is a piezoelectric modulus and we must have

$$S_{ij} = \delta^k_{ij} h_k. \tag{119}$$

Relation (119) has two very distinct meanings: First, if the values of i, j, and k are fixed, modulus δ^k_{ij} may be defined by a simultaneous measurement of S_{ij} and h_k. It should be noted that as

$$S_{ij} = S_{ji} \quad \text{and} \quad \delta^k_{ij} = \delta^k_{ji},$$

there are really only 18 distinct moduli. However, for further discussion and because of the symmetry of notations, it is much more convenient to assume that there are nominally 27 moduli.

Second, if we consider k as a mute index, (119) represents 9 equations, each having three terms in its right-hand member.

As homogeneity is essential from the tensorial point of view, and as (119) represents a contracted multiplication, the 27 moduli S^k_{ij} are the components of a third-order tensor, twice covariant, and once contravariant.

This makes it possible, when the 27 moduli are known in one system, to calculate them in another system through the relation

$$\bar{\delta}^p_{mn} = \alpha^i_m \alpha^j_n \beta^p_k \delta^k_{ij}, \tag{120}$$

which in rectangular coordinates is

$$\bar{\delta}^p_{mn} = \sum_{ijk} \alpha^i_m \alpha^j_n \alpha^k_p \delta^k_{ij}. \tag{121}$$

These relations also represent the numerical values of moduli at a point $\bar{M}$ when they are known at point M.

In a medium completely without symmetry, as in the hemihedrism form of the triclinic system, all 27 moduli can exist. This means that an electric field oriented in any manner can cause the appearance of all the elastic deformations.

In a medium having symmetry, the necessary relations exist between the moduli so that, if $\bar{M}$ is symmetrical with respect to M, there must be 27 such relations.

$$\bar{\delta}^p_{mn} = \delta^p_{mn}, \tag{122}$$

and, by associating them with (120), the relations between moduli at point M are obtained.

$$\delta^p_{mn} = \sum_{ijk} \alpha^i_m \alpha^j_n \alpha^k_p \delta^k_{ij}, \tag{123}$$

the α's being the coefficients of the transformation matrix of the coordinates of one point in coordinates of the symmetrical point for the symmetry considered.

If the body has a center of symmetry, the transformation matrix is (-1), i.e.,

$$\alpha_1^1 = \alpha_2^2 = \alpha_3^3 = -1,$$

and (123) can be written

$$\delta_{mn}^p = \alpha_m^m \alpha_n^n \alpha_p^p \delta_{mn}^p = -\delta_{mn}^p = 0. \tag{124}$$

All moduli are zero; the phenomenon cannot occur in a medium having a center of symmetry.

We shall now review the various possible cases of symmetry, whose association will enable us to find the possible piezoelectric properties in all crystalline systems.

9.1 Case of Complete Nonsymmetry

The tables of moduli for a case of complete nonsymmetry follow.

δ_{11}^1	δ_{12}^1	δ_{13}^1
δ_{12}^1	δ_{22}^1	δ_{23}^1
δ_{13}^1	δ_{23}^1	δ_{33}^1

δ_{11}^2	δ_{12}^2	δ_{13}^2
δ_{12}^2	δ_{22}^2	δ_{23}^2
δ_{13}^2	δ_{23}^2	δ_{33}^2

δ_{11}^3	δ_{12}^3	δ_{13}^3
δ_{12}^3	δ_{22}^3	δ_{23}^3
δ_{13}^3	δ_{23}^3	δ_{33}^3

9.2 Case of an Axis Passing Through ox_3, Nonbinary

The transformation matrix is

c	$-s$	
s	c	
		1

To effect the tensorial transformation

$$\bar{\delta}_{ij}^k = \sum_{lmn} \alpha_i^l \alpha_j^m \alpha_k^n \delta_{lm}^n$$

and write the relation

$$\delta_{ij}^k = \sum_{lmn} \alpha_i^l \alpha_j^m \alpha_k^n \delta_{lm}^n, \tag{125}$$

we shall calculate all the possible terms equal to $\alpha_i^l \alpha_j^m$ by letting

$$\gamma_r^s = \alpha_i^l \alpha_j^m,$$

where r is a number designating the product ij, and s is a number designating the product lm.

As i, j, l, and m vary from 1 to 3, r and s vary from 1 to 9; then transformation (123) may be written

$$\delta_r^k = \sum_s \gamma_r^s \alpha_k^r \delta_s^n. \tag{126}$$

This transformation can be effected by a matrix product as shown previously and (126) may be written in the form

$$(\delta) = (\gamma)(\delta)(\alpha^T). \tag{127}$$

The terms of tensor δ_{ij}^k may be arranged in any order, provided matrix γ corresponds. In the matrix adopted here, lines 21, 31, and 32 are omitted as they give only equations already written. Consequently, the corresponding lines may be omitted in matrix γ, reducing it to 9 lines and 9 columns.

	1	2	3
11	δ_{11}^1	δ_{11}^2	δ_{11}^3
12	δ_{12}^1	δ_{12}^2	δ_{12}^3
22	δ_{22}^1	δ_{22}^2	δ_{22}^3
13	δ_{13}^1	δ_{13}^2	δ_{13}^3
23	δ_{23}^1	δ_{23}^2	δ_{23}^3
33	δ_{33}^1	δ_{33}^2	δ_{33}^3

To take account of the terms represented by the omitted lines 21, 31, and 32, matrix γ columns 12 and 21 are replaced by a column designated 12, which contains the sum of the terms of 12 and 21. Similar treatment is provided for 13 and 31, and for 23 and 32. The following arrangement is thus obtained for (127).

$$\begin{array}{c|ccc|} & 1 & 2 & 3 \\ \hline 11 & a & b & \gamma \\ 12 & d & e & f \\ 22 & g & h & i \\ 13 & k & l & m \\ 23 & n & \omega & p \\ 33 & q & r & t \\ \hline \end{array} = \begin{bmatrix} c^2 & -2cs & s^2 & & & \\ cs & c^2-s^2 & -c & & & \\ s^2 & 2cs & c^2 & & & \\ & & & c & -s & \\ & & & s & c & \\ & & & & & 1 \end{bmatrix} \times \begin{bmatrix} a & b & \gamma \\ d & e & f \\ g & h & i \\ k & l & m \\ n & o & p \\ q & r & t \end{bmatrix} \times \begin{bmatrix} c & s & \\ -s & c & \\ & & 1 \end{bmatrix}. \tag{128}$$

It is evident that the matrix products given above are of the form

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} c & 0 \\ 0 & \alpha \end{bmatrix} \times \begin{bmatrix} A \\ B \end{bmatrix} \times \begin{bmatrix} \alpha^T \end{bmatrix}, \tag{129}$$

which gives two groups of equations.

$$\left.\begin{array}{l} (A)=(C)(A)(\alpha^T) \\ (B)=(\alpha)(B)(\alpha^T). \end{array}\right\} \tag{130}$$

The second group has already been obtained for the case of two-index tensors and gives the relations:

$$k=\omega, \qquad n=-l, \qquad t=t,$$

and

$$m=p=q=r=0.$$

Also

$$\delta^1_{13}=\delta^2_{23}, \quad \delta^2_{13}=-\delta^1_{23}, \quad \delta^3_{13}=\delta^3_{23}=\delta^1_{33}=\delta^2_{33}=0,$$

and

$$\delta^{3'}_{33}=\delta^3_{33}.$$

The equations of the first group must now be developed.

$$\begin{bmatrix} a & b & \gamma \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} c^2 & -2cs & s^2 \\ c\delta & c^2-s^2 & -cs \\ \delta^2 & 2cs & c^2 \end{bmatrix} \times \begin{bmatrix} a & b & \gamma \\ d & e & f \\ g & h & i \end{bmatrix} \times \begin{bmatrix} c & \delta & \\ -s & c & \\ & & 1 \end{bmatrix}. \tag{131}$$

$$\begin{bmatrix} a & b & \gamma \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} c^2a-2csd+s^2g & c^2b-2csxe+s^2h & c^2\gamma-2csf+s^2i \\ csa+(c^2-\delta^2)d-csh & csb+(c^2-s^2)e-csh & cs\gamma+(c^2s^2)f-csi \\ s^2a+2csd+c^2g & s^2b+2cse+c^2h & s^2\gamma+2csf+c^2i \end{bmatrix} \times \begin{bmatrix} c & s & \\ -s & c & \\ & & 1 \end{bmatrix}, \tag{132}$$

from which we derive:

$$\left.\begin{aligned}(a+g)(c-1)-s(b+h)&=0\\(b+h)(c-1)+s(a+g)&=0\\(i-\gamma)s-2fc&=0\\-(i\gamma)c-2fs&=0\\(a+e)(c-1)-(d+h)s&=0\\-(a+e)s-(d+h)(c-1)&=0.\end{aligned}\right\}\qquad(133)$$

Hence,

$$a=-g=-e,\qquad -b=+h=-d,$$
$$i=\gamma,\qquad \text{and}\qquad f=0,$$

which gives the following arrangement of the table of moduli.

δ^1_{11}	$-\delta^2_{22}$	δ^1_{13}
$-\delta^2_{22}$	$-\delta^1_{11}$	δ^1_{23}
δ^1_{13}	δ^1_{23}	

1

$-\delta^2_{22}$	$-\delta^1_{11}$	$-\delta^1_{23}$
$-\delta^1_{11}$	δ^2_{22}	δ^1_{13}
$-\delta^1_{23}$	δ^1_{13}	

2

δ^3_{11}		
	δ^3_{11}	
		δ^3_{33}

3

(134)

9.3 Case of Ternary Axis, Parallel to Diagonal of Cube Whose Sides are Parallel to Axes of Coordinates

In the case of a ternary axis that is parallel to the diagonal of a cube, the sides of which are parallel to the axes of coordinates, the relation (125) becomes

$$\delta^k_{ij}=\alpha_i^{i+1}\alpha_j^{j+1}\alpha_k^{k+1}\delta^{(k+1)}_{(i+1)(j+1)}.\qquad(135)$$

The three coefficients of α being equal to unity, the relation is

$$\delta^k_{ij}=\delta^{(k+1)}_{(i+1)(j+1)}.$$

The terms of tables 2 and 3, given below, are derived from the terms of table 1 by circular permutation.

δ^1_{11}	δ^1_{12}	δ^1_{13}
δ^1_{12}	δ^1_{22}	δ^1_{23}
δ^1_{13}	δ^1_{23}	δ^1_{33}

1

δ^1_{33}	δ^1_{13}	δ^1_{23}
δ^1_{13}	δ^1_{11}	δ^1_{12}
δ^1_{23}	δ^1_{12}	δ^1_{22}

2

δ^1_{22}	δ^1_{23}	δ^1_{12}
δ^1_{23}	δ^1_{33}	δ^1_{13}
δ^1_{12}	δ^1_{13}	δ^1_{11}

3

9.4 Alternate Plane

The transformation matrix for the alternate plane is

	1	2	3
1		-1	
2	1		
3			-1

.

The relations (124) have only one term in their right-hand members, the index of which is derived by a circular permutation of the indices of the left-hand member.

For certain terms, such as δ^1_{11}, we find that $\delta^1_{11}=-\delta^2_{22}$ and $\delta^2_{22}=+\delta^1_{11}$, whence $\delta^1_{11}=\delta^2_{22}=0$. By acting in this manner on all terms, the following tables are obtained.

	1	2	3
1	0	0	δ^1_{13}
2	0	0	δ^1_{23}
3	δ^1_{13}	δ^1_{23}	

1

1	2	3
0	0	δ^1_{23}
0	0	$-\delta^1_{13}$
δ^1_{23}	$-\delta^1_{13}$	

2

1	2	3
δ^3_{11}	δ^3_{12}	0
δ^3_{21}	$-\delta^3_{11}$	0
0	0	0

3

9.5 Symmetry With Respect to Binary Axes

Symmetry with respect to the binary axes manifests itself by the suppression of certain moduli. If we assume the binary axis L_1^2, the transformation matrix and (124) become

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & & \\ 2 & & -1 & \\ 3 & & & -1 \end{array}, \qquad (131)$$

$\delta_{ij}^k = \pm\delta_{ij}^k$. Every time the product $\alpha_i^i\alpha_j^j\alpha_k^l = -1$, the modulus cannot exist.

A grid may be formed by superposing 3 tables having numbered boxes of which those corresponding to a + sign have been cut out. The following arrangement is obtained for axis L_1^2.

	1	2	3
1	▨		
2		▨	▨
3		▨	▨

1

	1	2	3
1		▨	▨
2	▨		
3	▨		

2

	1	2	3
1		▨	▨
2	▨		
3	▨		

3

The transformation matrix and grids for axis $L_3^{(2)}$ are as follows.

$$\begin{array}{ccc} -1 & & \\ & -1 & \\ & & 1 \end{array}$$

▨	▨	
▨	▨	
		▨

		▨
		▨
▨	▨	

		▨
		▨
▨	▨	

The grids corresponding to symmetries with respect to the planes are complementary to the preceding ones.

Plane P_1

	▨	▨
▨		
▨		

Plane P_1

▨		
	▨	▨
	▨	▨

Plane P_1

▨		
	▨	▨
	▨	▨

Plane P_3

		▨
		▨
▨	▨	

Plane P_3

▨	▨	▨
▨	▨	
		▨

Plane P_3

▨	▨	
▨	▨	
		▨

With these four tables of moduli and grids, it is easy to determine the piezoelectric properties of all crystalline groups by the moduli corresponding to the relation between the deformations and the fields.

We can represent, in a similar way, the relation between dielectric polarizations and stresses in a solid.

9.6 Expression of Stresses in Material

Consider a material in a state of stress and also in equilibrium. The stress may be represented by a small parallelepiped isolated from the rest of the material and maintained in the same condition of constraint by applying forces to its faces.

Thus, in Fig. 5, on face $BCDE$, a force FA is applied at the center of F and, in general, is oriented obliquely with respect to the normal of the face.

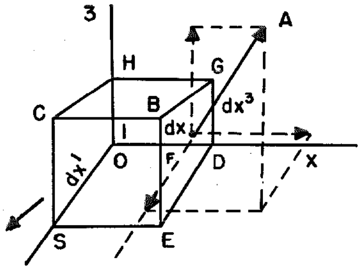


Fig. 5.

This force, relative to one face whose normal is directed along ox^2, is numbered $\vec{F}_2$ and has three components designated F_{12}, F_{22}, and F_{32}. These components are proportional to the areas of the surfaces to which they are applied, and their quotients by these areas represent effectively the stress condition at the point considered. This quotient is represented by T.

$$T_{ij} = \frac{F_{ij}}{d\sigma j}. \tag{136}$$

On the face $GHIS$, a force is exerted that is equal and opposite in sign to FA.

If we designate the area $d\sigma$ as $+$ when the normal out of the cube through that surface is oriented in the positive direction of the axes, and designate the opposite case as $-$, the stresses T_{ij} are expressed in the same manner on two opposite faces and, therefore, nine quantities T_{ij} will be sufficient to represent the stresses at a point which is the center of the cube, when the dimensions of the cube are decreased.

There are really only six distinct quantities T_{ij}, i.e.,

$$T_{ij} = T_{ji} \tag{137}$$

for, if the system is in equilibrium, there is no resultant couple. The forces acting on the faces, projected on AB and OC, Fig. 6, are $T_{12}dx^1dx^3$, and their moments with respect to the axis of the cube parallel to ox^3 are

$$T = T_{12}dx^1dx^3dx^2 = T_{12}d\sigma.$$

Similarly, the forces applied to the faces projected on CA and BC have a resultant moment

$$T = T_{21}d\sigma.$$

As the system is in equilibrium,

$$T_{12} = T_{21}.$$

These nine components constitute a tensor. If u^1 is the displacement of straight line OA after deformation, that of straight line BC is $u_1^1 + du_1^1$, and the work of deformation effected by the forces oriented along axis 1 is

$$dw_1 = \bar{F}_1 du^1$$

since

$$F_1 = T_{1j}d\sigma^j$$

and

$$dw_1 = T_{ij}du^1d\sigma^j.$$

For the whole volume, the work is

$$dw = T_{ij}du^1d\sigma^j. \tag{138}$$

We shall represent all quantities in rectangular coordinates, and the coefficients of the transformation matrixes α are the directing cosines of the second system of axes with respect to the first. We know that under such conditions, the areas of the surfaces are transformed in accordance with the values of their normals. We thus have a right to consider the areas as being once contravariant; the displacements du, homogeneous to lengths, are also once contravariant; therefore the stresses are twice covariant.

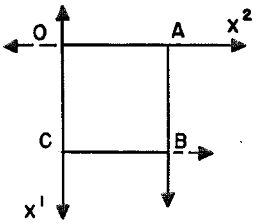


Fig. 6.

The piezoelectric relation can be written as follows.

In a constrained body, there appears a dielectric polarization that is a linear function of the deformations. The dielectric polarization is related to the induction

$$\vec{b} = K_0\vec{h} + 4\pi\vec{P}, \tag{139}$$

$$P_i = \delta_i^{jk} T_{jk}. \tag{140}$$

Lippmann showed by thermodynamic analogy the identity of one of the moduli δ_1^{11} with the modulus previously described. The same reasoning may be extended to all moduli, the δ_1^{jk} and the δ_{jk}^i are really the covariant and contravariant components of one tensor, components which are numerically equal in trirectangular axes.

We can easily see that the general relations (139) and (140) can assume the shape given originally by Curie for one particular case.

If we assume a slab of quartz cut perpendicularly to the electric axis and subjected to a compressional stress in a direction normal to the force, we have

$$P_1 = \delta_1^{11} T_{11}, \tag{141}$$

and if at the same time a potential difference V is applied between the electrodes covering the slab, we have

$$b_1 = Kh_1 + 4\pi\delta_1^{11} T_{11}. \tag{142}$$

But $b = 4\pi\sigma$, where σ designates the surface density of the charges, hence

$$\sigma = \frac{K}{4\pi}\frac{v}{e} + \delta_1^{11} T_{11}, \tag{143}$$

and if the capacitor has an area S,

$$Q = \sigma s = \frac{KS}{4\pi e} V + \delta_1^{11} F_1, \tag{144}$$

$$Q = CV + \delta F, \tag{145}$$

which is the relation given initially by Curie.

10. Elasticity of Solids, Hooke's Law, Tensorial Moduli With Four Indices

Hooke's Law states that there exists a linear relation between the stresses and the strains in an elastic body.

Any particular stress T_{ij} caused in an anisotropic body by any particular strain T_{kl}, results in a modulus defined by

$$T_{ij} = C_{ij}^{kl} \delta_{kl}. \tag{146}$$

If the body is completely anisotropic, all the constraints are linear functions of all the strains, and (146) then represents a system of nine equations, each having nine terms in its right-hand member. Thus there are 81 moduli C_{ij}^{kl} forming the components of a tensor of the fourth order, twice covariant, and twice contravariant.

If we know the components of the tensor at point M, we can calculate the components at point $\bar{M}$, whose coordinates are derived from those of point M by means of the linear transformation matrix α and the general relation

$$\overline{C_{mn}^{pq}} = \alpha_m^i \alpha_n^j \beta_k^p \beta_p^q C_{ij}^{kl}, \tag{147}$$

or

$$C_{mn}^{pq} = \sum_{ijkl} \alpha_m^1 \alpha_n^j \alpha_p^k \alpha_q^l C_{ij}^{kl}. \tag{148}$$

Actually, there are not 81 independent moduli even in a completely anisotropic body.

As $\delta_{kl} = \delta_{lk}$ and $T_{ij} = T_{ji}$, we derive from (146) and for particular values of i, j, k, and l

$$C_{ij}^{kl} = C_{ji}^{kl} = C_{ij}^{lk} = C_{ji}^{lk}. \tag{149}$$

As the strain potential energy, which is invariant, can be determined by

$$dw = \sum_{ij} T_{ij} S_{ij},$$

we deduce

$$dw = \sum_{(ij)(kl)} C_{ij}^{kl} S_{ij} S_k^l. \tag{150}$$

In the latter sum, we can interchange the groups (ij) and (kl) and obtain

$$C_{ij}^{kl} = C_{kl}^{ij},$$

hence

$$C_{ij}^{kl} = C_{ji}^{kl} = C_{ij}^{kl} = C_{ji}^{kl} = C_{kl}^{ij} = C_{kl}^{ji} = C_{lk}^{ji} = C_{lk}^{ij} = C_{lk}^{ji}.$$

There remain, finally, only 21 independent moduli. There are actually only six components S_{ij} for the strain, and all the terms form a homogeneous second-degree polynomial with respect to these six quantities and, thus, include $6(6+1)/2 = 21$ independent coefficients.

For these, it is customary to use a notation having a reduced number of indices, but we prefer to keep the four distinct indices to facilitate tensorial transformations and circular permutations, which are fundamental in expressing certain properties of symmetry.

We shall represent the moduli in tables having six lines and six columns.

To calculate the relation between moduli where there is a nonbinary symmetry axis ox^3, we can use a method similar to that for the three-index moduli; by letting $\gamma_r^s = \alpha_m^i \alpha_n^j$ and $\gamma_t^u = \alpha_p^k \alpha_q^l$, (148) becomes

$$C_r^s = \sum_{su} \gamma_r^s \gamma_t^u C_s^u, \tag{151}$$

which can be calculated by a matrix product

$$(C) = (\gamma)(C)(\gamma^T). \tag{152}$$

Developing and designating momentarily the moduli a, b, etc., (their indices correspond to the numbers at the heads of the lines and columns) we have

Designating matrixes corresponding to one fourth of each of the above matrixes by A, B, C, D, E, the operations are symbolized by the relation

$$\begin{array}{|c|c|}\hline A & B \\ \hline B^T & C \\ \hline\end{array}$$

$$= \begin{array}{|c|c|}\hline D & O \\ \hline O & E \\ \hline\end{array} \times \begin{array}{|c|c|}\hline A & B \\ \hline B^T & C \\ \hline\end{array} \times \begin{array}{|c|c|}\hline D^T & \\ \hline & E^T \\ \hline\end{array},$$

which gives four groups of relations: $(A) = (D)(A)(D^T)$, $(B) = (D)(B)(E^T)$, $(C) = (E)(C)(E^T)$, and $(B)^T = (E)(B)^T(D^T)$. The fourth relation gives the same equations as the second.

But the fourth group is similar to (110), which enabled us to represent the properties of the two-index tensor, and the third group is similar to (131) of the three-index tensor; we derive from it

$$P = \delta, \quad Q = -q = 0, \quad r = t = 0, \quad u = u,$$

$$\begin{array}{c|cccccc|} & 11 & 12 & 22 & 13 & 23 & 33 \\ \hline 11 & a & b & \gamma & d & e & f \\ 12 & b & g & h & i & j & k \\ 22 & & h & l & m & n & o \\ 13 & d & i & m & p & q & r \\ 23 & e & j & n & q & s & t \\ 33 & f & k & o & r & t & u \\ \hline \end{array} = \begin{array}{|cccccc|}\hline c^2 & -2cs & s^2 & & & \\ cs & c^2-s^2 & -cs & & & \\ s^2 & 2cs & c^2 & & & \\ & & & c & s & \\ & & & -s & c & \\ & & & & & 1 \\ \hline\end{array} \times \begin{array}{|cccccc|}\hline a & b & \gamma & d & e & f \\ b & g & h & i & j & k \\ \gamma & h & l & m & n & o \\ d & i & m & p & q & r \\ e & j & n & q & s & t \\ f & k & o & r & t & u \\ \hline\end{array}$$

$$\times \begin{array}{|cccccc|}\hline c^2 & cs & s^2 & & & \\ -2cs & c^2-s^2 & +2cs & & & \\ s^2 & -cs & c_2 & & & \\ & & & c & -s & \\ & & & s & c & \\ & & & & & 1 \\ \hline\end{array}.$$

and

$$d = -m = -j, \quad n = -e = -i, \quad \sigma = j, \quad \text{and} \quad K = 0.$$

That is,

$$C_{11}^{13} = -C_{22}^{13} = -C_{12}^{23},$$
$$C_{22}^{23} = -C_{11}^{23} = -C_{12}^{13},$$
$$C_{22}^{33} = C_{11}^{33} = C_{12}^{33} = 0.$$

There remains only to calculate the equations:

$$\begin{array}{c|c|c|c|} & 11 & 12 & 22 \\ \hline 11 & a & b & \gamma \\ \hline 12 & b & g & h \\ \hline 22 & \gamma & h & l \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline c^2 & -2cs & s^2 \\ \hline cs & c^2-s^2 & -cs \\ \hline s^2 & 2cs & c^2 \\ \hline \end{array} \times \begin{array}{|c|c|c|} \hline a & b & \gamma \\ \hline b & g & h \\ \hline \gamma & h & l \\ \hline \end{array} \times \begin{array}{|c|c|c|} \hline c^2 & cs & s^2 \\ \hline -2cs & c^2-s^2 & 2cs \\ \hline s^2 & -cs & c^2 \\ \hline \end{array}$$

$$= \begin{array}{|c|c|c|} \hline c^2a-2bcs+\gamma s^2 & c^2b-2gc\delta+hs^2 & \gamma c^2-2hcs+ls^2 \\ \hline acs+b(c^2-s^2)-\gamma cs & bcs+g(c^2-s^2)-hcs & \gamma c\delta+h(c^2-s^2)-lcs \\ \hline s^2a+2bcs+\gamma c^2 & bs^2+2gcs+hc^2 & \gamma s^2+2hcs+lc^2 \\ \hline \end{array} \times \begin{array}{|c|c|c|} \hline c^2 & cs & s^2 \\ \hline -2cs & c^2-s^2 & 2cs \\ \hline s^2 & -cs & c^2 \\ \hline \end{array}.$$

Calculating the sum $a+\gamma$, as being the first and third terms of the first line, we see that all we have to do is to multiply the first line of the first matrix by the sum of the first and third columns of the second, i.e.,

$$\begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline 1 \\ \hline \end{array}.$$

We thus derive:

from $a+\gamma$,

$$0=\delta(l-a)-2(b+h)c;$$

from g,

$$(a-\gamma-2g)c\delta+(b-h)(c^2-\delta^2)=0;$$

from $b+h$,

$$0=-c(l-a)-2(b+h)\delta;$$

from $b-h$,

$$(a-\gamma-2g)(c^2-\delta^2)-(b-h)c\delta=0;$$

whence

$$l=a;\quad b=h=0;\quad g=\frac{a-\gamma}{2};$$

$$C_{11}^{11}=C_{22}^{22};\quad C_{11}^{12}=C_{12}^{22}=0;\quad \text{and}\quad C_{12}^{12}=\frac{C_{11}^{11}-C_{11}^{22}}{2}.$$

The table of moduli follows.

	11	12	22	13	23	33
11	C_{11}^{11}		C_{11}^{22}	C_{11}^{13}	$-C_{22}^{23}$	C_{11}^{33}
12		$\frac{C_{11}^{11}-C_{11}^{22}}{2}$		$-C_{22}^{23}$	$-C_{11}^{13}$	
22			C_{11}^{11}	$-C_{11}^{13}$	C_{22}^{23}	C_{11}^{33}
13				C_{13}^{13}		
23					C_{13}^{13}	
33						C_{33}^{33}

There are only seven distinct moduli. This is the case of a ternary axis. The condensed table of moduli for a ternary axis is

	11	12	13	22	23	33
11	C_{11}^{11}	0	C_{11}^{13}	C_{11}^{22}	C_{11}^{23}	C_{11}^{33}
12		$\frac{C_{11}^{11}-C_{11}^{22}}{2}$	$-C_{11}^{23}$	0	$-C_{11}^{13}$	0
13			C_{13}^{13}	$-C_{11}^{13}$	0	0
22				C_{11}^{11}	$-C_{11}^{23}$	C_{11}^{33}
23					C_{13}^{13}	0
33						C_{33}^{33}

For a quaternary axis, the transformation matrix is

	-1	
$+1$		
		1

which gives the tables:

	11	12	13	22	23	33
11	C_{11}^{11}	C_{11}^{12}		C_{11}^{22}		C_{11}^{33}
12		C_{12}^{12}		$-C_{11}^{12}$		
13			C_{13}^{13}			
22				C_{11}^{11}		C_{11}^{33}
23					C_{13}^{13}	
33						C_{33}^{33}

Quaternary Axis

	11	12	13	22	23	33
11	C_{11}^{11}	C_{11}^{12}	C_{11}^{13}	C_{11}^{22}	C_{11}^{23}	C_{11}^{22}
12		C_{12}^{12}	C_{12}^{13}	C_{11}^{13}	C_{12}^{13}	C_{11}^{23}
13			C_{12}^{12}	C_{11}^{23}	C_{12}^{13}	C_{11}^{12}
22				C_{11}^{11}	C_{11}^{12}	C_{11}^{22}
23					C_{12}^{12}	C_{11}^{13}
33						C_{11}^{11}

Ternary Axis of the Cube

The table of moduli, in the case of symmetry about the diagonal of the cube, is derived from the table for complete nonsymmetry by circular permutation; the arrangement is obtained as follows.

When there exists an alternate plane of an even order, such as $\pi/2$, the symmetry about the binary axis L_3 makes the moduli having an odd number of indices vanish. The transformation matrix differs from that of the quaternary axis only in the sign of α_3^3, the table of moduli is the same as in the case of the quaternary axis, since α_3^3 is only involved by its square.

The grids corresponding to axis 1 and to the plane P_1 are the same as those of axis 3 and plane P_3, by virtue of the fact that the number of indices is even.

For symmetry about axis ox_1:

$$\alpha_1^1 = +1, \quad \alpha_2^2 = \alpha_3^3 - 1$$

for plane P_1,

$$\alpha_1^1 = -1, \quad \alpha_2^2 = \alpha_3^3 = +1.$$

If we consider the groups having an even number of "one" and if we express the symmetry, the relation will be of the form:

$$C_{ij}^{11} = \alpha_i^i \alpha_j^j C_{ij}^{11},$$

or

$$C_{ij}^{11} = (-1)^2 C_{ij}^{11}$$

with respect to the axis, and

$$C_{ij}^{11} = (+1)^2 C_{ij}^{11}$$

with respect to the plane.

If the group of indices contains index 1 only once,

$$C_{ij}^{lk} = (-1)^3 C_{ij}^{lk}$$

with respect to the axis, and

$$C_{ij}^{lk} = -1 C_{ij}^{lk}$$

with respect to the plane. This gives the same distribution of the signs + and −1.

	11	12	13	22	23	33
11	▨			▨	▨	▨
12		▨	▨			
13			▨			
22				▨	▨	▨
23					▨	▨
33						▨

L_1 Axis and Plane P_1

	11	12	13	22	23	33
11	▨	▨		▨		▨
12		▨		▨		▨
13			▨		▨	
22				▨		▨
23					▨	
33						▨

These tables give us information regarding the elastic properties of a material.

In the case of quartz, for instance, characterized by a ternary axis ox_3 and a binary axis ox_1, we obtain the following table:

	11	12	13	22	23	33
11	C_{11}^{11}			C_{11}^{22}	C_{11}^{23}	C_{11}^{33}
12		$\frac{C_{11}^{11} - C_{11}^{22}}{2}$	$-C_{11}^{23}$			
13			C_{13}^{13}			
22				C_{11}^{11}	$-C_{11}^{23}$	C_{11}^{33}
23					C_{13}^{13}	
33						C_{33}^{33}

We see, for instance, that a tensional stress oriented along the optical axis ox_3 can cause a strain in the directions 3 and δ_{33}, as well as in the directions 1 and 2, these two latter strains being equal, $\delta_{11}=\delta_{22}$.

Similarly, a stress of type T_{11} causes a strain δ_{11}, δ_{22}, and δ_{33}, and also a strain of type δ_{23}, etc.

We also see that a stress of type 12, caused for instance by a tensional torque whose moment is oriented along the electrical axis, causes a type-12 strain with a modulus

$$C_{12}^{12}=\frac{C_{11}^{11}-C_{11}^{22}}{2},$$

but also a type-11 strain, i.e., a change of length in the direction of axis 1, as would have been caused by a compression in the direction of this axis.

10.1 Case of Isotropic Materials

An isotropic material has an infinity of axes and of planes of symmetry; to reduce the number of moduli, we shall use the table for the ternary axis and superimpose the grids for binary axes L_1 and L_3; we have then found the arrangement and since there exists a symmetry about a ternary axis oriented as the cube diagonal, we must have equality between the moduli by circular permutation, which gives the tables shown below.

	11	12	13	22	23	33
11	C_{11}^{11}			C_{11}^{22}		C_{11}^{33}
12		$\frac{C_{11}^{11}-C_{12}^{11}}{2}$				
13			C_{13}^{13}			
22				C_{11}^{11}		C_{11}^{33}
23					C_{13}^{13}	
33						C_{33}^{33}

	11	12	13	22	23	33
11	C_{11}^{11}			C_{11}^{22}		C_{11}^{22}
12		$\frac{C_{11}^{11}-C_{11}^{22}}{2}$				
13			$\frac{C_{11}^{11}-C_{11}^{12}}{2}$			
22				C_{11}^{11}		C_{11}^{22}
23					$\frac{C_{11}^{11}-C_{11}^{22}}{2}$	
33						C_{11}^{11}

In the case of isotropic materials, there are only two distinct moduli, C^{11}_{11} and C^{22}_{11}. The first one corresponds to the proportionality relation between stress and strain in the direction of the stress. It has the same value along the three axes, and, therefore in all directions in space. It is Young's modulus E. The second one corresponds to a lengthening strain in a direction perpendicular to the tensional stress; it has the same value along the three axes, i.e., the lengthening is the same in all directions in space perpendicular to the stress. Poisson's coefficient gives the ratio between modulus C^{22}_{11} and Young's modulus. $C^{22}_{11}=\sigma E$. Finally, modulus C^{12}_{12} expresses the relation between a torque and the tension in a wire, it is proportional to Coulomb's modulus, but we see that it depends on moduli C^{11}_{11} and C^{22}_{11}, so that there are actually only two characteristic moduli for any isotropic material.

10.2 THERMAL EXPANSION OF SOLIDS

The change in length with temperature of a bar of length x may be expressed by the expansion binomial

$$x=x_0(1+A\theta),$$

where $\theta=t-t_0$, which can be written

$$\frac{x-x_0}{x_0}=A\theta.$$

But $(x-x_0)/x_0$ is the relative deformation in direction x, δ_{11} for instance. We can generalize and state that in an anisotropic body, a change in temperature may bring about a displacement of all points; these displacements are characterized by the tensor of the 3 and must produce a tensorial relation. The temperature change θ being a scalar $\delta_{ij}=A_{ij}x\theta$, i.e., a tensor of order zero, the expansion coefficient A forms a tensor with two covariant indices and six distinct components. The characteristic tables for the expansion coefficient for the various symmetries are the same as those for the specific inductive powers.

The tables of constants give only two temperature coefficients for certain crystalline substances; this constitutes a gap. In fact, by an appropriate choice of axes, we can always reduce the components of the tensor of the δ_{ij} to three components, $\bar{\delta}_{11}$, $\bar{\delta}_{22}$, $\bar{\delta}_{23}$, but in such a system there still exist three distinct components of the tensor of expansions.

10.3 THERMAL CONDUCTIVITY

Thermal conductivity may be defined by the amount of heat passing in one unit of time through an area σ^i under the action of a temperature gradient;

$$\frac{dq}{dt}=x^j_i\sigma^i\left(\frac{d\theta}{\partial x}\right)_j.$$

This gradient, in which θ has a zero variance and x is contravariant, is itself once covariant, and since dq/dt is invariant, the thermal conductivity must be measured in an anisotropic body by nine components out of which six are distinct.

The reductions caused by symmetry are the same as for the dielectric constant.

11. Piezomagnetism

By analogy with piezoelectricity, Voigt has studied the possibility of a deformation of matter placed in a magnetic field. If the phenomenon exists, its effect is very small and, to date, it does not seem to have been demonstrated clearly. It is interesting, however, to estimate which crystalline groups might exhibit this behavior.

We shall assume that the presence in the medium of a magnetic induction creates deformations. To find the relation between the moduli, we must first define the tensorial character of magnetic induction

Assume a charge e, moving at a velocity $\bar{v}$ in a magnetic field; the force applied to the mass is

$$\vec{f}=e\vec{v}A\vec{\beta},$$

and the work, which is an invariant, for a displacement dx of the charge is

$$d\mathcal{T}=\vec{f}\cdot\overrightarrow{dx},$$

$$d\mathcal{T}=(v^3\beta_3-v^2\beta_2)dx^1+(v^3\beta_1-v^1\beta_3)dx^2 \\ +(v^1\beta_2-v^2\beta_1)dx^3.$$

This expression is of the form:

$$d\mathcal{T}=\beta_{ij}dx^iv^j,$$

with

$$\beta_{12}=\beta_3=-\beta_{21},$$
$$\beta_{23}=\beta_1=-\beta_{32},$$
$$\beta_{13}=-\beta_2=-\beta_{31}.$$

Magnetic induction appears here as a tensor of the second order, nonsymmetrical, twice covariant.

The phenomenon of piezomagnetism would be represented by moduli M^{kl}_{ij} such that

$$S_{ij} = M^{kl}_{ij} \beta_{kl}.$$

We see that

$$M^{kl}_{ij} = -M^{lk}_{ij} = M^{kl}_{ji} = -M^{pk}_{ji} \quad \text{and} \quad M^{ll}_{ij} = 0.$$

There are 18 distinct moduli in the case of complete nonsymmetry. In this case, the table of moduli is

	11	12	13	21	22	23	31	32	33
11		M^{12}_{11}	M^{13}_{11}	$-M^{12}_{11}$		M^{11}_{23}	$-M^{13}_{11}$	$-M^{11}_{23}$	
12		M^{12}_{12}	M^{13}_{12}	$-M^{12}_{12}$		M^{23}_{12}	$-M^{13}_{12}$	$-M^{22}_{12}$	
13		M^{12}_{13}	M^{13}_{13}	$-M^{12}_{13}$		M^{23}_{13}	$-M^{13}_{13}$	$-M^{23}_{13}$	
21		M^{12}_{12}	M^{13}_{12}	$-M^{12}_{12}$		M^{23}_{12}	$-M^{23}_{12}$	$-M^{13}_{12}$	
22		M^{12}_{22}	M^{13}_{22}	$-M^{12}_{22}$		M^{23}_{22}	$-M^{13}_{22}$	$-M^{23}_{22}$	
23		M^{12}_{23}	M^{13}_{23}	$-M^{12}_{23}$		M^{23}_{23}	$-M^{13}_{23}$	$-M^{23}_{23}$	
31		M^{12}_{13}	M^{13}_{13}	$-M^{12}_{13}$		M^{23}_{13}	$-M^{13}_{13}$	$-M^{23}_{13}$	
32		M^{12}_{23}	M^{13}_{23}	$-M^{12}_{23}$		M^{23}_{23}	$-M^{13}_{23}$	$-M^{23}_{23}$	
33		M^{12}_{33}	M^{13}_{33}	$-M^{12}_{33}$		M^{23}_{33}	$-M^{13}_{33}$	$-M^{23}_{33}$	

11.1 REDUCTION OF MODULI FOR A NONBINARY AXIS

As in the case of elasticity, we group the indices in pairs and set the coefficients of a matrix γ so that

$$\gamma^r_s = \alpha^m_i \alpha^n_j.$$

The relations involved for the particular case of symmetry are written

$$(M) = (\gamma)(M)(\gamma^T).$$

The table of components of tensor M consists of nine lines and nine columns, but, because of nonsymmetry with respect to the upper indices, we need consider only three columns. Similarly, the symmetry of the lower indices, $\delta_{ij} = \delta_{ji}$, make necessary consideration of only six lines. This reduces matrix γ to only six lines and six columns, obtained by adding the terms of columns 1.2 +2.1, etc.In matrix (γ^T), we need consider only three lines and three columns. The equations are written

$$\begin{array}{|c|c|c|}\hline a & b & \gamma \\ \hline d & e & f \\ \hline g & h & i \\ \hline k & l & m \\ \hline n & \omega & p \\ \hline q & r & \sigma \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|}\hline c^2 & -2cs & s^2 & & & \\ \hline +cs & c^2-s^2 & -cs & & & \\ \hline s^2 & +2cs & c^2 & & & \\ \hline & & & c & -s & \\ \hline & & & s & c & \\ \hline & & & & & 1 \\ \hline \end{array} \times \begin{array}{|c|c|c|}\hline a & b & \gamma \\ \hline d & e & f \\ \hline g & h & i \\ \hline k & l & m \\ \hline n & \omega & p \\ \hline q & r & \sigma \\ \hline \end{array} \times \begin{array}{|c|c|c|}\hline 1 & & \\ \hline & c & s \\ \hline & -s & c \\ \hline \end{array},$$

$$\begin{array}{|c|c|c|}\hline ac^2-2dcs+gs^2 & bc^2-2ecs+hs^2 & \gamma c^2-2fcs+is^2 \\ \hline (a-g)cs+d(c^2-s^2) & (b-h)cse(c^2-s^2) & (\gamma-i)cs+f(c^2-s^2) \\ \hline as^2+2dcs+gc^2 & bs^2+2ecs+gc^2 & (\gamma s^2)+2fcs+ic^2 \\ \hline kc-ns & lc-\omega s & mc-ps \\ \hline ks+nc & ls+\omega c & ms+pc \\ \hline q & r & \sigma \\ \hline \end{array} \times \begin{array}{|c|c|c|}\hline 1 & & \\ \hline & c & s \\ \hline & -s & c \\ \hline \end{array}.$$

Whence the equations:

$$\begin{array}{r|r|r|} (g-a)s-2dc=0 & (b+h)(c-1)-(\gamma+i)s=0 & (\gamma-e)(c-1)+s(h-f)=0 \\ -(g-a)c-2ds=0 & (b+h)s+(\gamma+i)(c-1)=0 & (\gamma-e)s-(h-f)(c-1)=0 \\ k(c-1)-ns=0 & +(p-l)c+(\omega+m)s=0 & \\ ks+n(c-1)=0 & r(c-1)-\sigma s=0 & \\ q=q & rs+\sigma(c-1)=0 & \end{array}$$

$$a=g, \quad b=h=-f, \quad \gamma=e=-i, \quad p=l, \quad \omega=-m, \quad h=f, \quad d=k=n=p=\sigma=0.$$

The tables corresponding to the symmetry of the various axes are:

	12	13	23
11	M_{11}^{12}	M_{11}^{13}	M_{11}^{23}
12		M_{11}^{23}	$-M_{11}^{13}$
22	M_{11}^{12}	$-M_{11}^{13}$	$-M_{11}^{23}$
13		M_{13}^{13}	M_{13}^{23}
23		$-M_{13}^{23}$	M_{13}^{13}
33	M_{33}^{12}		

Ternary Axis

	12	13	23
11	M_{11}^{12}	M_{11}^{13}	M_{11}^{23}
12	M_{12}^{12}	M_{12}^{13}	M_{12}^{23}
22	M_{22}^{12}	M_{22}^{13}	M_{22}^{23}
13	M_{13}^{12}	M_{13}^{13}	M_{13}^{23}
23	M_{23}^{13}	M_{23}^{13}	M_{23}^{23}
33	M_{33}^{12}	M_{33}^{13}	M_{33}^{23}

Nonsymmetry

M^{12}_{11}	$-M^{12}_{22}$	M^{12}_{33}
M^{12}_{12}	$-M^{12}_{23}$	M^{12}_{13}
M^{12}_{22}	$-M^{12}_{33}$	M^{12}_{11}
M^{12}_{13}	$-M^{12}_{12}$	M^{12}_{13}
M^{12}_{23}	$-M^{12}_{13}$	M^{12}_{12}
M^{12}_{33}	$-M^{12}_{11}$	M^{12}_{22}

Cube Diagonal

M^{12}_{11}		
M^{12}_{11}		
	M^{13}_{13}	
		M^{13}_{13}
M^{12}_{33}		

Alternate Plane π_2

The grids for planes P_1 and P_3 are the same as those for the perpendicular binary axes.

	12	13	23
11			▨
12	▨	▨	
22			▨
13	▨	▨	
23			▨
33			▨

	12	13	23
11	▨		
12	▨		
22	▨		
13		▨	▨
23		▨	▨
33	▨		

12. *Physical Properties of Bodies Crystallizing in Various Systems*

12.1 CRYSTALLINE NETWORK

A crystal is capable of cleavage in several directions; the faces appearing by cleavage are plane. A crystalline network is constituted by three systems of planes that limit the smallest possible volume that can be isolated. These planes are called *reticular planes*, the smallest elementary volume is the *mesh*, and the apexes are the *nodes*. The arrangement of atoms inside the mesh is the *pattern*. The crystal network, most generally, presents a certain symmetry. The symmetry of the network is generally greater than that of the crystalline medium.

When the crystalline medium has the symmetry of its network, it is said to be holohedral. When the symmetry of the crystal is lower than that of the network, we have a merihedrism.

This merihedrism is a hemihedrism, tetartohedrism, or ogdohedrism when the crystal symmetry is one half, one fourth, or one eighth that of the network.

The number of natural faces of the crystal is equal to the number of faces of the network in the case of holohedrism, of twice, four times, or eight times that number in the case of hemihedrism, tetartohedrism, and ogdohedrism.

12.2 CRYSTALLINE SYSTEMS

The 32 classes of symmetry in crystals are divided into seven systems according to the symmetry of the crystal networks.

1. Triclinic System. The network has no axis of symmetry.
2. Binary or Monoclinic System. The network has one binary axis.
3. Orthorhombic or Terbinary System. The network has three binary axes, rectangular two by two.
4. Rhombohedral or Ternary System. The network has one ternary axis.
5. Quadratic System. The network has one quaternary axis.
6. Senary or Hexagonal System. The network has one senary axis.
7. Cubic System. The network has four ternary axes, three quaternary axes, and six binary axes.

We shall review rapidly these various systems as well as their classes of symmetry, indicating for each class a particular form and the table of components of the various characteristic tensors. We shall begin with the least nonsymmetric system.

12.2.1 Triclinic System

12.2.1.1 Holohedrism C

This form has no symmetry axis but has one center of symmetry, as shown in Fig. 7. All odd-order tensors are zero; it has no pyroelectric properties.

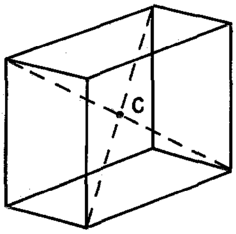


Fig. 7.

Dielectric Susceptance

x_1^1	x_1^2	x_1^3
x_2^1	x_2^2	x_2^3
x_3^1	x_3^2	x_3^3

9 Constants

Elasticity

	11	12	22	13	23	33
11	C_{11}^{11}	C_{11}^{12}	C_{11}^{22}	C_{11}^{13}	C_{11}^{23}	C_{11}^{33}
	12	C_{12}^{12}	C_{12}^{22}	C_{12}^{13}	C_{12}^{23}	C_{12}^{33}
		22	C_{22}^{22}	C_{22}^{13}	C_{22}^{23}	C_{22}^{33}
			13	C_{13}^{13}	C_{13}^{23}	C_{13}^{33}
				23	C_{23}^{23}	C_{23}^{33}
					33	C_{33}^{33}

21 Constants

Examples: albite, disthene.

12.2.1.2 Hemihedrism

This form has no symmetry. All even- or odd-order tensors are complete, the material may exhibit pyroelectricity or piezoelectricity.

Pyroelectricity

A_1
A_2
A_3

Piezoelectricity

δ_{11}^1	δ_{12}^1	δ_{13}^1
δ_{12}^1	δ_{22}^1	δ_{23}^1
δ_{13}^1	δ_{23}^1	δ_{33}^1

1

δ_{11}^2	δ_{12}^2	δ_{13}^2
δ_{12}^2	δ_{22}^2	δ_{23}^2
δ_{13}^2	δ_{23}^2	δ_{33}^2

2

δ_{11}^3	δ_{12}^3	δ_{13}^3
δ_{12}^3	δ_{22}^3	δ_{23}^3
δ_{13}^3	δ_{23}^3	δ_{33}^3

3

The tensors for dielectric susceptance and for elasticity are the same as in the preceding case. Example: axinite.

12.2.2 Monoclinic or Clinorhombohedral System

12.2.2.1 Holohedrism L^2PC

The holohedral form is a straight prism on a parallelogram as a base, as may be seen in Fig. 8.

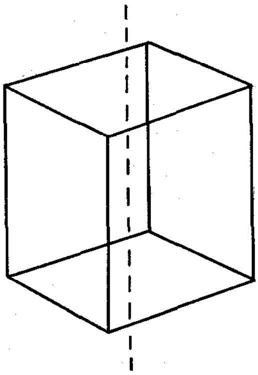


Fig. 8.

Dielectric Susceptance

5 Moduli, Double-Refracted Biaxial Crystals

x_1^1	x_1^2	
x_2^1	x_2^2	
		x_3^3

Elasticity

C_{11}^{11}		C_{11}^{22}			C_{11}^{33}
	C_{12}^{12}				
		C_{22}^{22}			C_{22}^{33}
			C_{13}^{13}		
					C_{23}^{33}

8 Distinct Moduli

Neither pyroelectric nor piezoelectric properties exist.

12.2.2.2 *Holoaxial Enantiomorphous Hemihedrism*

This form has one binary axis $L_3^{(2)}$. The structure is shown in Fig. 9.

Pyroelectricity

π_3

1 Modulus

Dielectric Susceptance

x_1^1	x_1^2	
x_2^1	x_2^2	
		x_3^3

5 Moduli

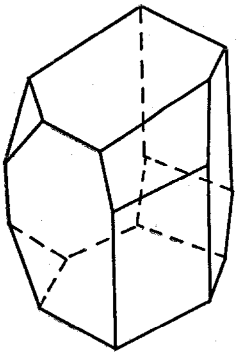


Fig. 9.

Elasticity

C_{11}^{11}	C_{11}^{12}	C_{11}^{22}			C_{11}^{33}
	C_{12}^{12}	C_{12}^{22}			
		C_{22}^{22}			C_{22}^{33}
			C_{13}^{13}	C_{13}^{23}	
					C_{33}^{33}

11 Moduli

Pyroelectricity

	1	2	3
1	δ_{11}^1	δ_{12}^1	
2	δ_{12}^1	δ_{22}^1	
3			δ_{33}^1

	1	2	3
1			δ_{13}^2
2			δ_{23}^2
3	δ_{13}^2	δ_{23}^2	

	1	2	3
1			δ_{13}^3
2			δ_{23}^3
3	δ_{13}^3	δ_{23}^3	

13 Moduli

Examples: tartaric acid, sugar, lithium sulfate.

12.2.2.3 *Antihemihedrism*

The crystalline form is shown in Fig. 10.

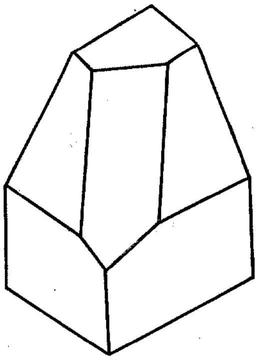


Fig. 10.

Pyroelectricity

π_2
π_3

2 Moduli

Dielectric Susceptance

	x_1^2	x_1^3
x_2^1		
x_3^1		

4 Moduli

One Plane P

	δ^1_{12}	δ^1_{13}
δ^1_{12}		
δ^1_{13}		

1

δ^2_{11}		
	δ^2_{22}	δ^2_{23}
	δ^2_{23}	δ^2_{33}

2

δ^3_{11}		
	δ^3_{22}	δ^3_{23}
	δ^3_{23}	δ^3_{33}

3

14 Moduli

Elasticity

11	12	22	13	23	33
C^{11}_{11}		C^{22}_{11}		C^{23}_{11}	C^{33}_{11}
	C^{12}_{12}		C^{13}_{12}		
		C^{22}_{22}		C^{23}_{22}	C^{33}_{22}
			C^{13}_{13}		
				C^{23}_{23}	C^{33}_{23}
					C^{33}_{33}

13 Moduli

Examples: organic compounds, copper sulfate.

12.2.3 *Orthorhombohedral System*

12.2.3.1 *Holohedrism*

This form has three perpendicular binary axes and one center as may be seen in Fig. 11. The original solid is an orthorhombohedral prism. All pyroelectric and piezoelectric moduli are zero.

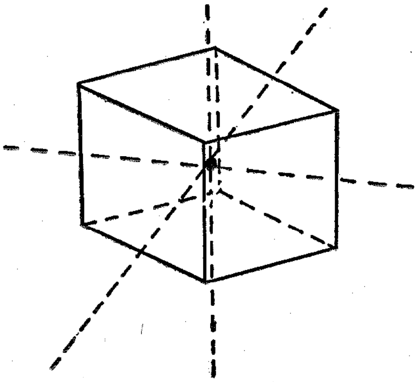


Fig. 11.

Dielectric Susceptance

x^1_1		
	x^2_2	
		x^3_3

3 Moduli

No Double Refraction

Elasticity

C^{11}_{11}		C^{22}_{11}			C^{33}_{11}
	C^{12}_{12}				
		C^{22}_{22}			C^{33}_{22}
			C^{12}_{13}		
					C^{33}_{23}

8 Moduli

Examples: sulphur, tridymite, aragonite.

12.2.3.2 *Holoaxial Hemihedrism* $L^{(2)}_3 L^{(2)}_1$

This form has no center and no pyroelectric properties. It is shown in Fig. 12.

Dielectric Susceptance

x^1_1		
	x^2_2	
		x^3_3

3 Moduli

	1	2	3
1			
2			δ^1_{23}
3		δ^1_{23}	

1

		δ^2_{13}
δ^2_{13}		

2

	δ^3_{12}	
δ^3_{12}		

3

3 Moduli of Type δ^1_{23}

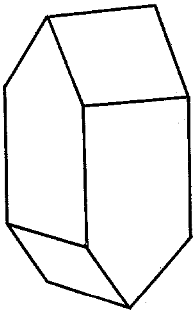


Fig. 12.

The electric field oriented in the direction of one of the crystallographic axes causes an angular strain in the perpendicular plane. For elasticity, it has the same moduli as holohedrism.

Examples: Rochelle salt, epsomite (magnesium sulphate).

12.2.3.3 Antihemihedrism $L_3^{(2)}P_1$

The crystalline form is shown in Fig. 13.

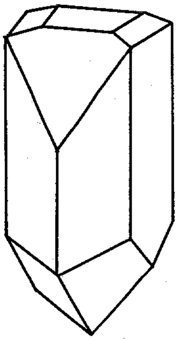


Fig. 13.

Pyroelectricity

π_3

1 modulus

Susceptance

	x_1^2	
x_2^1		

Piezoelectricity

		δ_{13}^1
δ_{13}^1		

1

		δ_{23}^2
	δ_{23}^2	

2

δ_{11}^3		
	δ_{22}^3	
		δ_{33}^3

3

5 Moduli

Examples: calamine, topaz, picric acid.

12.2.4 Rhombohedral System

12.2.4.1 Holohedrism $CL_3^3P_1$

The network is a rhombohedron as is evident from Fig. 14.

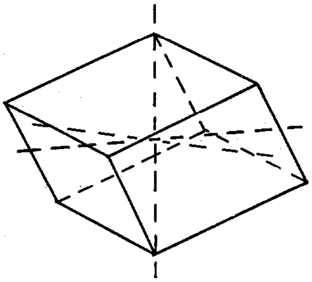


Fig. 14.

Dielectric Susceptance

	x_1^2	
$-x_1^2$		

Elasticity

C^{11}_{11}		C^{22}_{11}		$-C^{23}_{22}$	C^{33}_{11}
	$\frac{C^{11}_{11}-C^{22}_{11}}{2}$		$-C^{23}_{22}$		
		C^{11}_{11}		C^{23}_{22}	C^{33}_{11}
			C^{13}_{13}		
				C^{13}_{13}	
					C^{33}_{33}

7 Distinct Moduli

Examples: calcite, sodium nitrate, corindon, etc.

12.2.4.2 Holoaxial Hemihedrism $L^{(3)}_3 L^{(2)}_1$

This form displays no pyroelectric properties. It is shown in Figs. 15 and 16. For elasticity, it has the same moduli as holohedrism.

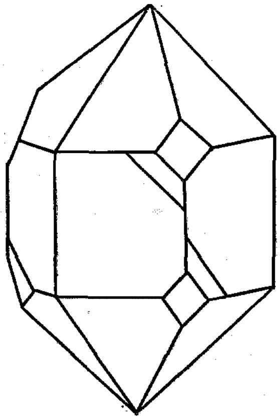


Fig. 15.

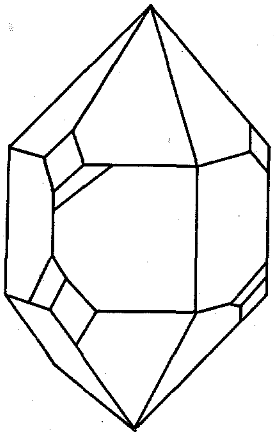


Fig. 16.

Dielectric Susceptance

x^1_1		
	x^1_1	
		x^3_3

3 Moduli

Piezoelectricity

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & \delta^1_{11} & & \\ 2 & & -\delta^1_{11} & \delta^1_{23} \\ 3 & & \delta^1_{23} & \end{array}$$

1

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & & -\delta^1_{11} & -\delta^1_{23} \\ 2 & -\delta^1_{11} & & \\ 3 & -\delta^1_{23} & & \end{array}$$

2

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & & & \\ 2 & & & \\ 3 & & & \end{array}$$

3

2 Distinct Moduli

Example: quartz.

12.2.4.3 Antihemidrism $L_3^3P_1$

Pyroelectricity

π_3

Dielectric Susceptance

x_1^1		
	x_1^1	
		x_3^3

Piezoelectricity

	δ_{22}^2	δ_{13}^1
δ_{22}^2		
δ_{13}^1		

1

$-\delta_{22}^2$		
	δ_{22}^2	δ_{13}^1
	δ_{13}^1	

2

δ_{11}^3		
	δ_{11}^3	
		δ_{33}^3

3

For piezoelectricity, the electric field oriented along axis ox^3 causes a strain of the type δ_{33} and two equal strains in the perpendicular directions $\delta_{11}=\delta_{22}$, i.e., a tourmaline disk cut perpendicularly to axis ox^3 is compressed in the electric field at the same time it increases in width, remaining circular.

This form has the same moduli of elasticity as quartz. The structure is shown in Figs. 17, 18, and 19.

Examples: tourmaline, carborundum, potassium bromate.

12.2.4.4 Tetartohedrism L_3^3

The form of this crystal is shown in Fig. 20.

Pyroelectricity

π_3

Dielectric Susceptance

x_1^1	x_1^2	
$-x_1^2$	x_1^1	
		x_3^3

Piezoelectricity

δ_{11}^1	δ_{22}^2	δ_{13}^1
δ_{22}^2	$-\delta_{11}^1$	δ_{23}^1
δ_{13}^1	δ_{23}^1	

1

$-\delta_{22}^2$	$-\delta_{11}^1$	$-\delta_{23}^1$
$-\delta_{11}^1$	δ_{22}^2	δ_{13}^1
$-\delta_{23}^1$	δ_{13}^1	

2

δ_{11}^3		
	δ_{11}^3	
		δ_{33}^3

3

6 Moduli

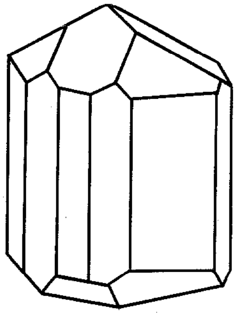


Fig. 17.

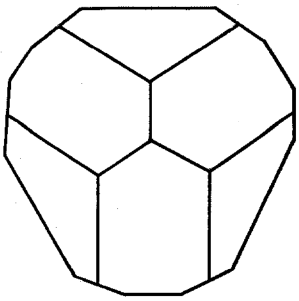


Fig. 18.

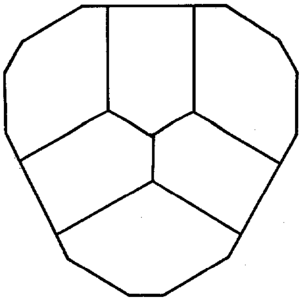


Fig. 19.

Elasticity

C^{11}_{11}		C^{22}_{11}	C^{13}_{11}	$-C^{23}_{22}$	C^{33}_{11}
	$\frac{C^{11}_{11}-C^{22}_{11}}{2}$		$-C^{23}_{22}$	$-C^{13}_{11}$	
		C^{11}_{11}	$-C^{13}_{11}$	C^{23}_{22}	C^{33}_{11}
			C^{13}_{13}		
				C^{13}_{13}	
					C^{33}_{33}

7 Moduli

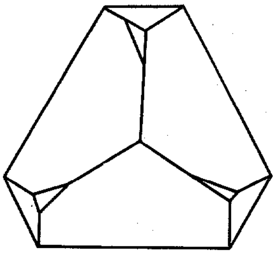


Fig. 20.

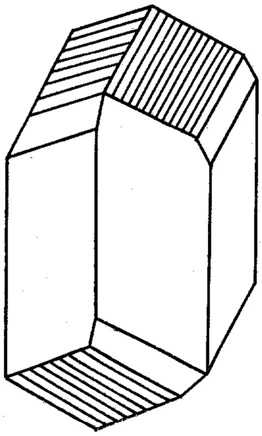


Fig. 21.

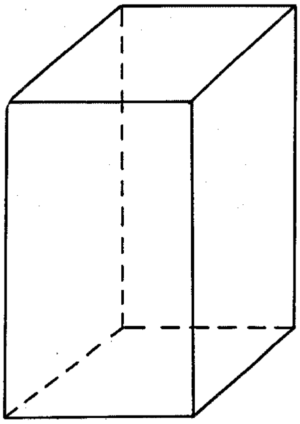


Fig. 22.

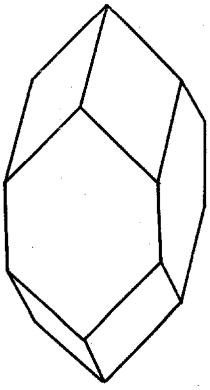


Fig. 23.

Example: sodium periodate.

12.2.4.5 Centered Hemihedrism L^3_3C

This form has no pyroelectric and no piezoelectric properties. Susceptance and elasticity are as for tetartohedrism. Fig. 21 shows the form of the crystal.

Example: dolomie.

12.2.5 Quadratic System

12.2.5.1 Holohedrism $L^{(4)}_3L^{(2)}_1P_1C$

The network consists of quadratic prisms. No pyroelectric and no piezoelectric properties exist. The crystalline form is shown in Fig. 22 and in Fig. 23.

Susceptance

x^1_1		
	x^1_1	
		x^3_3

Elasticity

	11	12	22	13	23	33
11	C^{11}_{11}		C^{22}_{11}			C^{33}_{11}
12		C^{12}_{12}				
22			C^{11}_{11}			C^{33}_{11}
13				C^{13}_{13}		
23						
33						C^{33}_{33}

7 Moduli

In the case of the quadratic system, the reduction of the moduli are the same as in the case of the hexagonal system, the 90-degree rotation gives the same reductions as a rotation of 120 degrees, and the additional 180-degree rotation is the same in both cases. The only difference is in the modulus of elasticity C^{12}_{12}.

Rotation about axis 3, of any angle, gives the relation

$$C^2\delta^2\left|C^{12}_{12}-\frac{C^{11}_{11}-C^{22}_{11}}{2}\right|=0.$$

But, here, for 90 or 180 degrees, $C_3=0$; the relation always holds and the condition

$$C^{12}_{12}=\frac{(C^{11}_{11}-C^{12}_{11})}{2}$$

is not necessary.

Example: calomel.

12.2.5.2 *Holoaxial Hemihedrism* $L^{(4)}_3L^2_1$

This form has no pyroelectricity. For dielectric susceptance, it has the same moduli as holohedrism. The structure of the crystal is shown in Fig. 24.

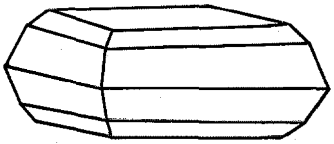


Fig. 24.

Piezoelectricity

		δ^1_{23}
	δ^1_{23}	

1

		$-\delta^1_{23}$
$-\delta^1_{23}$		

2

3

Elasticity

C^{11}_{11}		C^{22}_{11}			C^{33}_{11}
	C^{12}_{12}				
		C^{11}_{11}			C^{33}_{11}
			C^{13}_{13}		
				C^{13}_{13}	
					C^{33}_{33}

8 Moduli

Example: nickel sulphate.

12.2.5.3 *Antihemihedrism* $L^{(4)}_3P_1$

Pyroelectricity

π_3

Dielectric Susceptance

	x^2_1	
$-x^2_1$		

Piezoelectricity

		δ^1_{13}
δ^1_{13}		

		δ^1_{13}
	δ^1_{13}	

δ^3_{11}		
	δ^3_{11}	
		δ^3_{33}

This form has the same moduli of elasticity as the preceding class. Fig. 25 shows the crystalline form.

Example: chalcopyrite.

12.2.5.4 Parahemihedrism $\Lambda^4 C$

This form has no pyroelectricity and no piezoelectricity. It has the same dielectric susceptances as holohedrism and the same elasticity moduli as holoaxial hemihedrism. The form may be seen in Fig. 26.

Example: schellite.

12.2.5.5 Tetartohedrism With Quaternary Axis $L_3^{(4)}$

The shape of this crystal is shown in Fig. 27.

Pyroelectricity

π_3

Dielectric Susceptance

x_1^1		
	x_1^1	
		x_3^3

Piezoelectricity

		δ_{13}^1
		δ_{23}^1
δ_{13}^1	δ_{23}^1	

1

		δ_{23}^1
		δ_{13}^1
$-\delta_{23}^1$	δ_{13}^1	

2

δ_{11}^3		
	δ_{11}^3	
		δ_{33}^3

3

For elasticity, this form has the same moduli as hemihedrism.

Example: lead molybdate.

12.2.5.6 Sphenohedral Antihemihedrism $L_3^{(2)} L_1^{(2)} \bar{\omega}_2$

This form has no pyroelectric properties. It has the same elasticity moduli as hemihedrism. The form is shown in Fig. 28.

Dielectric Susceptance

x_1^1		
	x_2^2	
		x_3^3

Piezoelectricity

		δ_{23}^1
	δ_{23}^1	

1

		δ_{23}^1
δ_{23}^1		

2

	δ_{12}^3	
δ_{12}^3		

3

Example: mercuric cyanide, tetraethyl ammonium iodide in sphenoidal form.

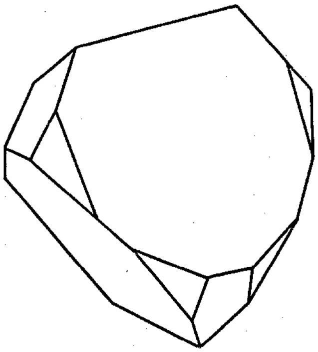


Fig. 25.

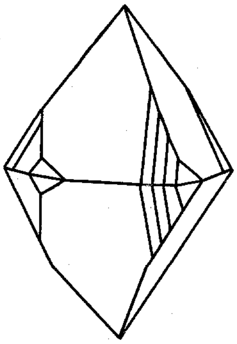


Fig. 26.

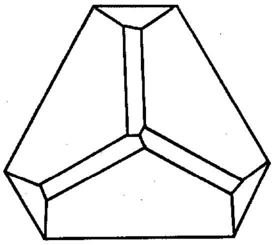


Fig. 27.

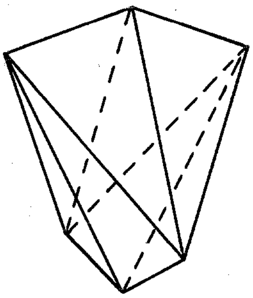


Fig. 28.

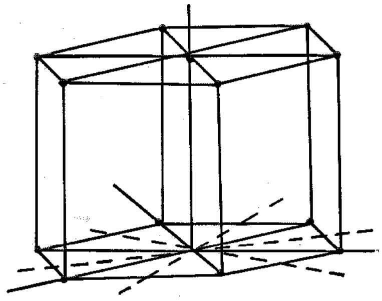


Fig. 30.

12.2.5.7 *Sphenohedral Tetartohedrism* $L_3^2\bar{\omega}_2$

No crystals exist in this class. It would have the shape shown in Fig. 29.

12.2.6 *Hexagonal System*

12.2.6.1 *Holohedrism* $L_3^6L_1P_1C$

In primitive form, it is a multiple mesh in the shape of a regular hexagonal prism as shown in Fig. 30. It has no pyroelectric and no piezoelectric properties.

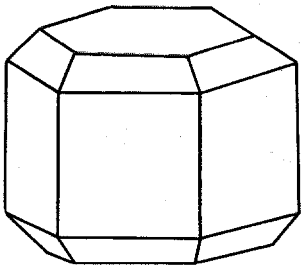


Fig. 31.

Dielectric Susceptance

x_1^1		
	x_1^1	
		x_3^3

Elasticity

	11	12	22	13	23	33
11	C_{11}^{11}		C_{11}^{22}			C_{11}^{33}
12		$\frac{C_{11}^{11}-C_{11}^{22}}{2}$				
22			C_{11}^{11}			C_{11}^{33}
13				C_{13}^{13}		
23					C_{13}^{13}	
33						C_{33}^{33}

5 Distinct Moduli

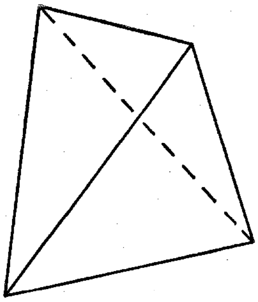


Fig. 29.

Example: magnesium. (See Fig. 31.)

12.2.6.2. *Holoaxial Hemihedrism* $L_3^{(6)}L_1^{(2)}$

This form has no pyroelectric properties. It has the same dielectric susceptances and the same elasticity moduli as holohedrism.

Piezoelectricity

		δ^1_{23}
	δ^1_{23}	

1

		$-\delta^1_{23}$
$-\delta^1_{23}$		

2

3

Example: β quartz above 575 degrees centigrade.

12.2.6.3 Parahemihedrism With Ternary Axis $L_3^{(6)}P_3C$

This form has no pyroelectric and no piezoelectric properties. It has the same dielectric susceptances and same elasticity moduli as holohedrism. The form is shown in Fig. 32.

Example: apatite.

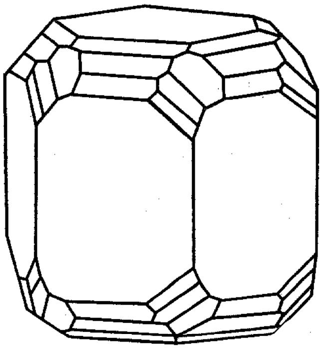


Fig. 32.

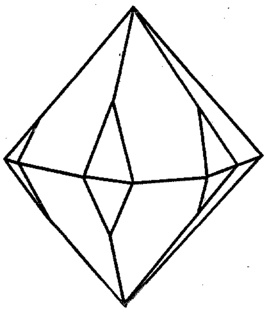


Fig. 33.

12.2.6.4 Antihemihedrism $\Lambda^{(6)}P_1$

Its dielectric susceptance and elasticity have the same moduli as holohedrism. The shape is shown in Fig. 33.

Pyroelectricity

π_3

Piezoelectricity

	1	2	3
1			δ^1_{13}
2			
3	δ^1_{13}		

1

	1	2	3
1			
2			δ^1_{13}
3		δ^1_{13}	

2

	1	2	3
1	δ^3_{11}		
2		δ^3_{11}	
3			δ^3_{33}

3

Examples: zincite, wurzite, iodyrite.

12.2.6.5 Tetartohedrism $L_3^{(6)}$

For elasticity, this form has the same moduli as holohedrism.

Pyroelectricity

π_3

Dielectric Susceptance

x^1_1	x^2_1	
$-x^2_1$	x^1_1	
		x^3_3

Piezoelectricity

		δ_{13}^{1}
		δ_{23}^{1}
δ_{13}^{1}	δ_{23}^{1}	

		$-\delta_{23}^{1}$
		δ_{13}^{1}
$-\delta_{23}^{1}$	δ_{13}^{1}	

δ_{11}^{3}		
	δ_{11}^{3}	
		δ_{33}^{3}

Examples: double lithium, potassium sulphate. (See Fig. 34.)

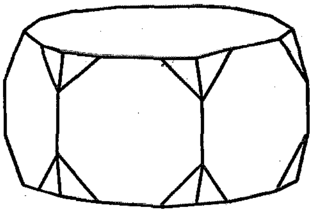


Fig. 34.

12.2.6.6 Trigonohedral Antihemihedrism $L_{(3)}^{3}L_{1}^{(2)}P_{3}$

This form has no pyroelectric properties. The shape is given in Fig. 35.

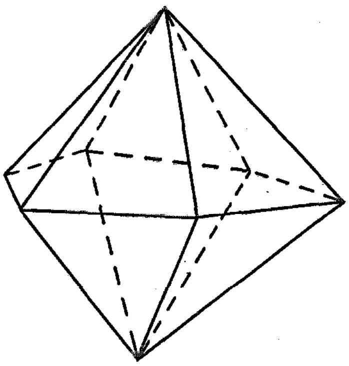


Fig. 35.

Dielectric Susceptance

x_1^1		
	x_1^1	
		x_3^3

Piezoelectricity

	11	12	13
11	δ_{11}^{1}		
12		$-\delta_{11}^{1}$	
13			

1

	$-\delta_{11}^{1}$	
$-\delta_{11}^{1}$		

2

3

Elasticity

C_{11}^{11}		C_{11}^{22}		$-C_{22}^{23}$	C_{11}^{33}
	$\frac{C_{11}^{11}-C_{11}^{22}}{2}$		$-C_{22}^{13}$		
		C_{11}^{11}		C_{22}^{23}	C_{11}^{23}
			C_{13}^{13}		
				C_{13}^{13}	
					C_{33}^{33}

Example: benitoite.

12.2.6.7 *Trigonohedral Antitetartohedrism* Λ^3P_3

Pyroelectricity

π_3

Dielectric Susceptance

x_1^1	x_1^2	
$-x_1^2$	x_1^1	
		x_3^3

Piezoelectricity

δ_{11}^1	δ_{22}^2	
δ_{22}^2	$-\delta_{11}^1$	

1

$-\delta_{22}^2$	$-\delta_{11}^1$	
$-\delta_{11}^1$	δ_{22}^2	

2

3

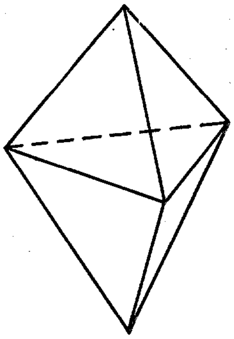


Fig. 36.

Example: This form has very rare symmetry and exists, perhaps, as silver phosphate. (See Fig. 36.)

12.2.7 *Cubic System*

12.2.7.1 *Holohedrism* $L_3^{(4)}L'^3_3L_1P_1C$

This form has no pyroelectric and no piezoelectric properties. The elasticity and dielectric susceptance moduli are the same as in an isotropic body. The crystalline form is shown in Fig. 37.

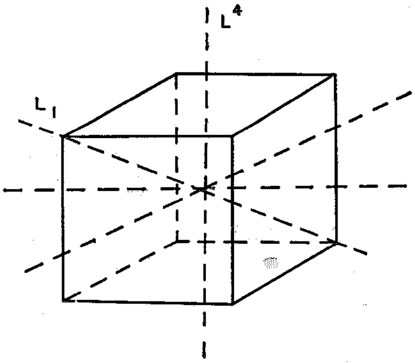


Fig. 37.

Dielectric Susceptance

x_1^1		
	x_1^1	
		x_1^1

Elasticity

C_{11}^{11}		C_{11}^{22}			C_{11}^{23}
	$\frac{C_{11}^{11}-C_{11}^{22}}{2}$				
		C_{11}^{11}			C_{11}^{23}
			C_{13}^{13}		
				C_{13}^{13}	
					C_{33}^{33}

Elasticity

11	12	22	13	23	33
C_{11}^{11}		C_{11}^{22}			C_{11}^{22}
	$\frac{C_{11}^{11}-C_{11}^{22}}{2}$				
		C_{11}^{11}			C_{11}^{22}
			$\frac{C_{11}^{11}-C_{11}^{22}}{2}$		
				$\frac{C_{11}^{11}-C_{11}^{22}}{2}$	
					C_{11}^{11}

Example: diamond, rock salt.

12.2.7.2 *Holoaxial Hemihedrism* $L_3^{(4)}L'^3L_1$

This form, shown in Fig. 38, has only one dielectric susceptance. It has no pyroelectric or piezoelectric properties.

Piezoelectricity

		δ_{23}^{1}
	δ_{23}^{1}	

1

		δ_{23}^{1}
	δ_{23}^{1}	

2

	δ_{23}^{1}	
δ_{23}^{1}		

3

Elasticity

C_{11}^{11}		C_{11}^{22}			C_{11}^{22}
	C_{12}^{12}				
		C_{11}^{11}			C_{11}^{22}
			C_{12}^{12}		
				C_{12}^{12}	
					C_{11}^{11}

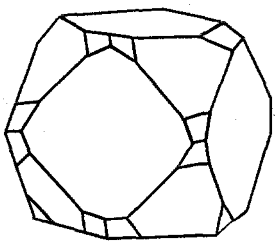


Fig. 38.

Example: sylvine.

12.2.7.3 *Parahemihedrism* $L_3^2L_3'P_3C$

This form has no piezoelectric properties. It has the same elastic constants as holoaxial hemihedrism. The structure is given in Fig. 39.

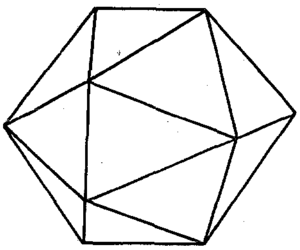


Fig. 39.

Examples: pyrite, cobaltine, alums.

12.2.7.4 *Antihemihedrism* $L_3^{(2)}L_3'\,\bar{\omega}_2P_1$

This form, the structure of which is given in Fig. 40, has no pyroelectric or piezoelectric properties. For elasticity, it has the same moduli as holo-axial hemihedrism.

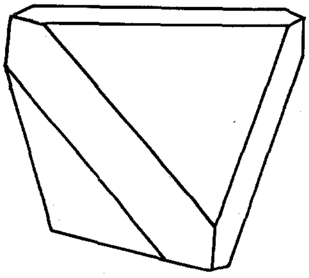


Fig. 40.

Example: blende.

12.2.7.5 Tetartohedrism $L_3^2 L_3' L_1$

This form has no pyroelectric or piezoelectric properties. For elasticity, it has the same moduli as holoaxial hemihedrism. Its structure may be seen in Fig. 41.

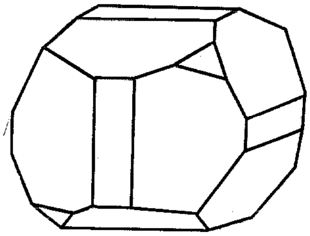


Fig. 41.

Example: sodium chlorate.

13. Conclusions

From these examples, it is evident that the particular symmetry of various crystalline systems indicate certain phenomena that are likely to take place in the crystallized material.

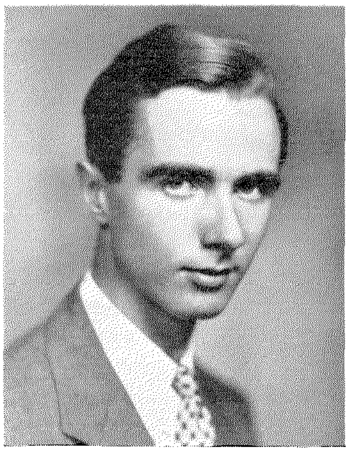
Tensor and matrix calculus permit the moduli linking cause and effect for all physical phenomena to be indicated for any crystalline system.

If a physical effect represented by a tensor of the order p is caused by the action represented by a tensor of the order q, the properties of the matrix act through a particular modulus that is a tensor of the order $p+q$.

The analytical transformations implied by this tensorial characteristic enable the relation between these moduli, as determined by the particular symmetry of the system, to be written. For instance, all tensorial moduli of odd order are zero if the crystalline substance has a center of symmetry.

It should be noted that the conclusions derived from this analysis indicate only the possibility of the existence of the various moduli. Such existence may not be stated with certainty. Only knowledge of the intimate structure of a crystal will allow its physical properties to be predicted.

Contributors to This Issue


Bernard T. Ellis, Jr.

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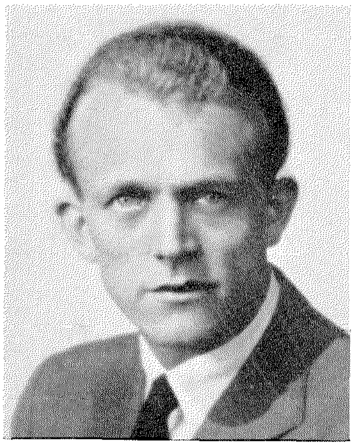
* * *

Fernand Mortiaux was born in Antwerp on March 3, 1906. He finished his studies in electromechanical engineering in 1929 at the University of Brussels.

Mr. Mortiaux entered the Institute National Belge de Radiodiffusion in 1930 and has been active in all phases of broadcasting. He was named chief engineer in 1935 and in 1946 became Administrateur-Directeur Général of the technical department.

Mr. Mortiaux is a reserve officer; he is a member of various technical committees, including the Committee on Television.


Fernand Mortiaux

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Albert Shadowitz was born on May 5, 1915 at Brooklyn, N. Y. He received the B.E.E. degree from Polytechnic Institute of Brooklyn, in 1935. After attending Harvard University during 1936 and 1937, he studied at the University of California from 1937 to 1940.

He served as an engineering employee at Aberdeen Proving Grounds from 1941 to 1943. In 1943, he joined Federal Telecommunication Laboratories as an engineer.

Mr. Shadowitz is an Associate Member of the Institute of Radio Engineers.

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Martin Silver graduated from New York University in 1941 with a Bachelor of Engineering degree. He then came to Federal Telecommunication Laboratories where he worked on communication-type jamming equipment. At the termination of the war, he was assigned to the development of frequency-modulation broadcast transmitters, monitors, links, and associated equipment.

Mr. Silver is an Associate Member of the Institute of Radio Engineers.

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Marcel Tournier was born in Paris on March 23, 1888. In 1909, he received a degree in engineering from Ecole de Physique et de Chimie Industrielles in Paris, to which he returned in 1911, after obtaining the grade of Licencié ès Sciences, as an assistant in general chemistry. He became chief of general physics laboratory studies in 1914.

He collaborated with Professor P. Langevin in Paris and at the Laboratoire du Centre d'Etudes de la Marine at Toulon. They developed a complete equipment for submarine detection incorporating piezoelectric apparatus. In 1928, he joined Les Laboratoires, Le Matériel Téléphonique as head of the piezoelectric quartz department.


Albert Shadowitz

In 1937, Mr. Tournier was named electrotechnical professor at the Ecole de Physique et de Chimie, and in 1941 conference lecturer of the radio section at the Ecole Supérieure d'Electricité. He is also serving as head of the research division of the Office National d'Etudes et de Recherches Aéronautiques.

He was awarded the Medal of the Société d'Encouragement pour l'Industrie Nationale for his early developments of the electronic organ. He is a Chevalier de la Légion d'Honneur and a member of the Société Française de Physique.

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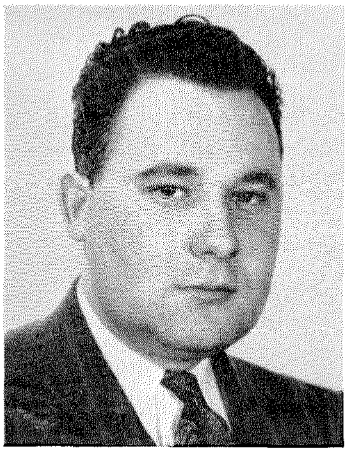
For the biography and photograph of E. P. G. Wright, see Volume 24, Number 1, page 128.


Martin Silver

INTERNATIONAL TELEPHONE AND TELEGRAPH CORPORATION

Associate Manufacturing and Sales Companies

United States of America

International Standard Electric Corporation, New York, New York

Federal Telephone and Radio Corporation, Newark and Clifton, New Jersey

Great Britain and Dominions

Standard Telephones and Cables, Limited, London, England

Branch Offices: Birmingham, Leeds, Manchester, England; Glasgow, Scotland; Dublin, Ireland; Cairo, Egypt; Calcutta, India; Johannesburg, South Africa

Creed and Company, Limited, Croydon, England

International Marine Radio Company Limited, Liverpool, England

Kolster-Brandes Limited, Sidcup, England

Standard Telephones and Cables Pty. Limited, Sydney, Australia

Branch Offices: Melbourne, Australia; Wellington, New Zealand

Silovac Electrical Products Pty. Limited, Sydney, Australia

New Zealand Electric Totalisators Limited, Wellington, New Zealand

Federal Electric Manufacturing Company, Ltd., Montreal, Canada

South America

Compañía Standard Electric Argentina, Sociedad Anónima, Industrial y Comercial, Buenos Aires, Argentina

Standard Electrica, S.A., Rio de Janeiro, Brazil

Compañía Standard Electric, S.A.C., Santiago, Chile

Europe and Far East

Vereinigte Telefon- und Telegraphenfabriks Aktien-Gesellschaft Czeija, Nissl and Company, Vienna, Austria

Bell Telephone Manufacturing Company, Antwerp, Belgium

China Electric Company, Limited, Shanghai, China

Standard Electric Doms A Spolecnost, Prague, Czechoslovakia

Standard Electric Aktieselskab, Copenhagen, Denmark

Compagnie Générale de Constructions Téléphoniques, Paris, France

Le Matériel Téléphonique, Paris, France

Les Téléimprimeurs, Paris, France

Lignes Télégraphiques et Téléphoniques, Paris, France

Ferdinand Schuchhardt Berliner Fernsprech- und Telegraphenwerk Aktiengesellschaft, Berlin, Germany

Lorenz, C., A.G. and Subsidiaries, Berlin, Germany

Mix & Genest Aktiengesellschaft and Subsidiaries, Berlin, Germany

Süddeutsche Apparatefabrik Gesellschaft M.B.H., Nuremberg, Germany

Telephonfabrik Berliner A.G. and Subsidiaries, Berlin, Germany

Nederlandsche Standard Electric Maatschappij N.V., Hague, Holland

Dial Telefonkereskedelmi Részvény Társaság, Budapest, Hungary

Standard Villamossági Részvény Társaság, Budapest, Hungary

Telefongyár R.T., Budapest, Hungary

Fabbrica Apparecchiature per Comunicazioni Elettriche, Milan, Italy

Standard Elettrica Italiana, Milan, Italy

Societa Italiana Reti Telefoniche Interurbane, Milan, Italy

Nippon Electric Company, Limited, Tokyo, Japan

Sumitomo Electric Industries, Limited, Osaka, Japan

Standard Telefon- og Kabelfabrik A/S, Oslo, Norway

Standard Electrica, Lisbon, Portugal

Standard Fabrica de Telefoane si Radio S.A., Bucharest, Rumania

Compañía Radio Aerea Maritima Española, Madrid, Spain

Standard Eléctrica, S.A., Madrid, Spain

Aktiebolaget Standard Radiofabrik, Stockholm, Sweden

Standard Telephone et Radio S.A., Zurich, Switzerland

Telephone Operating Systems

Compañía Telefónica Argentina, Buenos Aires, Argentina
Compañía Telegráfico-Telefónica Comercial, Buenos Aires, Argentina
Compañía Telegráfico-Telefónica del Plata, Buenos Aires, Argentina
Companhia Telefonica Paranaense S.A., Curitiba, Brazil
Companhia Telefonica Rio Grandense, Porto Alegre, Brazil
Compañía de Teléfonos de Chile, Santiago, Chile
Compañia Telefónica de Magallanes S.A., Punta Arenas, Chile
Cuban Telephone Company, Havana, Cuba
Cuban American Telephone and Telegraph Company, Havana, Cuba
Mexican Telephone and Telegraph Company, Mexico City, Mexico
Compañía Peruana de Teléfonos Limitada, Lima, Peru
Porto Rico Telephone Company, San Juan, Puerto Rico
Shanghai Telephone Company, Federal, Inc., U.S.A., Shanghai, China

Radiotelephone and Radiotelegraph Operating Companies

Compañía Internacional de Radio, Buenos Aires, Argentina

Compañía Internacional de Radio Boliviana, La Paz, Bolivia

Companhia Radio Internacional do Brasil, Rio de Janeiro, Brazil

Compañía Internacional de Radio, S.A., Santiago, Chile

Radio Corporation of Cuba, Havana, Cuba

Radio Corporation of Porto Rico, San Juan, Puerto Rico[1]

[1] Radiotelephone and Radio Broadcasting services.

Cable and Radiotelegraph Operating Companies

(Controlled by American Cable & Radio Corporation)

The Commercial Cable Company, New York, New York[2]

Mackay Radio and Telegraph Company, New York, New York[3]

All America Cables and Radio, Inc., New York, New York[4]

The Cuban All America Cables, Incorporated, Havana, Cuba[2]

Sociedad Anónima Radio Argentina, Buenos Aires, Argentina[5]

[2] Cable service. [3] International and Marine Radiotelegraph services.
[4] Cable and Radiotelegraph services. [5] Radiotelegraph service.

Laboratories

International Telecommunication Laboratories, Inc., New York, New York
Federal Telecommunication Laboratories, Inc., Nutley, New Jersey
Standard Telecommunication Laboratories Ltd., London, England
Laboratoire Central de Télécommunications, Paris, France