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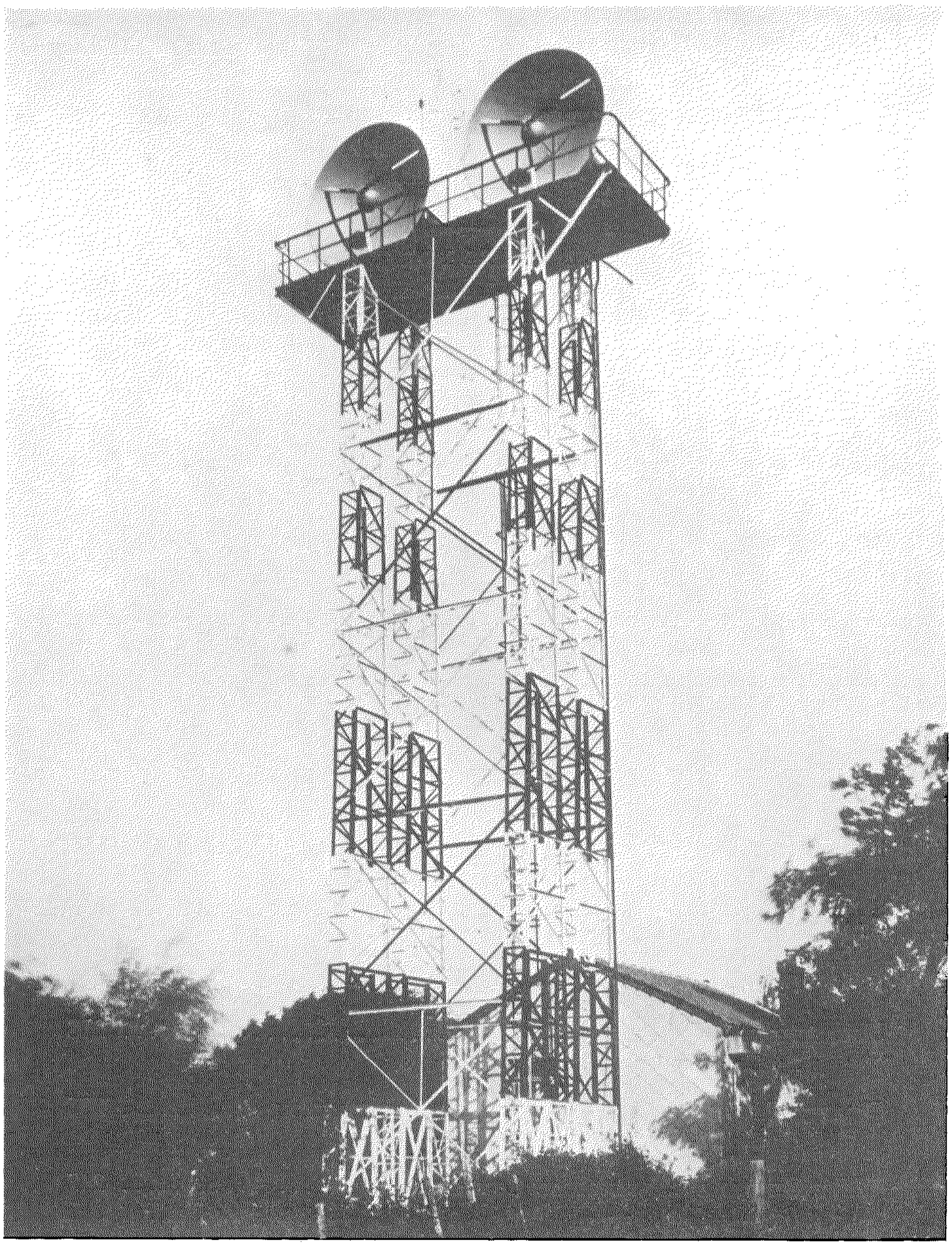
CONTENTS



MICRO-RAY TOWERS, ST. INGLEVERT, FRANCE, CONSTRUCTED FOR COMMERCIAL TRANSMISSIONS ACROSS THE ENGLISH CHANNEL. THE WORKING FREQUENCY WAS IN THE NEIGHBORHOOD OF 1700 MEGACYCLES AND THE BEAMS WERE VERY SHARPLY CONCENTRATED. THIS IMPORTANT DEVELOPMENT WAS DEMONSTRATED PUBLICLY IN 1931 (SEE ELECTRICAL COMMUNICATION, JULY, 1931).

Automatic Ticketing of Telephone Calls

By W. HATTON
Manager, Telephone Division
Federal Telephone & Radio Corporation, Newark, New Jersey
(Formerly Technical Director, Bell Telephone Manufacturing Company, Antwerp, Belgium)

IN view of the recent introduction of automatic ticketing facilities between Culver City and Los Angeles, California, by the Bell System,[1] a licensee under the International Standard Electric Corporation's U.S.A. patents on automatic ticketing systems, it is interesting to recall the public introduction of automatic ticketing of toll telephone calls in Bruges, Belgium, on December 7, 1936. Long a dream of communication engineers, and the first of its kind anywhere, the successful consummation for the Belgian Administration by the Bell Telephone Manufacturing Company, an I. T. & T. associate, of this method of handling toll calls on a fully automatic basis marked an important milestone in the evolution of automatic telephony.[2]

The significance of applications of automatic toll ticketing will be evident from the following quotation from an article in this journal.[3]

"Automatic Toll Ticketing demonstrated the possibility of offering to a telephone subscriber the advantages of full automatic dial service, toll as well as local, previously fully satisfactory only on local service. It makes available an individual and complete printed record of every toll call without participation by any human agency other than the calling and called subscribers.

"Without toll ticketing, it is necessary in the case of toll or long distance calls to resort to the preparation of toll tickets by an operator, a cumbersome and time consuming procedure, or the introduction of time and zone metering, furnishing merely an integrated record of unit charges and often involving burdensome equipment alterations. Automatic toll ticketing, on the other hand, can usually be introduced with minimum modification of existing systems."

The automatic printing register used in Belgium and applications of automatic ticketing of long distance telephone connections in Rotary automatic systems were described in April 1937, and January 1940, respectively.[4,5] Automatic ticketing applications to step-by-step systems[6] were described in January 1939.

The completely satisfactory service rendered by the automatic toll ticketing system in Bruges prompted the Belgian Administration to extend its application.[7] Installations subsequently installed included Brussels, the capital city, and a number of other towns in Belgium. Furthermore, plans were formulated by the Administration for full national dialing and international dialing between Brussels and The Hague, Holland, and Paris, France, but these were interrupted by the outbreak of war.

References

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2. "Automatic Ticketing of Telephone Toll Calls," by Leslie B. Haigh, *El. Com.*, Vol. 15, No. 4, 1937.
3. "Milestones of Communication Progress," by H. T. Kohlhaas, *El. Com.*, Vol. 20, No. 3, 1942.
4. "Automatic Printing Register for Telephone Call Recording," by L. Devaux, *El. Com.*, Vol. 15, No. 4, 1937.
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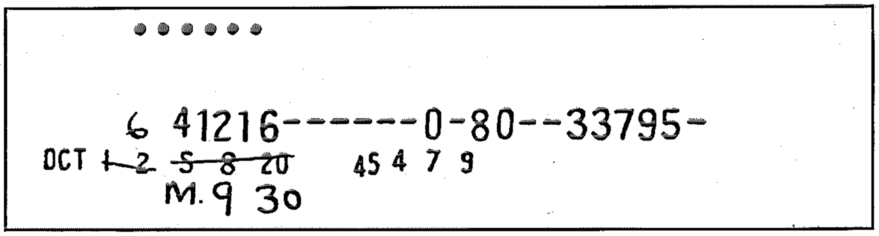
Record of the First Commercial Call Ticketed Automatically on October 6, 1936



Mr. J. A. Marchal, Régie Engineer in Charge of Ghent and Bruges, parted with the original ticket which he regarded "as a very precious possession likely to be of historical value."

How the ticket came to be produced two months before cut-over is a story in itself, and illustrates the immense value of the documentary evidence afforded by such a record. On the day in question, testing of the equipment was in progress in preparation for a demonstration arranged for October 8. Amongst the test tickets in the container, a ticket which carried two regular subscribers' numbers was noticed. The calling number was found upon investigation to belong to a railway depot in Bruges. This number was immediately called up by the Administration and the official who replied was informed that a call had been put through automatically to Blankenberghe railway station. This was, of course, contrary to regulations because the new service was not yet open to the public. The call was at first emphatically denied, as might have been expected, since only the person in charge of the depot is allowed to make calls outside the Bruges local area. Eventually, when the overwhelming evidence of the date, time, numbers involved, and duration of conversation was appreciated, the call was reluctantly admitted by an employee—and duly charged.

(The date and time mechanisms were not set when this call was put through; hence, the longhand corrections on the ticket.)

Standard Telephones and Cables, Limited, London—60th Anniversary

By C. W. EVE
Commercial Director, S. T. & C., Ltd., London

SIXTY years ago, on the second of May, 1883, was laid the foundation of what is today the premier telecommunication engineering and manufacturing organisation in the British Empire, when Mr. J. E. Kingsbury [1] with a staff of two opened an office in Moorgate, London, as representative and agent of the Western Electric Company. But for the exigencies of the war, this anniversary would have been marked by suitable ceremonies; present conditions, however, permit only this brief record of the event.

Before the General Post Office, in 1911, absorbed the old National Telephone Company (of which Sir Frank Gill, now Chairman of the Board of Directors of S. T. & C., was then Chief Engineer), Western Electric's British manufactures had been very largely confined to the cable field. About this time, H. M. Pease began to make his influence felt. While serving as Chief Engineer, then as Sales Manager, and before becoming Managing Director in 1918, he did much towards establishing an enviable *esprit de corps* and was mainly instrumental in promoting, both in Government and telecommunication circles, the spirit of confidence that made possible the later rapprochement between the Government and industry on a common policy of national development.

During World War I, as in World War II, the Company devoted its entire energies to the provision of equipment for the Allied Nations. While it would be gratifying to indicate the extent of its present efforts, discretion must be exercised; but it can be truthfully stated that those who, in 1918, rightly regarded the Company's contribution to Victory as substantial, now realize that it was incomparable with the current effort.

With H. M. Pease in control, the years 1918–1928 witnessed expansion in Company plant capacity and the taking up of many new lines of business. To its primary products of telephone equipment, and telephone and power cables, were added radio and transmission systems. In addition to the original North Woolwich plant, the new Southgate factory was constructed and further premises were rented at Hendon (the latter were subsequently relinquished when the Southgate and Woolwich plants were enlarged).

In 1925 the London Company (Western Electric Company, Limited) became associated with the International Telephone & Telegraph Corporation and its name was changed to Standard Telephones & Cables, Limited. Standard Telephones & Cables opened branches in many places and has sales offices in Birmingham, Brighton, Bristol, Leeds and Manchester in England; Glasgow in Scotland; Dublin in Eire; Cairo in Egypt; Calcutta in India; and Pretoria in South Africa.

In 1928, H. M. Pease was appointed European General Manager of the International Standard Electric Corporation [2] and E. S. Byng became Managing Director of Standard Telephones and Cables, Limited. Mr. Byng held this position until 1935, when he became Vice-Chairman of the Company. He was succeeded by T. G. Spencer, the present Managing Director who, following his appointment as General Manager in 1932, had been Joint Managing Director since 1933.

Standard Telephones and Cables, Limited, like other organisations, was handicapped seriously by the world economic depression of the nineteen-thirties. Nevertheless, under the able leadership of T. G. Spencer and the close teamwork of his staff, the Company achieved the high water mark in its history during the last years of peace. At the outbreak of war in September, 1939, the number of employees was 78 percent greater

[1] J. E. Kingsbury, M. I. E. E., is the author of the notable and authoritative book: "The Telephone and Telephone Exchanges—Their Invention and Development," published by Longmans, Green & Co., 1915.

[2] In 1925, the International Telephone and Telegraph Corporation acquired the International Western Electric Company and changed its name to International Standard Electric Corporation.

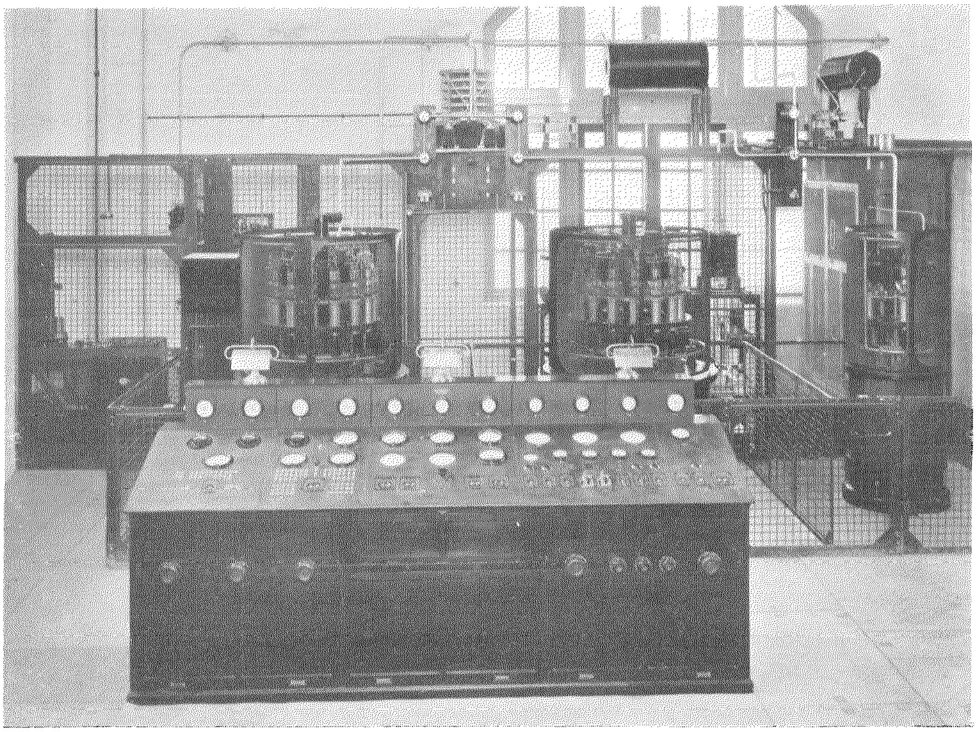


Fig. 1—View of power amplifier and control table of the first radio telephone transmitter supplied for the General Post Office Transatlantic Services at Rugby, England, through Standard Telephones and Cables, Ltd., London. The two 15-tube, circular power amplifier units are shown behind the control table. Either one of these units can be operated separately or, alternatively, the thirty 10-kilowatt tubes can be operated in parallel at from 40 to 75 kilocycles. The transmitter was designed for single-sideband operation.

than in 1938, and, in the same period, the business had increased 112 percent.

With its highly skilled technical, manufacturing and commercial staff, Standard Telephones and Cables, Limited, has entered the field of many ancillary lines with the result that the organisation is now engaged in the manufacture and supply of some sixteen major products involving over a hundred specialised fields. Each, in many organisations, would be handled by a separate company with corresponding additional overhead.

In addition to its various plants, Standard Telephones and Cables, Limited, has two subsidiaries: Kolster-Brandes, Limited, which manufactures and distributes all types of Radio Broadcast Receiving Sets and moulded products, and Stanelco Products, which merchandises various specialty products. It is no exaggeration to say that Standard Telephones and Cables, Limited, is the only organisation within the British Empire that, under one management, covers the whole gamut of telecommunications.

Following September, 1939, Standard Telephones and Cables activities were vastly expanded; the story of the amazing developments of the subsequent war years must be deferred pending resumption of peace. Moreover, exemplification of outstanding contracts and developments consummated over a period of sixty years is difficult. In retrospect, especially because of the introduction of increasingly complex equipment, accomplishments regarded as epoch-making in their day now may seem rather commonplace. Very often, too, the fundamental principles of the newer equipment date back to the simpler innovations of the past. The following very brief summary, however, serves to convey some indication of the Company's contributions to the advancement of the communication art:

1883–1943

1883 London office of Western Electric Company started at Moorgate Street with a total staff of three men.

1884 First multiple magneto switchboard in England installed at Liverpool.

1890 Revolutionary introduction in Europe of paper as dielectric in dry-core cable.

1897 Telephone system installed at Aldershot to facilitate movement of troops at Diamond Jubilee Review by Field Marshal, the Duke of Cambridge.

1900 First central battery switchboard installed at Bristol.

1914 First loaded telephone cable laid in England.

1915 London-Birmingham Cable of 104 wires designed on multiple twin system—the first of its kind in Europe.

1917 Demonstration of Nash Hydrophone Submarine Detector in Weymouth Bay.

1922 Supplied first broadcasting station at Birmingham for the then British Broadcasting Company.

1923 First trans-Atlantic, one-way radio speech transmitted from New York to New Southgate.
Introduced and manufactured in Great Britain the first superheterodyne receiver.
First British-made repeater installed at Fenny Stratford and Derby.

1924 Public Address System installed at the British Empire Exhibition, then the largest installation of its kind.
The first British-made iron dust loading coils were supplied and placed in service.

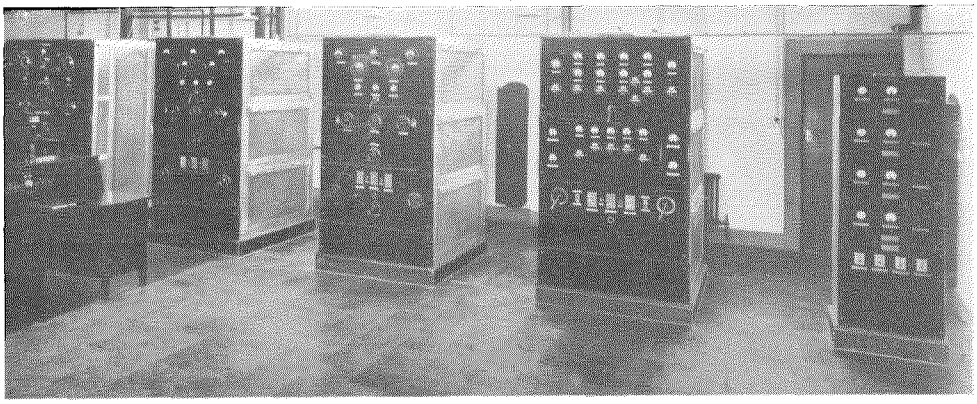


Fig. 2—View of one transmitter of the first Empire broadcaster designed and manufactured for the British Broadcasting Corporation by Standard Telephones and Cables, Ltd., London. The five radio units and control desk of a single transmitter are shown. The station contains two complete transmitters, each rated at 12 kilowatts output and capable of operation on any of six wavelengths from 16.9 to 50 meters; it was formally opened for operation December 19, 1932.

1925 Introduced the first water-cooled valve (vacuum tube) in Europe.
Introduced cone loudspeakers in Europe, the first step in high-fidelity reproduction.
Installed first ship-to-shore radio telephone equipment on Whaling Fleet in South Georgia.
Western Electric Company, Limited, became Standard Telephones and Cables, Limited.
Completion of the London–Glasgow Trunk Telephone Cable and its Repeater Stations, the backbone of the trunk telephone cable network of Great Britain.

1926 Two-way trans-Atlantic radio telephony demonstrated between Rugby and New York.

Fig. 4—View of the reflector towers at Lympne, England, terminus of the Anglo-French micro-ray link between Lympne and St. Inglevert, France. It represents the first commercial micro-ray installation, designed to operate on a 17-centimeter wavelength with sharply concentrated beams.

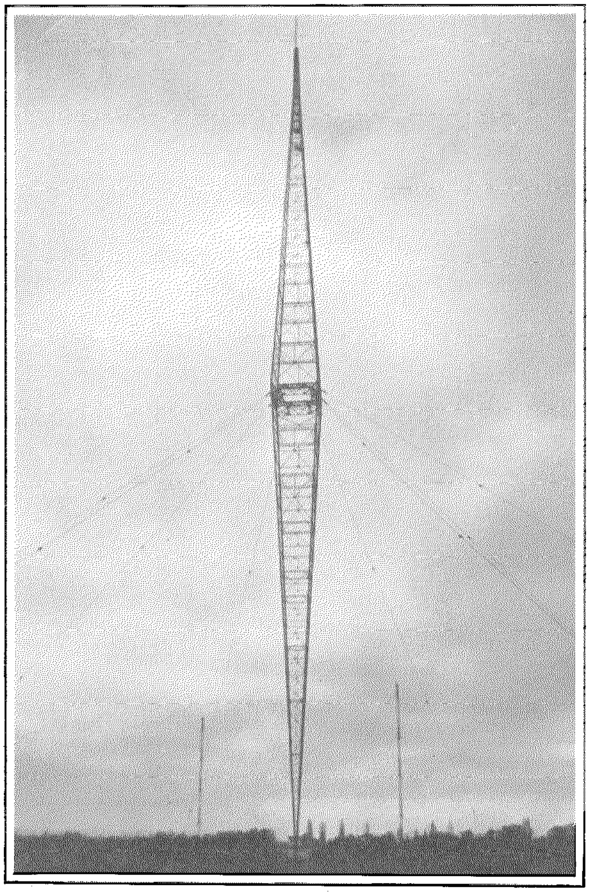


Fig. 3—Anti-fading antenna of the 120-kilowatt broadcasting station at Lakihegy near Budapest. Its height is 307 meters (1005 feet); it is higher than the Eiffel Tower, the highest structure in Europe and the tallest mast in the world.

Built the high power single-sideband long wave radio transmitter for the first G.P.O. trans-Atlantic service; operation started in 1927.
Supplied the first large broadcast transmitter installed in the Dominions.

1927 Installation of Kalundbord Radio Station, Denmark.
Installed first shortwave telegraph link between Spain and Buenos Aires.
Installed first shortwave telephone link between Spain and South America (service opened in 1929).

1928 Supplied the first shortwave telephone transmitter for G.P.O. trans-Atlantic Services at Rugby.

1929 First ship-to-shore demonstration of ship-to-shore telephony between S. S. Berengaria and the land telephone services.

1930 Inaugurated the first regular ship-to-shore Atlantic Radio Telephone Service by installing the shortwave transmitter for the G.P.O. at Rugby.

1931 Gave first public demonstration of Micro-Ray communication between Dover and Calais.

1932 First Empire Broadcaster designed and manufactured for the B.B.C. (This equipment relayed the Coronation Ceremonies to the Empire throughout the world.)
Installation of first voice frequency telegraph system between London-Dundee providing 18 carrier telegraph channels.
Installed first high power (120-kw) medium wave broadcasting station in Europe at Prague.

1933 In conjunction with the Blaw Knox Corporation, erected the highest anti-fading aerial in the world at Budapest (overall height 1005 feet).
Installed the first commercial Micro-Ray Radio Link with the shortest wave length used commercially, i.e., 17 centimeters, between Lympne, England and St. Inglevert, France.

1936 Manufactured and installed first 12-channel carrier cable telephone scheme between Bristol and Plymouth.
Manufactured and installed first co-axial cable in Great Britain for the G.P.O. telephone network between London and Birmingham.
Completed first 9-channel ultra-shortwave radio link for the British Post Office between Belfast and Stranraer.
Australia–Tasmania cable scheme placed in service, providing carrier telephone and telegraph channels, together with a channel for the transmission of broadcasting programmes.

1937 Cunard White Star R. M. S. Queen Mary radio equipment manufactured and installed for the International Marine Radio Company.
Public Address Equipment installed in Westminster Abbey for the Coronation of Their Majesties, King George VI and Queen Elizabeth.

1938 First installation of instrument landing equipment at three major London Airports.
Shortwave broadcasters installed for the B.B.C. Short Wave Empire service.
Installation of Moscow–Khabarovsk carrier telephone system comprising the longest telephone line in the world, i.e., 6,000 miles.

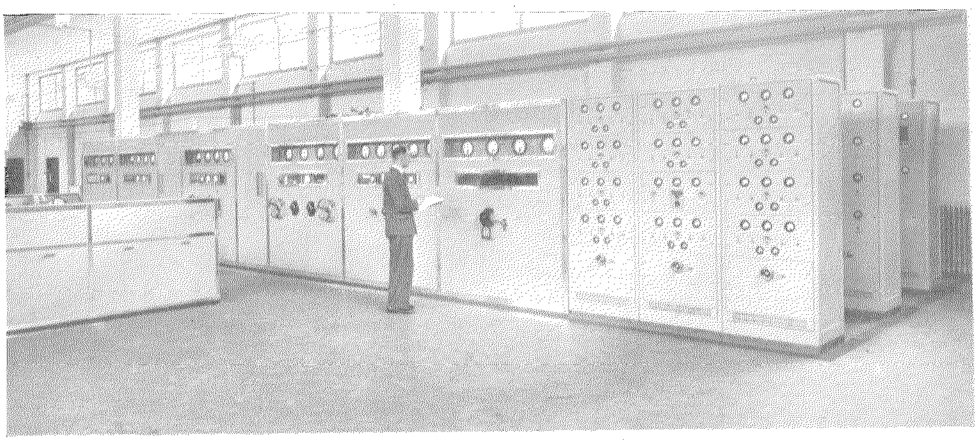


Fig. 5—View of the two 50-kilowatt, 22-megacycle shortwave broadcasters supplied by Standard Telephones and Cables, Ltd., London, and installed at Daventry for the British Broadcasting Corporation. Work on these transmitters was expedited and they were completed in time for the world-wide broadcasting of the Coronation Ceremonies of H.M. King George VI, May 12, 1937.

Microphones and Receivers*

With Special Reference to Speech Communication

By L. C. POCOCK, M.Sc., A.M.I.E.E.
Standard Telephones and Cables, Limited, London, England

THE author is frequently asked how many volts such and such microphone will deliver to amplifier and radio equipment. Such a question is not easily answered in concise and definite terms, and it is accordingly one of the objects of this paper to collect and describe the data which are required to answer the question as fully as possible. There is, of course, a corresponding question regarding the power required at the reproducing end, and this too will be dealt with.

The data presented relate to speech; reproduction of music, with its studio technique and many special requirements, has become an almost specialised side of radio with its own special standards and literature. This paper is addressed in large part to those concerned in a great variety of radio communications, where broadcasting standards are economically impossible.

Speech consists of syllables and words with short pauses between them; an average duration for a syllable is about 1/5 second. During a syllable certain frequencies more or less constant and harmonically related are present; any one sound is characterised by a number of frequencies and by their amplitudes.

During a continuous passage of speech many different sounds with their corresponding frequency content are used; some frequencies recur more frequently than others as different sounds are made, and some sounds with their corresponding frequencies occur more frequently than others. It follows that if analysis by means of filters is made, a speech spectrum can be constructed showing how the speech power is divided between the frequency bands used in the analysis. It is important to remember, however, that such a spectrum is weighted by the frequency of occurrence of speech frequencies so that, for example, the high levels shown at the low frequency end of the spectrum owe their magnitude to frequently occurring low frequency vowel tones rather than to intrinsically high intensities, and, therefore, the speech spectrum is not the appropriate data to use where a question is to be answered bearing on instantaneous peak values; such questions must be answered by data founded on a study of the peak intensity.

Measurements of Speech

Acoustic measurements are made by electrical means with the exception of measurements made with a Rayleigh disc, and this is quite unsuitable for the purposes to be discussed; it is, therefore, to be understood in what follows, that speech is directed into a high quality microphone of known sensitivity and that all measurements are made on the electrical output of the microphone. More directly acoustic measurements are also made by probe microphone and by noise level meter, both of which are sufficiently well known in principle to need no description.

It is not uncommon in specifications and elsewhere to find references to speech voltages, but this practice should be deprecated; a voltmeter connected across a speech circuit, if slightly damped, varies over a wide range, and if well damped it wanders and drifts in a very imprecise way and no real value can be attached to any such measurement.

A better indicator of speech level in a circuit is the volume indicator; the instrument first and still almost exclusively known under this name gives a series of ballistic readings; the damping and period of the meter have been adjusted specially for speech measurement and it has been found, as this paper will show, that the volume indicator generally gives a significant indication of speech voltage or power. Readings are obtained in decibels above or below a fixed datum

* Paper read before London Section, April 30th, 1943, and before North-Eastern Section, June 4th, 1943, British Institution of Radio Engineers. Reprinted from Jr. of Brit. Inst. of Radio Engrs., June–August 1943. As far as possible the terminology used is in accordance with British Standard glossary of acoustical terms and definitions.

(usually 6 milliwatts) in a 600-ohm circuit. In this paper, volume indicator measurements are given relative to 1 milliwatt in a 600-ohm circuit to facilitate comparison with other measurements.

Another useful instrument is the peak indicator which has a quicker response than the volume indicator and longer relaxation time so that it gives a tolerably steady indication of the highest peak occurring during a fairly definite short interval; this instrument is useful when data are required to determine the grid swing that must be allowed for in a given circuit.

A very useful piece of laboratory equipment is the speech energy meter which measures the total electrical speech energy delivered to it in a timed interval. Arrangements for this purpose employing a thermocouple and fluxmeter have been described,[2] but the apparatus used by the author integrates the electrical speech power by storing it as heat in the cathode of a valve.

By means of a pre-heating (or bias) current the initial temperature is brought to the threshold of sensitivity so that the temperature to which the cathode rises can be measured by the maximum plate current recorded at the end of the period of integration; rapid cooling in readiness for the next reading is effected by interrupting the pre-heating current.*

In order to determine the speech power output obtainable from various microphones, they were connected to the speech energy meter with suitable amplifiers when necessary and the whole of a short paragraph was spoken. In this case, therefore, the energy measured corresponds with a certain amount of speech of a certain intensity. The paragraph used was that constructed for a similar purpose by Dunn and Farnsworth [1]; it contains 64 words and 92 syllables and takes from 17 to 20 seconds to repeat, the variation in time depending mainly on the weight given to the full stops and breath intervals. Speech was at a fairly conversational level, such as to give a reading of about 64 db on a noise level meter at 50 inches in a small room.

Table I shows the results obtained and, for comparison, the results of volume indicator readings (across a 600-ohm circuit for which the volume indicator is calibrated). The microphone is to be regarded as coupled by a loss-free transformer to the 600-ohm circuit for this measurement.

It is seen from this table that the volume indicator reads 2 to 4 db higher than the directly measured electrical speech power and gives as good an indication on the commercial carbon microphone as on the moving-coil microphone. About 2 db may be allowed as the variation from person to person in the manner of reading a volume indicator, and, in any case, the volume indicator would be expected to be about 1 db high on account of the fact that its reading is independent of full stops and breath pauses.

* This apparatus was developed and designed by H. S. Leman in the S. T. & C. Ltd. Acoustics Laboratory.

TABLE I

Microphone	Free field sensitivity O.C. volts per dyne/cm²	Impedance ohms	Electrical average speech power	
			By integrating meter 0 db = 1 mw	By volume indicator 0 db = 1 mw
Moving-Coil Microphone (see Bibliography 9, Fig. 2b)				
at 2 inches	−82	23	−60.3	−56.6
at 12 inches	−82	23	−75.3	−73.0
Carbon Microphone. Standard P.O. handset, with 50 ma at ½ inch	—	100	− 4.6	− 1.7
Condenser Microphone with amplifier having 22db voltage gain and output impedance 600 ohms at 1⅜ inches	−22.2	600	−18.0	−11.2
Commercial quality. Moving-Coil Microphone with mouthpiece (see Fig. 4)				
Talking very close	—	50	−45.8	−44.2
Talking at 2 inches	—	50	−70.0	−66.2

An exception to the agreement noted between the two methods occurs with the condenser microphone; this has been noticed repeatedly and is thought to be due to the rising free field characteristic of this microphone.

These measurements of speech power have a real meaning and use; they are the basis for determining the power capacity of valves used for amplifying speech. Naturally, generous margins must be provided for the possible occurrence of louder speech. Harvey Fletcher in his well-known book indicates that about 7 percent of speakers talk as much as 12 db below the average and 4 percent rise to as much as 6 to 9 db above the average.

Louder speech, say 12 db above the average, is in the category of high-declamatory speech, which is accompanied by a change of spectrum and enhancement of upper frequencies at the expense of frequencies below 500 cps.

A complete distribution of calling levels is given by Sivian (Bibliography,[2] Fig. 2).

Apart from the consideration of whether the speech transmitted is declamatory or conversational, consideration must in some cases be given to the ambient noise level; when a speaker is immersed in noise he tends to speak louder. An approximate indication of the amount of increase in loudness is given in Table II.

TABLE II

EFFECT OF NOISE ON SPEAKING LEVEL

Noise level in db*	Increase in voice level db
40	average speech = 0
60	2
80	5
90	7.5
120	15

* Here and elsewhere noise level in db is relative to 10 [16] watts per square cm 1000 cps.

Whilst dealing with noise, it is convenient to remark here that noise entering the ear at the receiving end masks the received signals; whether noise is transmitted through the amplifier system with the signal, or originates at the receiving end, naturally makes no difference; on the other hand, the character of the noise affects the result. It may be inferred from the known fact that masking tones have more effect upon frequencies above them than on those below them, that large low frequency components have considerable adverse influence on articulation. On the other hand, the amplitude of the upper speech frequency components is small, so that they are relatively easily masked by high frequency noises.

As a particular example of the effect of noise on articulation, Fig. 1 is reproduced from data by Rocard (Bibliography [20]).

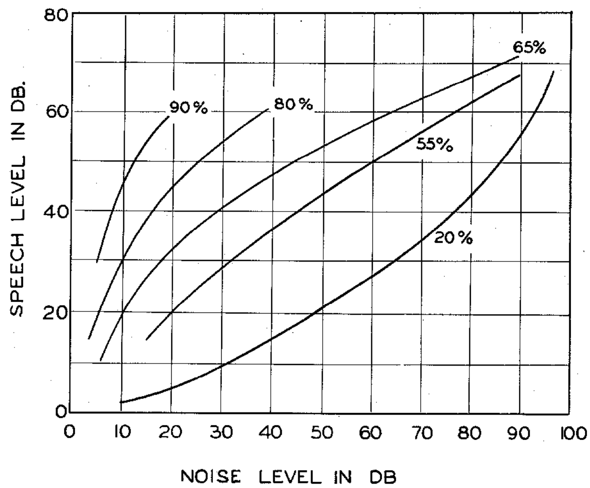


Fig. 1—Curves of Equal Articulation in the Presence of Noise at the Listening End (Rocard).

Another quantity important to the radio designer is speech peak voltage, the quantity which determines the grid swing that must be allowed for in every stage of the apparatus.

Sivian [2] shows that the peaks of speech pressure extend up to between 22 and 26 db above the average speech pressure, and that the highest peaks occur in the region of 250 to 500 cps although the peak level is well maintained within about 6 db up to 4,000 cps. More useful in a great many cases, where it is uneconomical to cater for the very highest peak pressures, is Sivian's (*loc. cit.*) curve showing the percentage of the time that the instantaneous power in speech exceeds the average power. Table III has been constructed from this curve and shows the percentage of time that peaks will be clipped if amplifying equipment has power capacity reck-

TABLE III

Percentage of time peaks are clipped	Amplifier grid swing capacity in db above average speech power requirements
0	20
5	9
10	5
15	2
18	0

oned on the basis of grid swing, limited to any number of decibels above the average speech power.*

The foregoing considerations, with the exception of those related to average speech pressure, may be quite properly based on the speech power measurements given in Table I, and they may be extended to other high quality microphones of known sensitivity and impedance by means of the results given for the moving-coil microphone standing first in Table I.

It may be of interest to record that measurements of pressure at the face of this microphone with single tones showed that the rms pressures producing the same microphone power output as average speech were 5.3 dynes/cm^2 for speaking at 2 inches and .78 dynes/cm^2 for speaking at 12 inches. This is in reasonable agreement with Dunn and Farnsworth's results [1] of 5.6 and 1.1 dynes/cm^2 for the same distances respectively, using only a single speaker.

Speech Power for Reception

The speech power to be delivered to a telephone receiver or loudspeaker depends of course on the loudness desired. As microphone output has been quoted for normal conversational speech defined as giving a reading of 64 db on a noise meter at 50 inches, it is reasonable to evaluate the power into the receiver to produce the same loudness. It will be assumed that moving-coil apparatus is correctly matched and that electromagnetic receivers are in accordance with prevailing practice matched at their 800 cps impedance. Further, it is useful to know that all well made electromagnetic receivers of the ordinary type and those acoustically equalised have within a few decibels the same order of efficiency.

Experiments have been made to determine the required data; in the case of loudspeakers, all that was necessary was to measure the electrical speech power delivered and the loudness of the output at the listening point.

Some typical results are given in Table IV.

TABLE IV

POWER REQUIREMENTS FOR LOUD SPEAKERS TO GIVE SPEECH AT 64 db LEVEL AT 50 INCHES ON THE AXIS

Type of moving coil loudspeaker	Volume indicator reading across a 1600-ohm circuit relative to 1 mw db	Actual speech Power relative to 1 mw db
Commercial 12-inch diaphragm, back closed	− 3.5	−6.7
Commercial 8-inch diaphragm, cabinet mounted	+ 8.9	+6.1
Laboratory model, 4-inch diaphragm, back closed	+12.8	+9.8

These results were obtained with speech from loudspeakers in a small room, with the noise meter placed 50 inches from the loudspeaker on the axis.

Amongst the various estimates that have been made of the amplifier output required by loudspeakers as a function of room size, the most satisfactory seems to be that published by G. E. Morison [8] who bases his argument on the direct radiation of the loudspeaker, showing that the build-up due to internal reflections in the room gives only a slight increase of sound level under the majority of ordinary conditions.[9] Hence, Morison concludes that if θ is the planar radiation angle of the loudspeaker and OP is the mean distance of the loudspeaker O from the observer P, taken as half the length of the room, then the area over which the radiated sound flows is

$$A = 2\pi (OP)^2 \left(1 - \text{Cos}\,\frac{\theta}{2}\right)$$

Assuming 100 db level for an orchestra reproduction and that this is equivalent to 10^{-6} (acoustic) watts per sq. cm., the acoustic power required to secure the 100 db level must be $10^{-6}A$. Then if the efficiency of the loudspeaker is known (say 5 percent for domestic loudspeaker) the power capacity of the amplifier output stage is

* It is unfortunate that no data seem to be available on the co-relation between articulation and peak clipping.

easily obtained. Actually, the amplifier must also be capable of reproducing, without distortion, higher peak power in accordance with the peak factors discussed elsewhere.

The drawback to this formula is its dependence on the angle θ which cannot be very precisely determined and is moreover a function of frequency. For speech the radiation angle at about 500 cps should be used.

To determine the power required by head receivers to produce a given loudness, the apparent loudness in the receivers was adjusted to sound equal to the loudness from a loudspeaker producing a measured level at the listening point.

On account of the inconstant impedance of electromagnetic receivers, it is more convenient and useful to measure the power available to them rather than the power actually taken; that is to say, if the receivers are intended to be used at a 200-ohm output they will ordinarily have approximately 200 ohms impedance at 800 to 1,000 cps, and it is useful to measure the power taken by a 200-ohm resistor connected in place of the receivers. Thus, in the present instance, after the electrical speech delivered to the receivers had been adjusted to loudness equal to the sound from the loudspeaker, the telephone receivers were replaced by resistance equal to the output impedance of the circuit.

Typical results are shown in Table V.

TABLE V

Power to be made available to telephone receivers in order that the speech received may be at the 64-db level (relative to 10^{-16} watts per sq. cm. at 1,000 cps.)

Type of receiver	Speech power taken by a resistance equivalent to the receiver impedance at 1,000 cps. db relative to 1 mw
P.O. handset receiver (resonant type)	−37
P.O. handset receiver (equalised type)	−37
Moving-coil receiver (high quality)	−33
Balanced armature receiver	−43

In connection with Table V, it must be pointed out that the figures are the power levels required to give a certain output and, therefore, the lower the power level, the more efficient the instrument concerned; thus, the balanced armature receiver, which has the construction of a miniature loudspeaker, is the most efficient.

When receivers are not closely coupled to the ears, but used in large volume ear pads, substantial losses occur of the order of 10 to 20 db.

For some unexplained reason it is customary for radio operators to use double head sets in contrast to the use of single head sets by the large majority of telephone operators. It would be difficult to determine whether there is any advantage due to psychological effects in the use of double head sets, but that there may be some advantage is borne out by the instinctive defence of the practice which many engineers put up.

The facts, as far as they have been ascertained, are that there is no difference in articulation when a given amount of signal power is shared between two receivers or heard in one receiver unless the receiving level be such as to approach overloading the receiver or the ear, in which case the overload is reduced by using two receivers and there is a gain in articulation. When using a single receiver, noise entering the free ear does not interfere with reception in the telephone ear unless it is substantially more than 60 db above the signal in the telephone ear.

Microphones

Microphones may be classed according to the principle on which they are based (carbon, moving-coil, etc.) and according to their quality. Generally speaking, and with the possible exception of early forms of broadcasting microphones, carbon microphones are not in the high quality class; they are not suitable for distant speaking because their characteristics are not linear and the output falls rapidly as the sound intensity diminishes, and also there is always a background noise that becomes troublesome. Types can be specially designed for distant speech (deaf aids and the like), but these will not be discussed here. The value of a carbon microphone lies in the fact that it draws its power from a battery, and so can be made to give out more energy than it receives. It can be designed to have a good flat frequency characteristic under specified measuring conditions, but the characteristic is likely to vary with intensity, and non-linearity is noticeable as a degradation of naturalness resulting mainly from overdriving at the frequencies where the diaphragm amplitudes are greatest.

One of the earliest high quality microphones was the condenser microphone, which in its usual form had a shallow front cavity which was sufficient to give rise to a measurable resonance; this was eliminated in a design described by Oliver.[5] The condenser microphone has the disadvantage of high impedance requiring good insulation and low capacity leads.

For practical purposes, where amplification is possible, microphones most extensively used are of the moving conductor type, a term used to describe moving-coil and ribbon construction; condenser microphones are still used, chiefly in laboratory work, and crystal microphones have a certain number of limited applications, especially when small dimensions are important as, for example, the microphone of a noise measuring set which should be so small as to have little disturbing effect in the noise field it is to measure.

Electromagnetic microphones also have limited and special uses, the principal advantage being extreme robustness.

Moving-conductor microphones are made up in a variety of forms and even in dual forms incorporating coil and ribbon in the same unit; the object of each design is to obtain some particular directional or non-directional effect, ranging from a single non-directional moving-coil unit housed in a spherical casing (a sphere has less disturbing effect on the sound field than any other shape) to "line" microphones.[22]

In a line microphone, highly directional properties are obtained by admitting the sound to the diaphragm through a bundle of tubes of different length having their open ends spaced near to the axial line; in this way sounds arriving from directions making an angle with the axis arrive at the actuating diaphragm out of phase; the spaced openings give somewhat the effect of a large diaphragm which has a certain amount of directional discrimination on account of the pressure not being uniform over its surface when waves arrive at an angle to the normal.

All such directional characteristics must necessarily be less effective at low frequencies unless very long tubes are used. For a given degree of directivity the overall length can be reduced by bending the tubes and making the acoustic path greater by proper amounts than the distance from the open end to the microphone; the sacrifice paid for this reduction of size is a loss of sensitivity at the upper frequencies. To overcome the difficulty of excessive length of tubes and to maintain sensitivity over a wide frequency range with directional properties almost independent of frequency, a combination of several microphones, each with its own bundle of tubes including acoustic delays, has been designed. Each distinct line microphone is designed for a part of the frequency range only and is connected through a separate electrical filter channel to the mixing circuit whence the output goes to the amplifier.

Reproducer Frequency Characteristics

The object of determining a frequency characteristic can only be to establish the performance of the instrument or system tested in relation to natural communication by air-borne speech or music.

Consider a system comprising a microphone and loudspeaker and let the microphone be placed in a single frequency sound field; it is desired to compare the sounds heard through the loudspeaker with the sounds that would be heard by the listener if he occupied the position of the microphone.

At the site of the microphone and before its introduction there existed an alternating acoustic pressure (characterising the "free field"); after the introduction of the microphone, the pressure operating on the microphone will be different (Fig. 2).

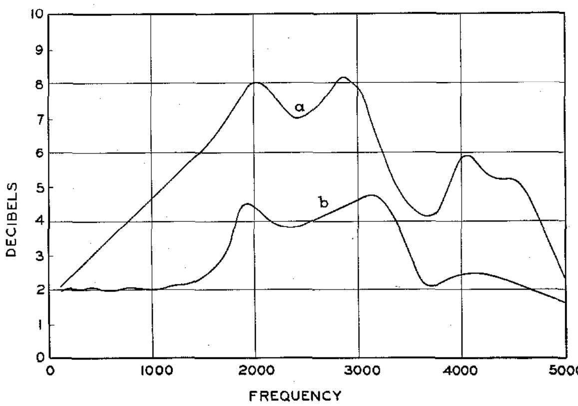


Fig. 2—Increase in Pressure Due to Placing a Cylindrical Microphone 8 Centimeters Diameter, 6 Centimeters Long, in a Sound Field Radiating from an Orifice 1.7 Centimeters Diameter, Distance 5 Centimeters—(a) at Centre of Microphone Face, (b) at Edge of Microphone Face.

When the listener himself occupies the position of the microphone, the pressure at his external ear will have yet another value due to the disturbance of the sound field by the listener.

The disturbance of the free field brought about by the introduction of the microphone must be regarded as part of the microphone frequency characteristic, and, therefore, for most purposes, the free field frequency characteristic of a microphone is of importance rather than its pressure frequency response. In order that the free field frequency characteristic of the microphone added to the frequency characteristic of the loudspeaker may define the overall performance, the loudspeaker characteristic must be measured in "free field," that is to say, by means of a microphone or measuring device of such small dimensions as not to disturb the sound field, and the sound field must be free from standing waves; other restrictions also are included in a full definition.

It is obvious that the aim must be to make the frequency characteristics of microphone and loudspeakers "flat."

When the sounds to be picked up by a microphone are to be reproduced by a head receiver,

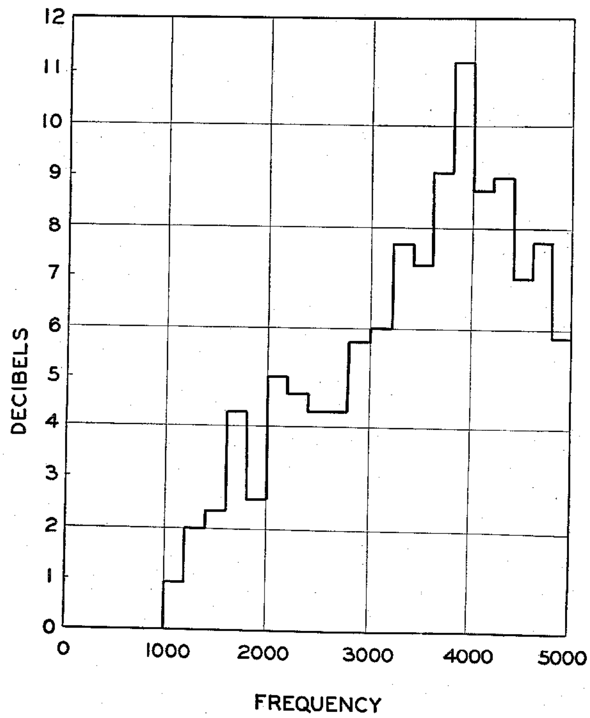


Fig. 3—Increase of Pressure at the Ear Due to Presence of the Head in a Field of Thermal Noise. Listening Point 50 Inches from 12-Inch Loudspeaker in a Small Room.

the conditions are different from those for loudspeaker reproduction. Sounds heard naturally—that is, by placing the head in a sound field—suffer "distortion." The head disturbs the sound field so that the pressure at the location of the listener's ears is changed by the presence of the listener. The disturbance produced by the listener's head (and ears) may not be very large in an idealised case with the listener facing the sound source, but under the conditions in which speech is normally heard, that is to say, in rooms not excessively damped, the effect may be considerable (Fig. 3). It therefore is of some importance to determine whether, or how much of, the disturbance due to the head ought to be artificially introduced by a head receiver to compensate for the lack of the diffraction effect accompanying ordinary hearing. Data is not yet available to enable this question to be answered except by the surmise that any other frequency characteristic than a flat one is likely to introduce unwanted transient distortions.

Considering then, that at present the aim is to make head receivers with "flat" frequency characteristics, the question arises, how the frequency characteristic is to be defined and how measured. The object is to measure the pressure produced by the receiver in the outer ear and to aim at making this constant with frequency when constant voltage is applied in the receiver circuit. The measurement is usually made by coupling the receiver to a microphone with an enclosed air volume of 2 to 6 cubic centimeters; the device is called an artificial ear and the results ought to be comparable with those obtained when the pressure is measured in a human ear to which the receiver under test is applied. Measurements of the pressure produced in human ears present various difficulties, and show considerable differences between one listener and another, especially at frequencies above 3,000 cps.

It is not surprising, therefore, that there is no complete specification for an artificial ear, and after various attempts to produce elaborate simulations of a human ear the practical tendency has been to make measurements of pressure in a small enclosed air space representing the volume enclosed between the ear cap and the ear. A certain amount of work has been done by aural balancing and by pressure measurements to con-

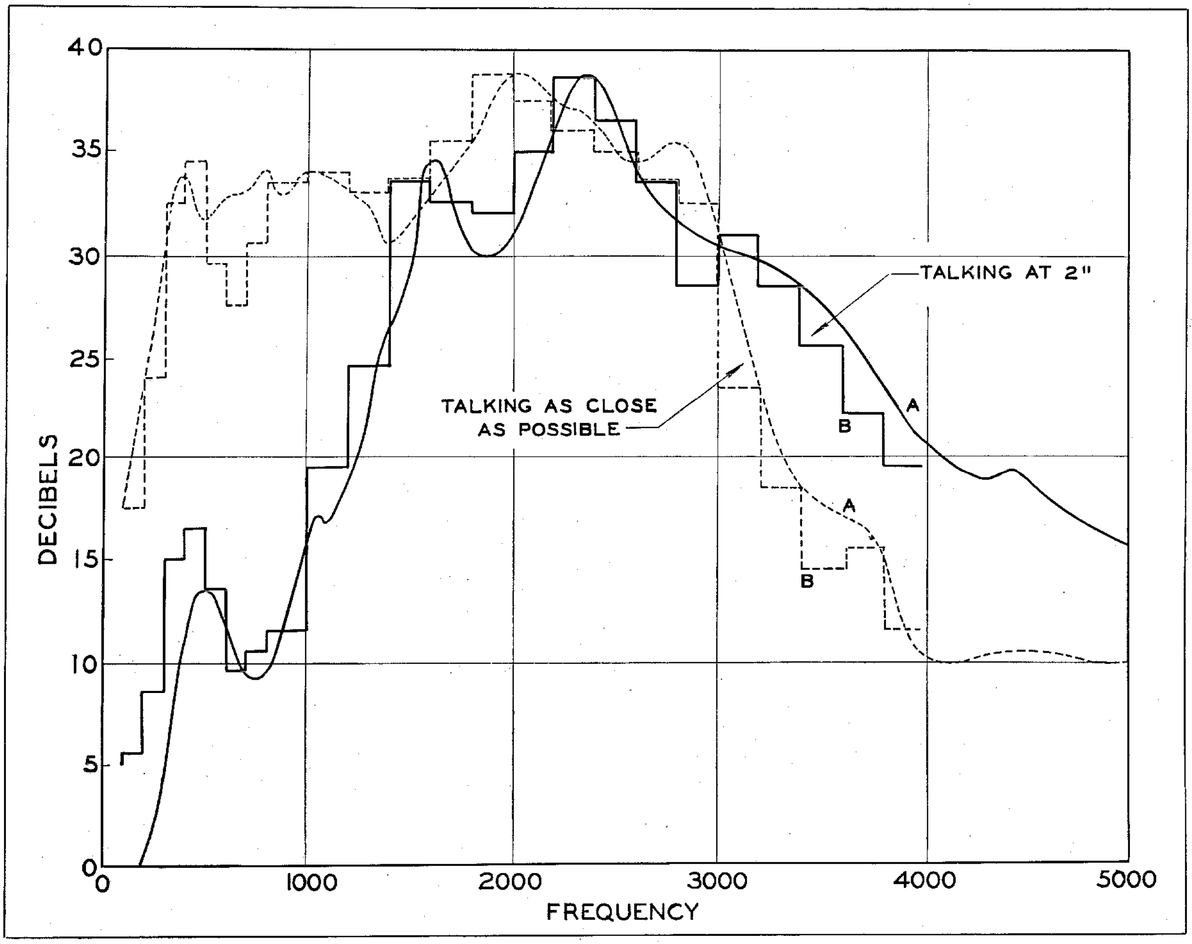


Fig. 4—Frequency Characteristics of a Moving-Coil Microphone with Mouthpiece (a) by Single-Tone Method, (b) by Speech-Analysis Method.

firm that with proper design of the artificial ear the pressure measured is in reasonable agreement with pressures produced in real ears up to about 3,000 cps. At higher frequencies the differences between individual ears make it difficult to describe the performance of an average ear, and the best that can be said is that the artificial ear measurements accord with measurements made on some real ears, but not on all.

Microphone Frequency Characteristics

Reference has been made to the speech energy spectrum which can be obtained by measuring the speech energy passed by narrow band filters; a method of measuring microphone characteristics has been based on measurements of this type. Although for many purposes, free field or pressure calibrations made with single tones are satisfactory, they are not entirely satisfactory for carbon microphones which present difficulties on account of instability and non-linearity. When the conditions are unusual and not particularly simple—for example, determining the frequency characteristic of a close speaking microphone of any type—it is preferable to use a method more directly related to working conditions. In these cases frequency characteristics may be measured by speaking to the microphone under working conditions and analysing with filters the power output; for this purpose filters are required with good sharp cut-offs, attenuating the stopped ranges 60 to 70 db.*

* Such a set of filters has been used to obtain results described in this paper. The filters were designed by H. S. Leman in the S. T. & C. Ltd. Acoustics laboratory.

The spectrum obtained using the microphone under test must be compared with the spectrum obtained with a "flat" microphone, or, in other words, the spectrum for a "flat" microphone must be known and subtracted from the spectrum given by the test microphone in order to put the frequency characteristic into the usual form of output for constant input.

A recent determination of the spectrum of speech (i.e., with a "flat" microphone), in rather finer detail than has hitherto been published, is shown in Table VI and compared with another set of published data.

The agreement between the author's results and those of Dunn and Farnsworth is reasonably good, considering that the latter gave results for only one speaker; the author's data is for four speakers. As might be expected from the distinctive character of different voices, definite and reproducible differences of 10 db or more are found in the 200-cps wide bands of different voices.

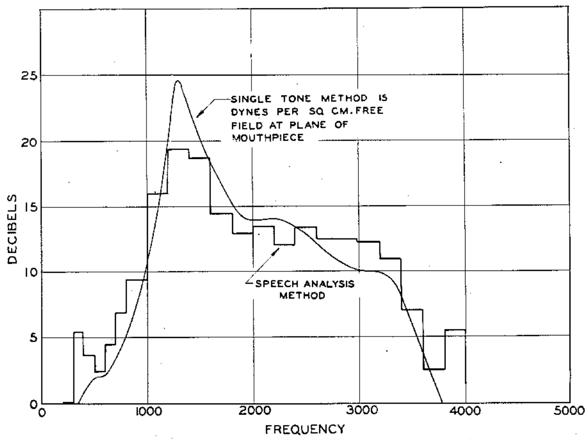


Fig. 5—Frequency Characteristic of P.O. Carbon Transmitter in Handset.

TABLE VI

Frequency bands (cycles)	Speech power for bands 1 cps. wide relative to total speech db	Comparison with Dunn and Farnsworth[1]		
		New data averaged over bands given by Dunn and Farnsworth		Data given by Dunn and Farnsworth (Table IV, Bibliography[1]) db relative to whole speech
		Frequency bands (cycles)	Power in band; db below whole speech	
100–200	−36.6	62.5–125	−18.6	−19.2
200–300	−35.6	125–250	−14.6	−12.3
300–400	−34.0	250–500	− 9.5	− 6.6
400–500	−30.9			
500–600	−29.7	500–700	− 6.8	− 5.6
600–700	−30.0			
700–800	−34.6	700–1000	−10.8	− 8.9
800–1000	−36.1			
1000–1200	−38.5	1000–1400	−13.0	−14.6
1200–1400	−39.4			
1400–1600	−39.7	1400–2000	−13.5	−14.9
1600–1800	−41.0			
1800–2000	−43.3			
2000–2200	−45.1	2000–2800	−16.3	−20.5
2200–2400	−45.1			
2400–2600	−45.5			
2600–2800	−45.5			
2800–3000	−49.6	2800–4000	−19.3	−20.9
3000–3200	−51.2			
3200–3400	−51.0			
3400–3600	−48.3			
3600–3800	−48.3			
3800–4000	−51.9			
4000–4200	−54.7	4000–5600	(−29.5)	−31.3
4200–4400	−57.5			
4400–4600	−59.1			
4600–4800	−60.5			
4800–5000	−62.5			

Frequency characteristics in Figs. 4 and 5 have been determined by the single tone method and by the speech analysis method for comparison.

Fig. 4 shows the performance of a moving-coil microphone used with a mouthpiece, and the purpose of the speech analysis test was to confirm that the single tone method of measuring the characteristic was in reasonable agreement with the true performance for speech, or, from another point of view, to ascertain whether the artificial head could be regarded as representing a talking head.

Fig. 5 shows the performance of a P.O. telephone handset transmitter; carbon transmitter characteristics measured with single tones vary widely in shape according to the sound intensity used, and the object of this test was to determine the sound intensity that would give the most satisfactory result by the single-tone method.

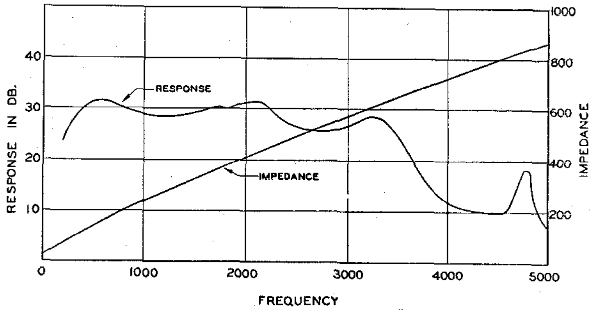


Fig. 6—Frequency Characteristic and Impedance of a Modern "Equalised" Telephone Receiver.

Coupling of Transducers

Some attention must be paid to the coupling between transducers and amplifier equipment; the point where two separate arts unite sometimes suffers from neglect through lack of coordination.

Loudspeakers, being nearly always of the moving-coil type, are in most cases readily matched to a fixed output impedance. Telephone receivers have, however, a variable impedance (Fig. 6). In commercial telephony it has been the practice to match these receivers at about their 800 or 1,000 cps impedance, but this practice is something of a hang-over, because, until recently, such receivers were resonant in this region, and loudness was all important, especially in the days before repeater amplifiers were available. Telephone receivers now made have comparatively flat frequency characteristics over the commercial speech range,[16, 18] and the continued practice of matching at 800–1,000 cps rests only upon the necessity for maintaining interchangeability between receivers of the new type and the old type. It should, however, be understood that the new type receivers are so designed as to give a relatively flat characteristic when operated out of an impedance approximately equal to their 1,000 cps impedance and departures from this practice will tilt the frequency characteristic.*

The introduction of feed-back has added complication to the coupling of instruments to amplifiers, and from this point of view it is desirable, though not always possible, that feed-back should not operate at the input or output of an amplifier system unless it be a complete self-contained system.

This aspiration is of the same character as the general principle of making every unit of a system with a flat frequency characteristic rather than to compensate rising characteristics in some units by falling characteristics in others. The arguments are the same in both cases, viz., the provision of equipment with the greatest possible range of application.

There are many commercial equipments for which it cannot be precisely foreseen how many microphones or receivers may be utilised or even what types of instrument may be required during the life of the equipment, and, therefore, unless decoupling or mixing devices are part of the outfit, operation should be as far as possible independent of the character of terminating impedances.

Amplifiers intended to operate reproducers should be designed with knowledge of the impedance variations which are to be encountered whether by variation of the load or because of the natural variations of the load impedance with frequency. The impedance of a large moving-coil loudspeaker may rise with frequency to many times the direct current resistance, while

* Frequency characteristics in ref. [16] are for constant volts applied to the telephone receiver; when correctly matched the characteristic of the equalised receiver appears as in Fig. 6.

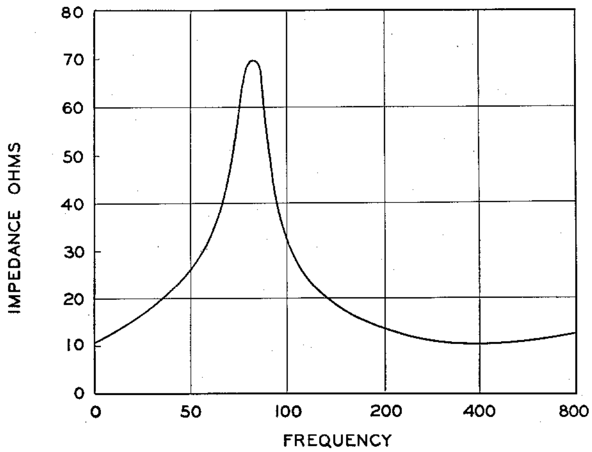


Fig. 7—Low Frequency Impedance of Moving-Coil Loudspeaker (Olsen).

the impedance of a small loudspeaker may not exceed the direct-current resistance, by 50 percent in the frequency reproduction range. In both instances, however, the motional impedance at some very low frequency is likely to be several times as great as the resistance (Fig. 7); the application of negative current feed-back will intensify the irregularities while negative voltage feed-back will very much diminish the irregularities, increase the damping, reduce the transient distortion [17] and reduce the amplifier harmonics.

Other types of feed-back are also known whereby the feed-back voltage is derived from a separate coil moving with the voice coil, and by this means the diaphragm velocity is held in more constant ratio to the signal voltage at the amplifier input. The result is an improvement in higher frequency response, but inasmuch as the low frequencies require constant amplitude rather than constant velocity, there is considerable falling off of the output at low frequencies where the radiation impedance is low and reactive; this requires compensation by a network preceding the amplifier.

A microphone also may be coupled to an amplifier in such a way as to produce a uniform voltage at the output for uniform pressure at the microphone; this device is useful in the laboratory, though it does not appear to have been extensively utilised elsewhere.

Distortion

Of the many distortions that may originate in electro acoustic transducers, the predominating types are well known. Frequency distortion need not be discussed here as other parts of this paper deal with frequency characteristics; it is sufficient to say that there is no technical obstacle in the way of producing apparatus with any required frequency characteristics; such frequency distortion as is found in modern equipment is there because of economic factors affecting the apparatus itself or the available frequency range of the equipment with which it is used.

Non-linear distortion in varying degrees is almost always present; at its worst in the carbon microphone, where current harmonics are bound to occur because the circuit resistance is modulated; actually, however, far more distortion is occasioned by the non-linear relations existing between displacement and carbon resistance.

In moving-coil devices, any asymmetry of the coil in relation to the magnetic field produces measurable harmonics; even in the case of a microphone where the actual movement is very small, harmonics at 10–15 db below the fundamental may be produced by loud sounds if the coil is not symmetrically placed. In moving-coil loudspeakers, the large amplitudes required call for special care in this matter of symmetrical placing to such a degree that the problem becomes one of providing a uniform magnetic field at every part of the field into which any part of the coil may move.

Direct acting loudspeakers also suffer from the limitation of their amplitude imposed by the centring devices; the functions of these are to impose rectilinear axial motion on the diaphragm and supply a slight restoring force. The restoring force is not, however, a linear function of the displacement on account of the tensile stresses produced.

It is often assumed that diaphragms and similar structures have a linear relation between deflection and load; for small deflections this must be true but, according to some recent measurements made by the author, it is doubtful whether the deflections are small enough in many practical cases to justify applying the assumption.

On the other hand, the treatment of large amplitudes generally introduces other non-linear variables which may be more important; thus, a telephone receiver operated at high output produces harmonics due to the change of flux with air gap and change of reluctance with flux, and, ultimately, due to the failure of the restoring force when the amplitude exceeds the initial deflection of the diaphragm under the steady pull of the permanent field; at ordinary telephone levels, the harmonic distortion of these receivers has been found to be small.

The author has long maintained that, so far as speech intelligibility was concerned, harmonic components arising from non-linearity had little effect, producing a loss of naturalness rather than a loss of intelligibility; loss of intelligibility due to non-linear (harmonic) distortion has never been convincingly proved. However this may be, non-linearity becomes important when noise is present because the noise then modulates the speech. In the same way, non-linearity may produce harmful modulation products between simultaneously existing speech or music tones.

Similar modulation products occur to a serious extent in a high-power, horn-type loudspeaker where the large diaphragm amplitudes occurring at low frequencies cause appreciable variation of the volume of the air chamber between the diaphragm and the throat of the horn, thereby modulating the higher frequencies.

It has been stated that intermodulation products amounting to only 2 percent can be detected by ear in a high quality system.

The effect of transient distortion on intelligibility has not been very fully investigated; in the first place, transient distortion is associated with and cannot be separated from frequency distortion, though it is not generally easy to appreciate the character of the transient distortion by looking at a frequency characteristic. It is to be expected, however, that frequency characteristics showing sharp peaks or sharp cut-offs indicate that impulse excitation will give rise to persistent tones in the frequency region concerned, and in bad cases these are wearisome as a perpetual tonal accompaniment of the reproduction.

The frequency characteristic does not, however, tell the whole story, for the presence of non-linear elements will affect the transient distortion in a way not determined by the steady state characteristic.

From the character of both speech and music there can be little doubt that good *transient* response is a desirable end in itself, and it is perhaps fortunate that extension of the frequency range reproduced reduces transient distortion.

Distortion Due to Wrong Level of Reproduction

Because the loudness of a tone is not linearly related to its intensity and the relation between loudness and intensity is a function of frequency, it follows that two tones of equal intensity will not generally sound equally loud and their relative loudness will depend upon the intensity. These effects are grouped and illustrated by the well-known equi-loudness contours. In application to sound reproduction, it is evident that to secure the right effect, reproduced sound (music in particular) must be at the intensity of the original sound, or at least it must be at a possible intensity, that is, at an intensity such as might exist in the vicinity of a listener not too unfavourably placed. To illustrate the effect of reproducing level, suppose that there are four tones of equal loudness in the original sound and these are reproduced with uniform amplification; Table VII shows the loudness when reproduced at different levels.

TABLE VII

Reproduction level in db relative to initial intensity	Loudness of each tone (db)			
	100 cps.	250 cps.	1,000 cps.	4,000 cps.
−20	31	43	50	49
0	70	70	70	70
+20	98	92	90	93

The original sounds were at the 70-db level; reproduction 20 db too high raises the virtual frequency characteristic 6 db at the lower end relative to 1,000 cps, while reproduction 20 db too low drops the virtual frequency characteristic at the lower end 12 db.

The listener should of course adjust the received volume (when it is under his control) until his æsthetic sense is satisfied as to the proper balance of intensities. Operation of a tone control knob might possibly produce the right balance at the wrong level, but it is the wrong thing to do; the writer considers that networks intended to increase selectivity should be confined to this function.

Equivalent Networks

Reproducers (including telephone receivers) and microphones of practically all types are mechanical vibrating systems that can be represented by equivalent electrical networks to a high degree of precision; some of the elements in these networks represent acoustic reactances and resistances which have to be represented first by their equivalent mechanical impedances; in general, all such acoustic impedances are multiplied by the square of the effective area of the diaphragm so that they are in the same mechanical impedance units as the purely mechanical elements of the system. A typical equivalent network for a high quality microphone is shown in Fig. 8; some or all of the elements shown occur in several types of moving-coil microphone.

It seems worth while pointing out that acoustic impedances depend upon the density and pressure of the air, so that apparatus designed to have certain definite frequency characteristics at sea level will have different frequency characteristics at high altitudes because the acoustic elements will change with altitude while the mechanical elements will not. Acoustic stiffnesses become less stiff at high altitudes in proportion to the barometric height, and acoustic masses become smaller in the same ratio.

Thus, for example, at Quito (10,000 feet) acoustic masses and stiffnesses have about 7/10 of their sea level values, and at twice this height the ratio is about 1/2. Acoustic resistances will also be reduced at high levels, probably in approximately the same proportion, but acoustic resistances are generally somewhat non-linear, values depending upon the air velocity.

Another point in connection with network representation of a microphone and receiver is that it may be used for studying transient distortion by electrical test methods, but it does not generally include non-linear elements, and therefore its use is mainly confined to studying the influence on the frequency characteristic of the various parts of the structure.

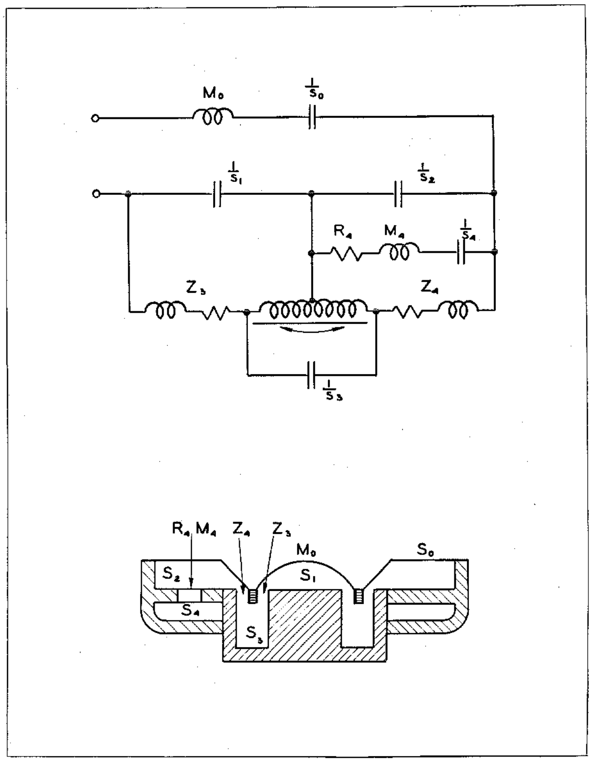


Fig. 8—Diagram and Equivalent Network of Moving-Coil Microphone. The diaphragm is assumed to move as a whole, but the areas on which s1 and s2 operate are different; this necessitates in the network an ideal auto transformer to couple z3, z4, and s3.

It should, however, be noticed that a non-linear element, if its law be known, can be replaced by a network of linear elements in so far as it affects any given particular transient.[21] This suggests a line of investigation that might be pursued.

Force Factor

A very important quantity in any conservative electro-acoustic system is the "force factor," which may be defined equally well as the open-circuit voltage produced by unit velocity of the diaphragm, or as the force exerted upon the diaphragm (held stationary) when unit current flows into the device. The force factor is, therefore, the measure of the coupling between the acoustic or mechanical side of the apparatus and the electrical or electromagnetic side. As progressive development tends to produce apparatus with higher force factors and correspondingly higher efficiency, the response of one side (e.g., the acoustic side) becomes more dependent upon the impedance of the other side (electrical), and it becomes necessary to design the apparatus with due consideration given to the electrical circuit. This is already of practical importance in application, especially to loudspeakers. A direct radiating moving-coil loudspeaker has a low frequency resonance and the radiation impedance of the air is predominantly reactive at low frequencies; the resonance excited by impulses does not die out quickly unless it is adequately damped. If the mechanical resistance of the loudspeaker and air load is R at resonance and the electrical resistance of the whole input circuit is r (both R and r in CGS units), the effective mechanical resistance at resonance will be

$$R+\frac{A^2}{r}$$

where A is the force factor. Hence, the importance of operating the loudspeaker from a circuit having an effective a-c resistance comparable with the resistance of the receiver coil in order to obtain a useful contribution to damping from the term A^2/r (see Bibliography,[15] Chapter 7). In other cases, e.g., electromagnetic receivers, the effective mechanical impedance of a system with mechanical impedance Z_m and electrical impedance $r+jl\omega$ is

$$Z_m+\frac{A^2}{r+jl\omega}=$$

$$Z_m+\frac{A^2(r-jl\omega)}{r^2+l^2\omega^2}$$

Whence it is seen that the inductance of the winding, operated on by the force factor, leads to a negative reactance in series with the mechanical impedance, and this may be made use of to annul the mechanical mass reactance in the upper frequency range.

Future Developments

Future development in microphones and reproducers may be expected to keep pace with any further developments in magnetic materials if the surprising advances of the last decade in this direction continue; smaller or more efficient microphones become possible as the materials available improve.

Loudspeakers, with the urge of large-scale manufacture, may be expected to improve in quality. While it is comparatively easy to secure a uniform manufactured product with the simplest construction, progressive requirements demand something less simple for their fulfilment. A practical step has been the use of two loudspeakers to cover different parts of the frequency range, but the tendency will probably be more towards better response in a single system. This entails essentially greater complication and more meshes in the equivalent network, and may take the form of either a conical radiator so constructed that it is in fact two coupled radiators or two coils elastically coupled, driving either a single or double conical radiator. Descriptions of both double-coil and double-cone loudspeakers will be found in the Bibliography.[15]

With the complete control of the equipment in the hands of the designer of domestic radio sets, there is every justification of the use of negative feed-back, in such a manner as to minimise harmonic distortion throughout and improve the loud speaker frequency and transient response.

Whatever means may be adopted, it is likely that the frequency range of high quality reproduction will extend to the utmost limits that congestion of wave lengths will allow. A recent pronouncement[19] gives 40 db as the volume range required for speech and 65 db for music with a frequency range 60 to 8,000 cps for speech and 100 to 7,000 cps for music. It is stated, however, that stereophonic transmission over two channels limited to 5,000 cps is preferred to single-channel reproduction up to 15,000 cps so that development of stereophonic reproduction must ultimately be expected as giving a better performance for a given wave length occupancy.

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Quartz Crystals—Development and Application

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Introduction

A QUARTZ crystal oscillator blank is a small, unimpressive looking device which can easily be mistaken for a piece of ground glass, but it is playing a role in World War II comparable to dive bombers and block busters. Since many such crystals are employed in the radio equipment of a single flying fortress, some idea can be formed of the quantity of quartz being used in the present conflict. As a matter of fact, it has been estimated that, in its single crystal plant in Newark, New Jersey, the Federal Telephone and Radio Corporation is now manufacturing quartz crystal oscillators at a rate exceeding the total combined production facilities of the U. S. A. prior to December, 1941.

The manufacture of quartz crystal oscillator blanks is a precision process as exacting as the cutting and grinding of a fine gem or expensive lens. Because of its hardness, tools similar to those used by the optician and lapidary are employed to convert the raw quartz into finished oscillator blanks. Early methods of manufacture depended largely on skilled hand methods. Rough blanks were cut from the raw quartz and these were ground to the desired frequency on flat glass or metal plates covered with an abrasive paste. Each operator was equipped with a miniature radio transmitter with indicating instruments to determine the frequency and activity of the crystal. This process was slow and tedious, requiring constant grinding, washing, and checking; frequently the quartz became inactive before the desired frequency was attained. Although an experienced lapper could often devise methods of edge grinding the crystals so that they would oscillate again, the process was one of "cut and try" and many blanks had to be discarded after hours had been spent on them. These methods naturally made crystal oscillators expensive and did not lend themselves to mass production.

While research was carried on to find improved cuts for crystals, much study was devoted to determining the causes of erratic action during the grinding process. Formulas were derived for the correct proportions of thickness, length, and width, as well as the proper cutting angle to produce the desired results. X-ray and polarized light tests were introduced to aid in selecting quartz that would produce the required characteristics, and to yield a greater quantity of finished crystals. These efforts have not only resulted in improved crystals, but crystal production has been rapidly accelerated.

This rationalization of quartz crystal oscillator manufacture has been extremely fortunate in connection with the war effort. Manufacturers of crystals have pooled their data, and crystals are now being produced by mass production methods, thus releasing considerable manpower. Where formerly a relatively small group of experts turned out crystals in small quantities by skilful hand methods, many new machines have taken their place. The whole process from raw crystal to the finished oscillator has been reduced to straight line production which is now conducted almost exclusively by unskilled female operators. Training problems have, consequently, been reduced to a minimum, and the work has become particularly appealing to women since it is mainly light, clean, and non-hazardous. Before the war, men were employed extensively, but the few skilled operators who have not been called to the armed forces are still employed for grinding precision crystals and standards which must be made by hand methods. Men also still handle and grade the heavy shipments of raw crystals.

Although the crystal industry is relatively new, it is already beginning to rival the fabulous traditions of the great lens makers whose achievements in developing exact formulas and high efficiency lenses make an arresting chapter in the history of the growth of civilization. To review the highlights of quartz crystal development and application, and to describe the current status of the manufacturing processes, is the purpose of the present article.

Uses

In World War I, the only application of oscillating quartz crystals was as supersonic resonators for submarine detection—crystal-controlled radio transmitters and receivers were not developed until after the war. In the supersonic resonator, the quartz crystal acts as a mechanical source of oscillations or vibrations, much like a loudspeaker diaphragm, but on higher frequencies than can be detected by the human ear. These supersonic vibrations are generated below the surface of the water, the water acting as an efficient medium for their transmission. When they strike an object, such as a submarine, they are reflected—the time lag in the return of the reflected waves to the point of origin determining the distance. Supersonic waves are employed in submarine detection and depth sounding since they are directional and can be concentrated into beams similar to light waves.

As is well known, quartz crystal oscillators now control the frequencies of broadcast transmitters, keeping each station exactly on its assigned frequency so that the station does not wander over the frequency spectrum and interfere with other programs. They are also used for frequency control of carrier telephone systems, making numerous simultaneous conversations on a single long-distance circuit possible without interference. Quartz crystal filters, in turn, help unscramble these conversations at the receiving end of the circuit so that each telephone subscriber hears only the conversation intended for him.

Quartz crystals have become indispensable to control frequency standards, since they permit a degree of accuracy obtainable in no other way. It is for that reason, since frequency is a function of time, that the most accurate master clocks are crystal-controlled. The precision attained in this manner is, in fact, such that radio time signals broadcast from the U. S. Bureau of Standards in Washington, D. C., are no longer considered of prime accuracy more than 150 miles away from the official time transmitting station. Although the radio waves travel with the speed of light, the lag beyond 150 miles becomes greater than the fundamental accuracy of the time signal.

Crystalline quartz is also employed in manometers to measure deviations in pressure. This use depends on the fact that an electrical potential is generated when the crystal is compressed—the magnitude of the voltage generated being proportional to the crystal pressure. This principle has an important wartime application in an instrument for measuring pressure developed in a gun barrel on discharge, making it possible to predetermine the probable distance a projectile will travel when propelled by a given charge on a specific trajectory.

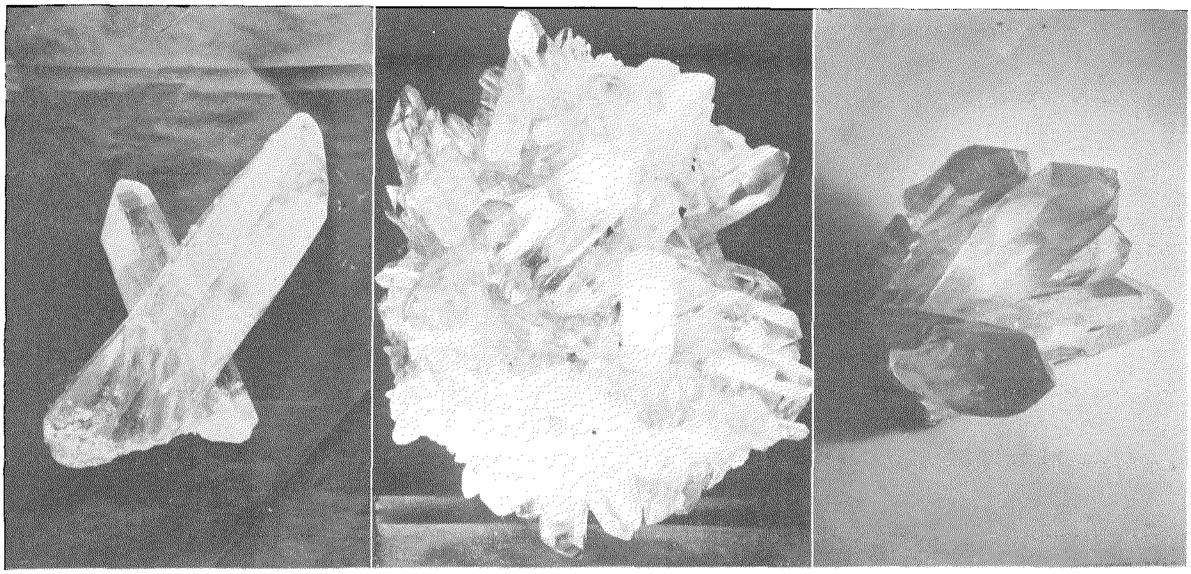
Besides their use as oscillator controls and filters in radio transmitters and receivers, frequency standards, carrier telephone circuits, and as pressure indicators, quartz crystals are employed in optical instruments as lenses and prisms, as pivot supports, and as balance weights for delicate apparatus. Several varieties are cut into gems and ornaments. Their high refractive value and their extreme transparency provide a sparkling jewel comparable to the diamond. In fact, the water-clear crystallized variety of quartz was known to the Greeks as "clear ice." It was supposed by them to have been formed by the intense cold of the Alps; hence the name crystal or, more commonly, rock-crystal applied to certain varieties. The name "quartz" is an old German word of uncertain origin first used by G. Agricola in 1529.

TABLE I

Percentage of Quartz Crystal Exports from Brazil

Year	U. S. A.	Japan	Germany	Great Britain	Others
	%	%	%	%	%
1924	2.03	64.28	27.98	3.44	2.27
1925	4.1	56.44	20.94	7.52	11.00
1926	7.36	52.78	19.94	6.57	13.35
1927	6.22	45.09	30.16	4.68	13.85
1928	8.35	51.70	11.93	6.39	21.63
1929	4.00	53.13	22.35	0.84	19.68
1930	6.24	58.51	7.63	3.70	23.92
1931	3.17	82.83	2.84	0.67	10.49
1932	2.74	86.45	5.31	0.72	4.78
1933	2.56	85.41	0.51	6.09	7.11
1934	3.24	78.09	2.12	3.82	12.73
1935	2.40	73.00	3.48	12.98	8.22
1936	5.52	68.18	10.30	5.67	10.33
1937	5.81	68.17	9.94	13.38	2.70
1938	4.36	58.62	12.88	21.93	2.21
1939	4.14	56.44	13.61	24.13	0.52
1940	5.50	40.52	3.27	47.35	3.36
1941*	41.42	21.80	14.99	21.51	0.28

* Cover period January–September only.



Courtesy of M.A.D.N.P.M.-Divisão de Geologia E Mineralogia, Brazil

Fig. 1—Several Interesting Formations of Crystals Found in the Quartz Mines of Brazil.

Brazil Principal Source of Quartz

Quartz is the commonest of minerals and is met with in a great variety of forms with very diverse modes of occurrence. Quartz crystals large enough to be used as mother crystal or raw material for radio oscillator plates have been found in the United States, Canada, Australia and Madagascar, but Brazil is the principal source of commercial quartz.

The importance of quartz in wartime is strikingly illustrated by Table I. Percentages of total exports from Brazil to countries directly involved in World War II show that shipments rose sharply in the war years.

Brazil evidently was an area of geologic disturbance particularly favorable to the formation of quartz crystals since not only are suitable crystals found there in greater quantities, but the largest crystals have also been discovered in that country. In Bahia, Brazil, two crystals were mined weighing 4,410 lbs. and 2,866 lbs., respectively. Another crystal, 45.276 inches long and weighing 1,945 lbs., is on permanent exhibition at the Bahia State Capital. The largest known quartz crystal was found at Teofilo Otoni in the state of Minas Geraes, Brazil, in 1939. This great crystal weighs 10,363 lbs. It is now being exhibited at the Belo Horizonte State Capital in Brazil.

But most crystals suitable for commercial use weigh from 100 grams to eight pounds and are usually not much larger than a man's fist. The mines yielding the greatest proportion of the commercial quartz vary in depth from about 20 to 80 feet. Some crystals are also found on the surfaces of dry river beds. The river bed crystal, called river quartz, is often preferred for oscillators since it tends to yield a higher percentage of finished oscillator blanks or plates. River quartz is usually reddish in color externally and, because it has been swirled in river beds by the waters of many centuries, its crystalline corners are rounded. Since it is not clear-surfaced and has no flat sides its optical axis is a little harder to find, making the determination of the proper cutting angle for river quartz more difficult. But its relative absence from twinning more than compensates for the difficulty of processing.*

Before the war seriously curtailed shipping from Brazil, raw quartz was sent in bulk to the U. S. A. where crystals suitable for oscillators were selected and the remainder used for other commercial purposes. Now, however, crystals are being selected in Brazil and, in certain emergencies, flown to the United States by air express, so pressing is the need for oscillator material.

* Twinning is the result of one crystal growing within another, either totally or partially. The twinned portions of crystals cannot be used for oscillators.

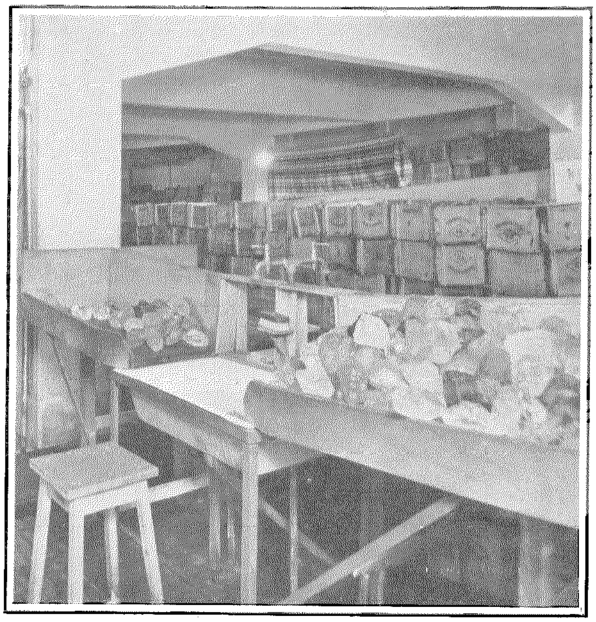
It is estimated that the output of quartz crystal oscillators in the U. S. A. in 1943 exceeded 10,000,000. The output in 1939 was only 20,000.

Properties

Quartz is a compound of silicon and oxygen (SiO_2), harder than glass and not so hard as the diamond. (Quartz has a rating of 7 in Moh's scale of hardness; the diamond 10.) Like the diamond, it is the result of a natural process of growth. Also, like the diamond, it is possible to produce quartz synthetically, but synthetic quartz cannot be used for oscillators, and this laboratory variety has been produced only in tiny beads at a much higher cost than natural quartz. Production of quartz in the laboratory has been recorded by at least thirty investigators in the last one hundred years. Crystals thus formed have varied in size from a few microns to eight millimeters.†

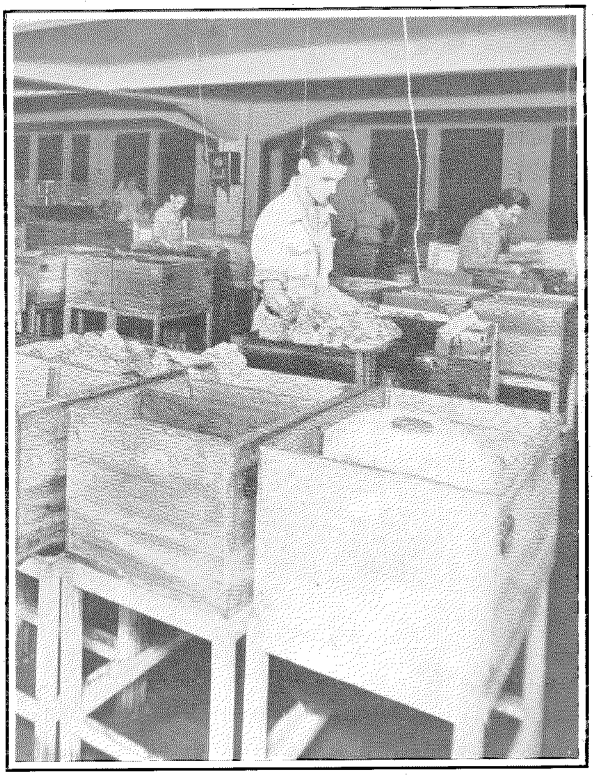
At high temperatures, quartz melts and fuses into a hard, glasslike substance having many uses. Quartz not suitable for other purposes is so fused and may then be drawn into the finest elastic fibers for suspending mirrors in precision indicating instruments. The fused quartz is also used for insulators and for the bulbs of mercury vapor lamps where its high melting point and its ability to transmit ultra-violet light make it greatly superior to glass. The hardness of quartz makes it a valuable grinding and polishing material for the abrasive in sandpaper and scouring soaps.

† "Recorded Experiments in the Production of Quartz," by Paul F. Kerr and Elizabeth Armstrong, Columbia University, April 1942.



Courtesy of Empresa Brasileira de Cristais, Brazil

Fig. 2—Suitable Quartz Crystals for Oscillators are Selected from the Bulk Quartz as it comes from the Mines. The Quartz is First Washed Clean to Free it from Mud and Clay as Shown Above.



Courtesy of Empresa Brasileira de Cristais, Brazil

Fig. 3—The Brazilian Operators are Examining the Crystals for Excessive Twinning and Other Flaws and Classifying them.

Piezo-Electric Characteristics

Quartz crystals sustain or control oscillations at fixed frequencies by means of what is known as their piezo-electric effect, that is, the ability of certain types of crystals to emit electric charges when pressure is applied to them. First public recognition of this phenomenon was recorded in a paper published by A. E. Becquerel in 1828 describing the study of measurements made by him of the electrical effects produced by different crystals when they were subjected to mechanical pressure. Evidently these experiments were considered of little value at the time, because it was not until 1880 that Jacques and Pierre Curie published their paper on the subject. The Curies had made actual measurements of the quantities of electricity produced by unit pressures parallel to the directions of the principal axes of various crystals. One year later, Gabriel Lippmann showed mathematically that the same substances probably would be mechanically deformed if exposed to an electrical field. The Curies promptly verified Lippmann's conclusions in their laboratory and this second phenomenon became known as the converse effect. It was not until after the later experiments of the Curies that the expression "piezo-electric" was applied to the mechano-electrical qualities of crystals by Dr. W. C. Hankel. Dr. Hankel obtained piezo from the Greek word *piezein*, meaning to press.

In a sense, the results of these experiments were surprising since quartz is one of the best known insulators of electricity. Oscillations in a quartz crystal are the result of both effects—the piezo-electric effect and the converse effect.

These reactions may be compared to that of a freely suspended pendulum. When the pendulum is raised, the work done lifting it is similar to the electric charge placed on the quartz crystal. When the pendulum is released, the stored energy is sufficient to carry the arm beyond the point of equilibrium so that oscillations occur, the period of oscillation being controlled by the length of the pendulum arm. In like manner, when the electric pressure on a crystal is released, the quartz attempts to return to its normal size and shape, but its elasticity carries it beyond the steady normal state, causing a slight distortion in the opposite direction which in turn generates

Courtesy of Empresa Brasileira de Cristais, Brazil

Fig. 4—After the Crystals Most Suitable for Oscillators are Selected, They are Weighed and Packed for Shipment.

an electrical potential opposite in polarity to the impressed potential. Like the pendulum, the crystal will continue to oscillate as long as the loss of energy in friction or heat is supplied from an external source; in the case of a crystal oscillator, from the vacuum tube circuit associated with it. The period of oscillation of the crystal, like that of the pendulum, is dependent upon the dimensions of the crystal.

While some forty types of crystals exhibit piezo-electric qualities, only three are commercially useful for this purpose. Rochelle salt crystals are as much as 1,000 times more piezo-electrically active than quartz, but rochelle salt is relatively soft, moisture sensitive, and has too low a frequency constant for high frequency applications. However, rochelle salt crystals have had wide commercial application in phonograph pickups, microphones, and even as the actuating mechanisms of small loud speakers and telephone receivers.

Tourmaline is another piezo-electric crystal which lies between rochelle salt and quartz in activity and stability. It is much more expensive than quartz and its only distinctive quality is that it has about a 50 percent higher rate of vibration for a given thickness than quartz. While it was at one time considered for high frequency oscillators, later research revealed that a quartz crystal could be made to vibrate at multiples of its fundamental frequency and quartz, therefore, took precedence over tourmaline except for certain special applications. Quartz, because of its hardness, permanence, temperature stability, and relatively low cost, is the preferred material for oscillator crystals and it is now being used extensively for this purpose.

Circuit Applications

Almost one hundred years passed after the work of Becquerel before the quartz crystal was used as a frequency controlling device. In 1922 Prof. W. G. Cady of Wesleyan University, Connecticut, U. S. A., developed his theory of the use of quartz crystals as resonators and applied them to frequency measurements. Cady found that a piezo-electric crystal could be made to maintain oscillations of a vacuum tube at the vibrating frequency of the crystal. Like many other developments in communications, this use of crystals had to await improvement of the vacuum tube and vacuum tube circuits. It was the relay or trigger action of the thermionic valve which made it possible to amplify the feeble charges generated by the vibrating crystal to useful proportions. Cady's first application of the quartz crystal oscillator was interesting since it depended upon the transfer of energy from electrical to mechanical and thence from mechanical to electrical through the crystal itself. That is, the crystal acted as a mechanical converter of electrical energy much like a motor-generator set. The crystal holder had four electrodes, one pair for input and the other pair for output. Actually, the vacuum tube was the oscillator and the crystal supported these oscillations at the crystal frequency. This method of employing the crystal was not, however, too reliable since it was possible to remove the crystal entirely and the tube would continue to oscillate so that, whether the crystal controlled oscillations or not, they could be sustained by the vacuum tube circuit alone. Cady later found that four electrodes were unnecessary—that the crystal would function with two electrodes, the single pair being employed both for input and output.

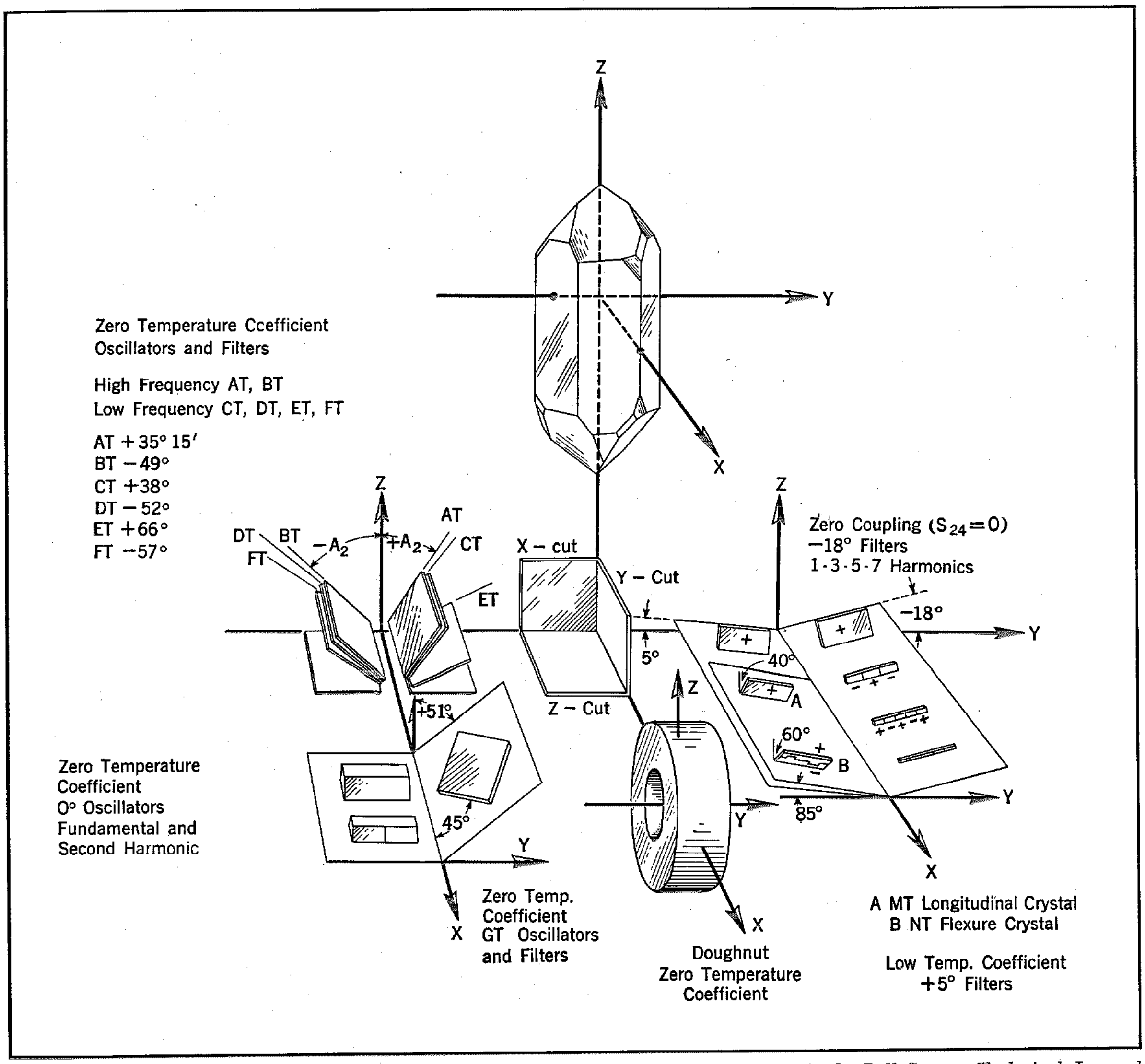
To overcome the difficulty of Cady's circuits, Prof. G. W. Pierce later connected the crystal in the vacuum tube circuit between the grid and filament with a resistance or choke in the plate circuit. Pierce's circuit could not oscillate without the crystal and it was simpler than the Cady circuits. Pierce further developed his circuit to permit higher output efficiency by substituting a circuit tuned to the crystal frequency for the choke or the resistor. Pierce also developed a circuit with the crystal connected between the grid and plate.

In 1927 Miller and Crosby modified the basic Pierce circuits by connecting the plate to a variable tap on the inductance in the output branch of the circuit to permit the impedance to be matched to the input of the following circuit so that maximum power transfer resulted. Later, harmonic amplifiers and cascade tuning, permitting doubler action in the oscillator circuit, resulted in improved efficiency and flexibility of crystal oscillators.

Temperature and its Effect on Frequency

The need of a hard, stable material for crystal oscillators is reflected in the necessity for physical stability. The importance of physical stability will be understood when the method of attaining and holding a certain frequency with a quartz crystal is considered. The natural frequency of an organ pipe is dependent largely upon the size of the air column enclosed by it (small pipes for the higher or treble frequencies; large pipes for the lower or bass frequencies). In the same manner, large bells emit deeper tones than smaller ones; and, similarly, the frequency of vibration of a tuning fork is usually inversely proportional to its size. All these instruments change tone or become detuned if subjected to extreme temperature variation, since a sufficiently large difference in temperature results in an appreciable change in the size of the vibrating medium. The frequency of a crystal is also determined by its size and, in like manner, by its temperature.

This tendency of quartz to change frequency as the temperature varies was a troublesome factor in early crystal oscillation circuits. The



Courtesy of The Bell System Technical Journal

Fig. 5—Oriented Quartz Crystal Cuts in Relation to the Natural Crystal.

crystal temperature goes up after the oscillator is turned on because vibration of the crystal causes friction between the molecules and develops heat. However, a high ambient temperature can also cause frequency variation. The temperature-frequency coefficient of the first crystals was about 0.0025 percent per degree Centigrade, too much for satisfactory radio application. Besides, action of the crystal was likely to be erratic at certain temperatures with the crystal frequency varying 0.0025 percent per degree Centigrade for a few degrees and then jumping one or two percent in the next degree.

A solution to temperature deviation was found in the development of ovens for the crystals. These ovens kept the temperature of the crystals above that likely to develop in use. Elaborate systems of maintaining this temperature within close limits tended to keep operation of the crystal extremely stable.

Broadcasting transmitters and other large fixed stations are equipped with such crystal ovens which, with heaters, thermometers, and control apparatus, make up a bulky portion of the transmitter. They do, however, maintain frequency stability within very close limits—as close as 0.0001 percent in some cases. Crystal frequency standards, the design of which emphasizes close frequency tolerance rather than power output, maintain their stability within even a smaller percentage. Since the weight and bulk of crystal ovens made them unsatisfactory for portable, and especially aircraft, radio equipment, it became desirable to produce a crystal with considerably less frequency variation due to temperature change. Work proceeded, therefore, on the crystal itself to see what could be done to reduce temperature variation so that use of ovens could be avoided.

Zero Temperature Coefficient Crystals

The first quartz crystal oscillator plates were cut parallel to the optical and the mechanical axes and were known as X-cut, Curie-cut, or zero-degree-cut crystals. They had good frequency stability but they were difficult to mount. Very loose mounting plates, leaving a small air space between the crystal and the plate, had to be employed so that the crystal could vibrate freely between them. Any application of pressure to the plates in order to mount the crystal more securely would interfere with free vibration and so prevent the crystal from going into oscillation.

The Y-cut, parallel-cut, or 30-degree-cut crystal was developed in 1925–26 to provide a more active oscillator. This crystal had a shear mode of vibration instead of the accordion-like vibration of the X-cut crystal. It could, therefore, be clamped securely between the electrodes so that the method of mounting was simpler and more rigid. The Y-cut crystal, however, possessed a relatively high temperature coefficient, 0.0085 percent per degree Centigrade change in temperature instead of 0.0025 percent per degree possible with X-cut crystals, and was more prone to spurious responses.

Between 1930 and 1936, extensive research was carried on with quartz crystals in laboratories all over the world. Experiments were made with crystals cut askew of normal axes to reduce temperature effect. During that period, the Bell Telephone Laboratories developed the A and B cut for crystals and the Radio Corporation of America V_1 and V_2 crystals. These were low drift crystals whose temperature coefficients were 0.0003, 0.0002, and even 0.0001 percent per degree Centigrade. Some crystals were cut that even had a practically zero temperature coefficient for narrow temperature ranges. Fig. 5 illustrates numerous possible cuts for producing various degrees of stability and activity.

The results of one series of tests to reduce the temperature effect by angular cutting have been described by Koga who pointed out that, since the temperature coefficients of frequency of X-cut and Y-cut quartz plates are negative and positive, respectively, the possibility of getting a zero temperature coefficient plate by cutting quartz in some orientation intermediate between the X-cut and Y-cut merited exploration.[1]

All these experiments resulted in the production of crystals stable enough to replace the oven-controlled type crystal in many forms of radio transmitters. Since satisfactory results were then possible without the oven and its associated control equipment, crystal operation of portable and aircraft transmitters and receivers became practical. The immediate value to aviation was to provide much more reliable communication facilities between the plane and the ground with

[1] "Notes on Piezo-electric Quartz Crystals," *Electrical Communication*, April, 1936.

radio equipment simple to operate and to maintain in operation without attention from the pilot whose time and efforts were taken up with the plane controls.

Absence of the oven left the crystal free to collect moisture, especially with the quick changes of altitude and temperature experienced in airplane operation. Since the slightest particle of moisture on a crystal will prevent oscillation, mountings sealing the crystals were evolved. Some of these mountings are merely airtight, while others are similar to metal vacuum tube enclosures where the crystals are inserted and the air evacuated or the enclosure is filled with an inert gas. Crystals mounted in this manner weigh no more than small vacuum tubes; they are rugged, easily changed to permit crystals of different frequencies to be used, and their temperature coefficients are very low so that they may be depended upon to maintain frequency under widely varying conditions.

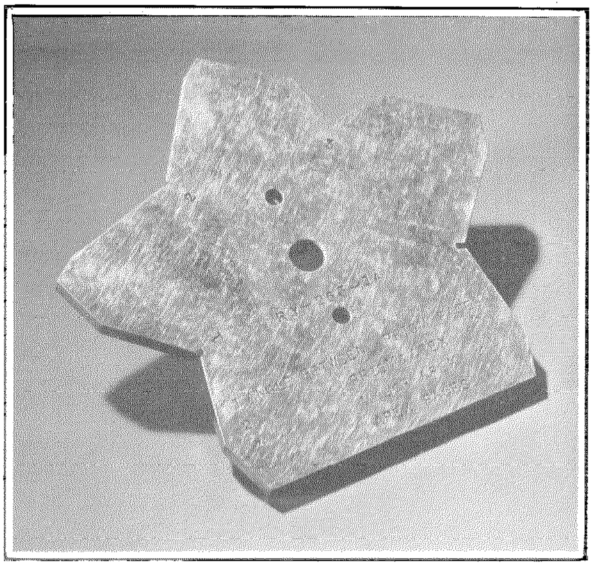


Fig. 6—Special Tool Developed by the Federal Telephone and Radio Corporation for Identifying the Faces of Quartz Crystals.

Mechanical Shop Production

The following brief description of crystal production at the Federal Telephone and Radio Corporation plant will illustrate the degree of development in crystal manufacture, since the methods used are typical of the industry.

Most of the raw crystals received from Brazil weigh under two pounds. While they have been selected for their possible yield of oscillators (crystals with obvious flaws, excess twinning, or other faults being rejected before shipment), they must still be sorted according to shape or size. While quartz crystals are hexagonal and uniaxial in shape, perfectly formed crystals are seldom found and often only a piece of the whole crystal is mined. Three different shapes are generally classified—pyramids, tips or caps, and irregulars, the latter being crystals having no distinct faces. They may also be classified as faced and unfaced. Many crystals are received with faces so mutilated that they are difficult to classify. For this purpose, Federal has developed a special tool, shown in Fig. 6, which is used as a template to identify the faces. As the angles between the various faces are determined by the crystaline structure, this tool provides a foolproof means of determining the nature of the raw crystal being checked. Federal's angle face tool has greatly speeded this process both at the Company's own plant and at a number of other crystal plants throughout the country.

Determination of the original shape of the uncut or raw crystal is necessary so that it may be mounted on the proper face for cutting. The crystals are fastened to glass blocks with a plastic cement. Irregulars, having no mounting faces, are tested with polarized light to determine their optical axis and a face is cut at right angles to this axis. On this face an X axis is determined so that a mounting surface may be cut along the optical axis and at right angles to the X axis.

Two other characteristics of the raw quartz must be determined before it can be set up and cut into blanks. Whether the plane of optical polarization rotates to the right or left, known as the crystal's "handedness," is found by cutting a small piece of the tip off the crystal and examining it in polarized light. Electrical polarity of the crystal is determined by pressing it along the X axis and noting the direction of the piezo-electric charge on a special vacuum tube voltmeter.

The rock crystals, all mounted on blocks and sorted according to shape, handedness, and polarity, are now ready to be cut into blanks. Each group requires special orientation in the slicing machine. Regular circular diamond cutting blades were heretofore used to slice the quartz. These blades had diamond dust moulded into slots on their periphery by combination of the diamond dust with a special plastic. However, a new type of blade, which was first tried at Federal and found to be much superior to the older type, is now used. The new blade is a steel disc with the diamond dust actually electroplated to its edge. The dust is deposited on the rim to form a cutting edge which wears away very much slower than on the older type and makes a quicker and much cleaner cut.

After the crystal is set up on the orientation head of the saw at the proper angle, a test cut is made. The piece cut, known as a wafer, is checked by means of a special X-ray machine designed for crystal production. X-rays give the exact relationships of the atomic planes in the crystal, and the correct angle to cut for a given type of crystal can be determined directly from the X-ray goniometer dial. The orientation head is then adjusted, if necessary, and a second cut made. The second wafer is X-rayed to check the adjustment. The third cut is also X-rayed and, if it checks for the proper cutting angle, which it should if adjustment has been correct, cutting is continued on the remainder of the raw quartz piece with every fifth or tenth wafer checked by X-raying. The ultimate frequency of the crystals roughly determines the thickness of the wafer.

The wafers are then cleaned with soap and water and etched. Formerly hydrofluoric acid was employed for etching, but hydrofluoric acid is hazardous to use and Federal has now substituted a product known as "Quartz-Etch"

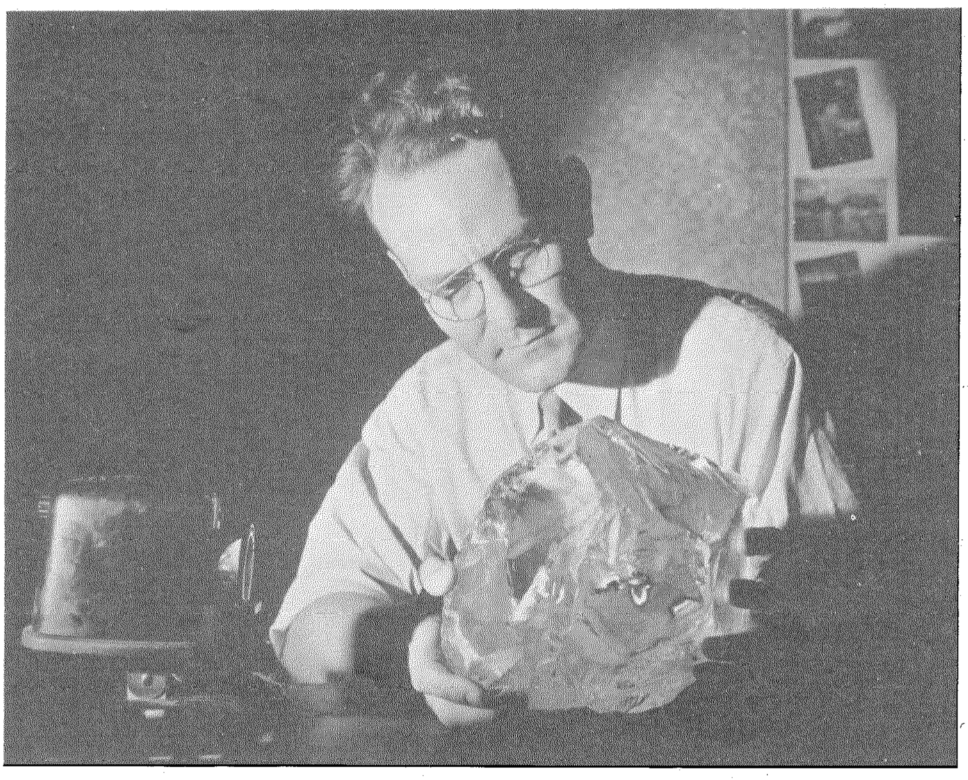


Fig. 7—Examining a Large Piece of Raw Quartz for Flaws Prior to Processing it at the Federal Telephone and Radio Corporation Crystal Laboratories in Newark, New Jersey.

for this step. Quartz-Etch presents no hazards in use and it provides an etch equal to that formerly obtained with hydrofluoric acid.

After the wafers are etched, they are examined in ordinary light adjusted at a critical angle to determine the usable portions. Twinned areas and flaws are marked out. The wafers are then placed under templates and as many squares as possible, slightly larger than the final blanks, are marked on the usable portions.

An operator then dices the wafers, that is, she scores them with a special cutter and breaks them up into the small squares or oblongs which were previously marked under the mask. Unusable material is discarded. The rough blanks are piled into small pillars and glued together with a special wax compound. Two sides of these pillars are ground square on a grinding wheel and the small blanks are separated again.

A new type machine is now in use at Federal to cut the edges of the blanks down exactly to size with all four edges square. These machines have surface wheels faced with diamond dust deposited electrically in the same manner as on the new diamond blades. It has also been found satisfactory to square and dimension all four edges while the blanks are still in pillar form. In this process, the first and third or second and fourth edges are automatically lapped exactly to size on rotary lapping machines. The crystal blanks, after being brought down to uniform size by either method, are ready to be surface machine ground or lapped, the process used to bring the thickness to a dimension known to be approximately correct for a given frequency.

The blanks are surface ground on a lapping machine designed especially for the purpose. Three steps of grinding are now employed using coarse, medium, and fine abrasive with each step timed by means of time switches on the lapping machines. The fine abrasive step is accurately timed so that the crystal frequency is brought to within fifty kilocycles of the ultimate frequency, that is, blanks coming from the operation vary in frequency from the correct frequency to minus 50 kilocycles of the correct frequency. This has been found to be the optimum tolerance for mechanical fabrication as the process has been developed at Federal.

Following surface lapping, the crystals are tumbled to round all edges slightly. This is accomplished by placing several hundred blanks into a long glass tube containing powdered abrasive. The tube is then fastened in a machine that teeters and revolves it thoroughly, mixing the crystal blanks and abrasive in much the same manner as a tumbling barrel used for removing burrs from small metal parts. The crystals with their edges rounded are easier to handle in later processes and less likely to be damaged by chipping.

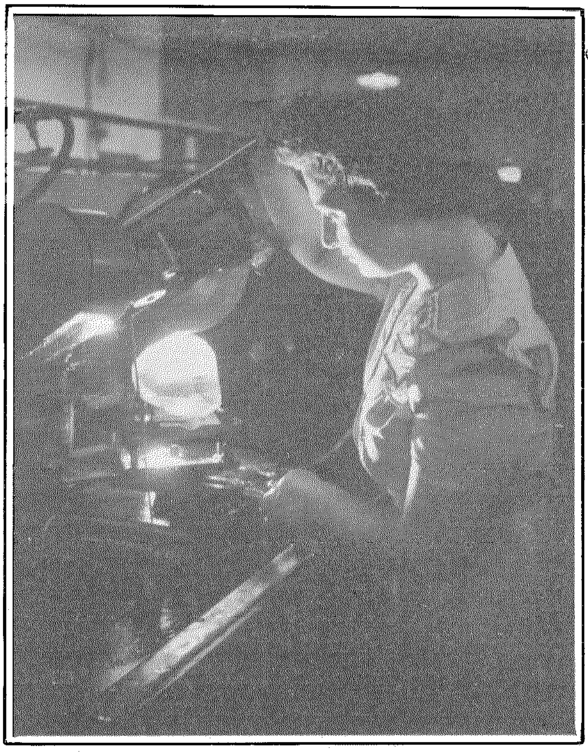


Fig. 8—Operator at the Federal Telephone and Radio Corporation Crystal Laboratories in Newark, New Jersey, Adjusting a High-Speed, Diamond-Impregnated Blade for the Initial Cutting Operation on a Piece of Raw Quartz.

After the final machine lapping and tumbling, their thickness is measured on an electro limit gage against standard quartz blanks cut to minimum and maximum mechanical limits. This special gage reads within plus or minus two one-hundred-thousandths of an inch.

Production up to this point is entirely mechanical with no attempt to provide anything but blanks accurately dimensioned and cut to the desired axis—both factors established empirically in the laboratory and checked or developed from time-to-time. As described so far, production is virtually an unskilled series of steps which can be carried out by operators who need have no knowledge of the nature of the crystal or its functions.

Electrical Shop Production

In the following steps, the crystal blanks are all checked electrically and physical measurements are no longer employed. In the first of the electrical steps, the crystal is checked against a standard crystal in a scanning radio receiver. The scanning receiver is equipped with a cathode ray tube which visually indicates the frequency of the standard and of the crystal under test, a scale on the C-R tube showing the deviation in kilocycles between the standard and the crystal being tested. The crystal blank must here measure within the 50-kilocycle tolerance established for mechanical fabrication. This is the first test of electrical output and oscillating capability. If the blank does not oscillate, it is turned over to operators who lap the edges to produce oscillation. Since much care has been used to reject material not likely to produce oscillating crystals, few blanks fail to pass the initial oscillating test. Further, relatively few blanks fail to oscillate after proper edge lapping. These few are, of course, rejected.

Up to a short time ago, at Federal, the crystals were then individually adjusted to frequency, an operation classified as semi-skilled. Each blank was hand lapped on a flat glass plate and periodically checked against a standard crystal until deviation from the standard was within 200 cycles when in its holder. The crystals were not actually fastened into the holders by the operators doing the lapping and checking. They were merely laid in the holders for checking; the final process of assembly was carried on in a separate assembly line, thereby allowing the lappers to devote all of their time to actual adjustment of the crystals. In the latter operation, the crystals were fastened into their proper holders, all moisture removed, and the holder sealed.

Recently, Federal has inaugurated a new method of bringing the crystals to frequency. This method substitutes etching for the former process of lapping on a flat glass plate. By etching the crystal instead of lapping it to frequency, the one remaining skilled operation in the entire process of fabricating crystal oscillator blanks has been eliminated. Special technique is no longer required. The crystal is checked against a standard as before to determine its frequency deviation. Then it is dipped into a container of Quartz-Etch. After a given time has elapsed, the operator removes the crystal from the Quartz-Etch, neutralizes the acid by dipping it into a weak soda solution, and washes it in running water. The crystal is then cleaned with a nylon bristle brush to remove contamination, rerinsed, and air-dried with compressed air. To retain cleanliness of the faces, they must not be touched by the operators' fingers. The crystal is then checked against the standard again while in a holder. If activity has dropped, it is restored by grinding the edges slightly on an emery board.

This newer method not only eliminates the need for skilled operators, but it prevents damage to the crystal by grinding the surfaces out of parallel to each other. It also permits an operator to finish many more crystals in a given time than with the old lapping method.

After the crystals finally check for frequency, they are fastened in their holders as previously and then submitted to preliminary temperature deviation tests. This is a belt-line operation with the crystals passed through first a hot box where they reach a temperature of plus 90 degrees C. At this temperature, they are lifted off the moving belt, checked for frequency and output, and returned to the belt. They then pass through a box to bring them to room temperature and are again checked. The final check on the belt line is made at minus 55 degrees C. after the crystals pass through a cold box to bring them down to this temperature. The speed of the belt is so controlled that the crystals take just the right time to bring them to the required temperature while passing through various temperature-changing boxes. Any crystals that fail the frequency or output tests along the line are, of course, rejected.

The next step is a final frequency and output test under varying conditions of temperature. It is the last operation before packing and shipping. At present fifty crystals are tested in a single run between minus 55 degrees C. and plus 90 degrees C. Low temperatures are obtained with dry ice and high temperatures with electric heaters. The whole process is continuous for the

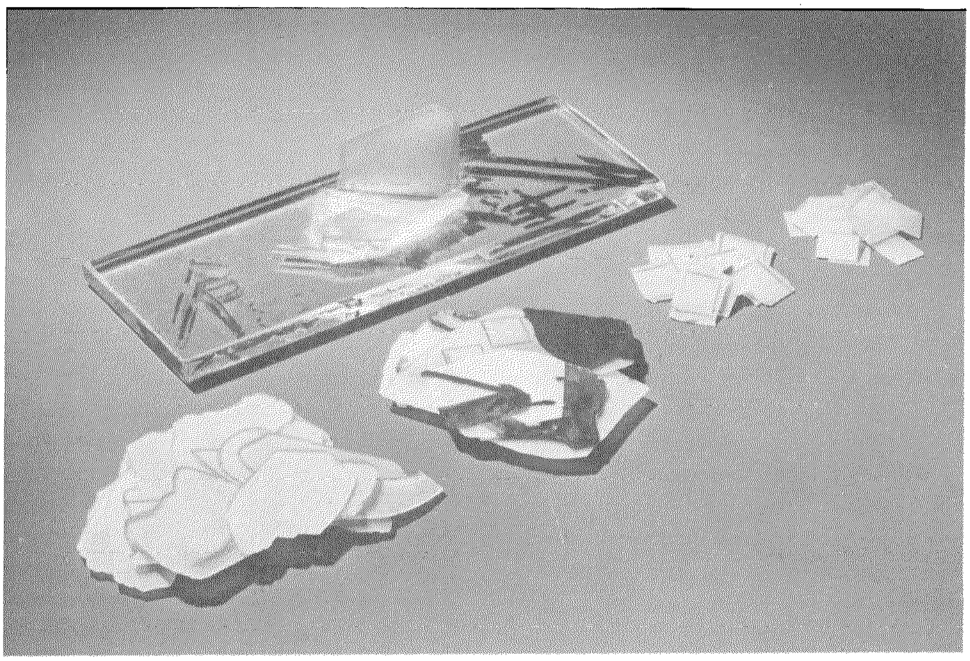


Fig. 9—Various Steps in Production from the Raw Crystal to the Finished Oscillator Blank are Shown Above. The Glass Block Illustrates how the Crystal Blanks are Sliced Out of Raw Quartz. On the Left are Rough Wafers Ready to be Classified. Next are Wafers with Twinned Portions and Flaws Blacked and with Framed Areas of Active Crystal Material. The Third Pile Shows the Diced or Roughed-Out Squares and the Last Pile the Finished Blanks.

fifty crystals in each run, with an operator checking the activity and frequency every two degrees. Rejects are noted on a chart and discarded after the run.

Post-War Developments

As present production of quartz crystal oscillators is entirely for military use many new applications must remain military secrets for the duration of the war. However, wartime use of crystals has stepped up production methods so that crystals manufactured completely on a mass production basis will be available after the war. Development is now being carried on that should soon permit production of crystal oscillators adjusted exactly to frequency by machinery, thus eliminating the few hand methods still being employed.

One development likely to hasten complete machine fabrication is the metal-plated quartz crystal. The crystal may be plated with any metal, but silver is used most often. While plating of quartz crystals is not in itself a new development, the process lends itself ideally to the final steps of machine fabrication. Quartz crystal blanks to be plated are machine-lapped to the permissible tolerance and then etched in quantity to a frequency slightly above the required frequency. Since the plating lowers the frequency, the metal deposit brings the blanks back to the exact frequency or slightly below frequency. Final frequency adjustment can then be accomplished simply by thinning the coating, removing part of the coating, or edge grinding the crystal. Even the edge grinding, in this case, would be done by automatic machines. If the plating is applied in very thin layers, the exact frequency can be reached without this final step. Plating provides a better type of crystal oscillator since it makes possible greatly simplified mountings permitting sturdy, reliable contact to the crystal without increasing the damping or reducing the freedom of the crystal to oscillate.

As each new method of fabricating quartz crystal oscillators materializes, it not only helps to increase production and reduce manpower needs, but it also brings down the cost of the finished product. Good crystals now sell at a fraction of their pre-war price and, through a continuation of the process of rationalization, the price of quartz crystals should reach a very low figure in the not too distant future. Hence, it will be economically practical to use more crystals in post-war radios without increasing their selling cost. For example, home radios could then be equipped with crystals to keep them exactly on the frequencies of the most popular local broadcasting stations. Thus, more reliable tuning would result with pushbutton controls, a feature heretofore provided only on expensive communications and aircraft receivers. This method of holding a receiver on the frequency of a transmitter would be particularly desirable for high frequency television and FM receivers, for, in the portions of the frequency spectrum used in these types of transmission, stability has been difficult to maintain by other means.

Another important line of development is in the design of very small, lightweight ovens for crystals so that temperature control of crystal oscillators becomes practical for aircraft. Thus, airplanes can be equipped with oscillators having stability and accuracy equivalent to secondary frequency standards. Extremely small ovens have been built which are quite as stable as the large ovens used at broadcast stations or in frequency standards. While such a degree of stability has certain essential wartime applications, it may also be expected to extend the usefulness and increase the reliability of aircraft radio aids to navigation. Post-war aircraft radio equipment will undoubtedly be operated on ultra-high frequencies and, with aircraft maintaining frequency stability to the same degree as the airport station, the whole radio system would be capable of completely unified operation within very close tolerances of assigned frequencies—an essential for efficient operation on ultra-high frequencies.

Further uses doubtless will be found for stabilized oscillators, and, for the present at least, such oscillators will derive their stability from the tiny piece of quartz which has become so important in the field of electronics and radio. As has been pointed out, the value of a quartz crystal oscillator blank is a direct function of intricate processes of selection, cutting, and grinding which may conceivably be supplanted by newer and simpler methods or a new substance may even take the place of quartz, one easier to fabricate and possibly synthetic in origin. Perhaps, for example, some plastic will eventually be found that will replace quartz—some substance that could be stamped into oscillator blanks as easily as a printed page can be turned out on a press. Until then we must depend upon nature for our source of raw material and all the acumen of science for the finished product.

Wave Guides in Electrical Communication

By JOHN KEMP, Member I. E. E.
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Editor's Note: This paper was published in The Journal of the Institution of Electrical Engineers, London, September 1943, Vol. 90, Part III (Communication Engineering), No. 11. As most technicians doubtless are aware, recent years have witnessed considerable development and application of wave guides but, for obvious reasons, the author has confined himself to a historical treatment of the subject up to about the period of the outbreak of the present war. It is felt, nevertheless, that the paper will be of interest to those who have not previously studied this subject and desire a general idea of its scope and possibilities. For this reason the I. T. & T. Corporation has reprinted the paper, copies of which will be supplied on request to engineers on the staff of the I. T. & T. and its associate companies, also to their friends, to the extent that they are available.

The following Summary and Table of Contents taken from the paper will give an idea of the scope and treatment of the subject:

Summary

An attempt is here made to survey the state of published knowledge of a branch of electrical engineering that has recently come into prominence and to present an introduction to it in general terms within a range sufficient to explain the development of what in effect is a new technique and its relation to other branches of electrical engineering.

The first part of the paper describes the salient properties of electromagnetic waves in hollow metal tubes and reveals the great extent to which the theory of the subject had its origin in the researches of Lord Rayleigh and other investigators who, during the closing years of the last century, worked in Great Britain. It also indicates to how great an extent the development of the theory and its practical application are due to researches carried out during recent years mainly in the United States.

The second part of the paper gives a systematic account of the development of elementary equipment appropriate to hollow-tube transmission with analogies from acoustics, telephony, and radio engineering.

The third part describes phenomena observed at the open ends of guides—flared or unflared—and exhibits the efficiency of these devices as radiators of energy into free space.

The prospective field of application of guides embraces systems of communication operating over any distance, and providing telephone and television channels in numbers vastly exceeding those of any system of established type. When flared into horns, guides may serve in systems for broadcasting of music or television, for blind landing of aeroplanes, for detecting, locating and manœuvring of ships, and for other purposes for which at present radiators and receivers of conventional type are used. Although the usefulness of such systems may thereby be enhanced or extended in range, their mode of operation is well known and hence their description is beyond the scope of this paper. The full extent of the field of application of guides will ultimately be governed by costs and must, for the present, remain a matter of conjecture; but, judged from the course the development has taken during recent years, conjecture suggests that the field will be large and attractive.

Contents

Intercoupled Transmission Lines at Radio Frequencies

By MORTON FUCHS

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Introduction

Although electrical phenomena often may be explained by the qualitative application of fundamental physical laws, mathematical analysis is usually required to obtain a precise picture of what is happening. This is particularly true of ordinary coaxial and open-wire radio frequency transmission lines. Phenomena of quarter wave resonances, standing waves, and impedance transformations along transmission lines are readily predictable by the application of classical transmission line equations, whereas it is not easy to procure a satisfactory explanation merely by simple picturization.

A similar situation obtains in the field of intercoupled transmission lines such as, for example, coupling power from a transmission line tank of a high frequency amplifier, or the use of a quarter wave coupled section as a cut-off filter. When one pair of lines is placed in proximity to another pair, interactions take place that are best explained by setting up transmission line equations which take this intercoupling into account. The purpose of the present paper is to formulate a system of equations applicable to the practical problems of intercoupling encountered by radio engineers.

The differential equations relating to voltage and current will be derived for a general system of lines and then applied in detail to a symmetrical configuration of four conductors. Boundary conditions will be introduced and typical cases of intercoupling analyzed.

Only steady state conditions will be considered. In order to simplify the equations, the conductors are assumed to be ideal and subject to the following qualifications:

a. Conductors are all parallel and circular in cross-section.

b. Current flows on the outside of conductor only, and is uniformly distributed around the circumference.

c. No series resistance in the conductors and no shunt conductance between conductors.

d. Spacing between conductors is small compared to the electrical wavelength, but reasonably large compared to the radii of the conductors.

e. The individual sum of the currents and charges on the conductors is equal to zero.

At radio frequencies, the above conditions are usually very well satisfied by physical lines.

Method of Derivation

Concepts of distributed inductance and capacitance have been successfully applied to the analysis of coaxial, single line to ground, and balanced transmission lines. In each of these systems the return circuit for the current is clearly defined, and one can set up a definition of conductor inductance and capacitance, depending only on the physical dimensions of the system. However, in a system of intercoupled transmission lines where the current on a particular conductor does not have a single return circuit, but returns on all of the other conductors, the ordinary concepts of distributed inductance and capacitance become inconvenient, and it is then desirable to analyze the system in a more fundamental manner. Differential equations giving the voltage drop along a conductor and the displacement current flowing from a conductor are formulated directly from the magnetic and potential coefficients.

The Magnetic Formula

Referring to Fig. 1, "n" represents the general conductor in a system of "N" conductors,

"p" is a particular conductor,

D_{nn} is the radius of conductor "n,"

D_{np} is the distance between centers of "n" and "p,"

I_n is the current flowing on "n."

At a radial distance "y" from conductor "p,"

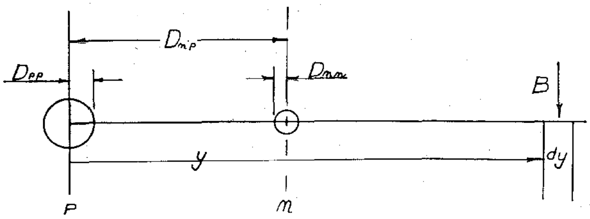

Fig. 1

the magnetic field produced by I_p and I_n is:

$$B=\frac{2I_p}{y}+\frac{2I_n}{y-D_{np}}.$$

The element of area normal to B per unit length of conductor is:

$$dA=dy.$$

The magnetic flux linking "p" is therefore:

$$\phi_p=\int_{D_{pp}}^{\infty}\frac{2I_p dy}{y}+{}_{n\neq p}\sum_{n=1}^{n=N}\int_{2D_{np}}^{\infty}\frac{2I_n dy}{y-D_{np}}.$$

Limits of Integration:

When "y" is less than D_{pp} flux from "p" does not link I_p which flows only on the surface of "p." When "y" is less than $2D_{np}$, flux from "n" does not link "p."

$$\phi_p=2I_p\log_e\left[\frac{\infty}{D_{pp}}\right]+{}_{n\neq p}\sum_{n=1}^{N}2I_n\log_e\left[\frac{\infty}{D_{np}}\right],$$

$$\phi_p=2\sum_{n=1}^{N}I_n\log_e[\infty]-2\sum_{n=1}^{N}I_n\log_e[D_{np}],$$

$$\sum_{n=1}^{N}I_n=0.$$

It may be demonstrated that

$$\sum_{n=1}^{N}I_n\log_e[\infty]=0.$$

Hence

$$\phi_p=-2\sum_{n=1}^{N}I_n\log_e[D_{np}]. \tag{1}$$

The total inductance of "p" may be defined as:

$$L_p=\frac{\phi_p}{I_p}=-\frac{2}{I_p}\sum_{n=1}^{N}I_n\log_e[D_{np}].$$

It is evident that L_p is a function of the currents on each line as well as of the spacing and radii of the lines. Since the currents are unknown, L_p is also unknown and no advantage is gained by its use.

The Electrostatic Formula

In connection with Fig. 2, q_n is the positive charge on a unit of length "n" and V_p is the voltage drop from "p" to infinity. The symbols used in connection with Fig. 1 also apply to Fig. 2.

At a radial distance "y" from conductor "p," the electric field produced by q_p and q_n is:

$$\mathcal{E}=\frac{2q_p}{y}+\frac{2q_n}{y+D_{np}}.$$

V_p is given by:

$$V_p=\int_{D_{pp}}^{\infty}\frac{2q_p dy}{y}+{}_{n\neq p}\sum_{n=1}^{N}\int_{D_{pp}}^{\infty}\frac{2q_n dy}{y+D_{np}},$$

$$V_p=2q_p\log_e\left[\frac{\infty}{D_{pp}}\right]+{}_{n\neq p}\sum_{n=1}^{N}2q_n\log_e\left[\frac{\infty}{D_{pp}+D_{np}}\right],$$

$$D_{pp}\ll D_{np},\quad \therefore\quad D_{pp}+D_{np}\approx D_{np},$$

$$V_p=2\sum_{n=1}^{N}q_n\log_e[\infty]-2\sum_{n=1}^{N}q_n\log_e[D_{np}],$$

$$\sum_{n=1}^{N}q_n=0$$

and it may be shown that

$$\sum_{n=1}^{N}q_n\log_e[\infty]=0,$$

$$V_p=-2\sum_{n=1}^{N}q_n\log_e[D_{np}]. \tag{2}$$

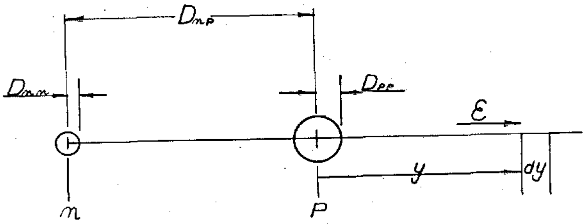

Fig. 2

The capacity of "p" may be defined as:

$$C_p = \frac{q_p}{V_p} = \frac{q_p}{-2\sum_{n=1}^{N} q_n \log_e [D_{np}]}.$$

As in the case of L_p, the capacity of "p" depends on the charges on all the other conductors as well as the physical dimensions of the system. Since q_n is unknown, C_p is also unknown and therefore is of no value.

The Differential Equations and Their Solution for the General Case

Consider the two general equations:

$$\phi_p = -2\sum_{n=1}^{N} I_n \log_e [D_{np}], \quad (1)$$

$$V_p = -2\sum_{n=1}^{N} q_n \log_e [D_{np}]. \quad (2)$$

For steady state conditions, ϕ, q, V, and I are harmonic functions of time; therefore,

$$\frac{\partial \phi}{\partial t} = j\omega\phi, \quad \frac{\partial q}{\partial t} = j\omega q,$$

$$j = \sqrt{-1}, \quad \omega = 2\pi f.$$

Let "x" represent the distance along each line measured from an arbitrary reference plane normal to the lines. At the point "x,"

$$\frac{\partial V_p}{\partial x} = \frac{\partial \phi_p}{\partial t} = j\omega\phi_p,$$

$$\frac{\partial I_n}{\partial x} = \frac{\partial q_n}{\partial t} = j\omega q_n.$$

Since time "t" has been eliminated from the equations, the only coordinate remaining is "x," and the partial derivatives may be written as total derivatives. Resubstituting the above equations in equations (1) and (2), and inserting appropriate electrostatic and electromagnetic units, heretofore neglected, we obtain:

$$\frac{dV_p}{dx} = -j\omega k_L \sum_{n=1}^{N} I_n \log_e [D_{np}], \quad (3)$$

$$V_p = -\frac{1}{j\omega k_C} \sum_{n=1}^{N} \frac{dI_n}{dx} \log_e [D_{np}]. \quad (4)$$

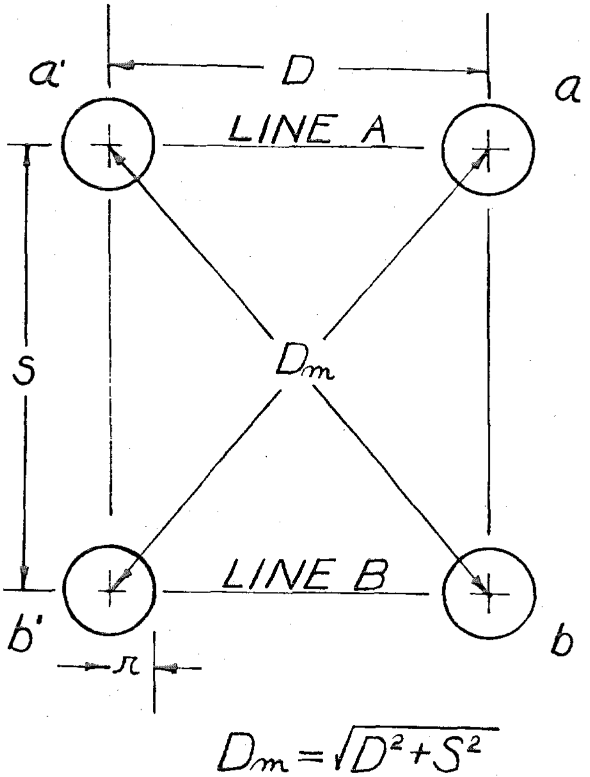

Fig. 3

For free space:

$$\sqrt{\frac{k_L}{k_C}} = 60, \quad \frac{1}{\sqrt{k_L k_C}} = c = 3 \times 10^{10} \frac{\text{cm}}{\text{sec}}.$$

Differentiating equation (4) and substituting (3) to eliminate terms involving voltage:

$$-j\omega k_L \sum_{n=1}^{N} I_n \log_e [D_{np}] = -\frac{1}{j\omega k_C} \sum_{n=1}^{N} \frac{d^2 I_n}{dx^2} \log_e [D_{np}],$$

$$\sum_{n=1}^{N} \log_e [D_{np}] \left[\frac{d^2 I_n}{dx^2} - (j\omega)^2 k_L k_C I_n\right] = 0,$$

$$I_n = C_n e^{j\omega x/c} + D_n e^{-j\omega x/c},$$

$$I_p = C_p e^{j\omega x/c} + D_p e^{-j\omega x/c}. \quad (5)$$

Differentiating equation (3) and substituting (4) to eliminate terms involving current:

$$\frac{d^2V_p}{dx^2}=(-j\omega k_L)(-j\omega k_C)V_p,$$

$$\frac{d^2V_p}{dx^2}-(j\omega)^2k_Lk_CV_p=0,$$

$$V_p=A_pe^{j\omega x/c}+B_pe^{-j\omega x/c}. \quad (6)$$

C_p and D_p may be evaluated in terms of A_p and B_p by resubstituting equations (5) and (6) in (3) and (4), and then equating the coefficients of like exponentials.

Analysis of Two Similar and Symmetrical Pairs of Transmission Lines

Lines A and B (Fig. 3) represent similar balanced open wire transmission lines. Because of geometrical symmetry:

$$V_a=-V_{a'}, \qquad I_a=-I_{a'},$$
$$V_b=-V_{b'}, \qquad I_b=-I_{b'}.$$

Also,

$$V_A=V_{aa'}=2V_a, \qquad I_A=I_a,$$
$$V_B=V_{bb'}=2V_b, \qquad I_B=I_b.$$

Applying equations (3) and (4), and making use of the above indicated relations:

$$\frac{dV_A}{dx}=2j\omega k_L\left[I_A\log_e\left(\frac{D}{r}\right)+I_B\log_e\left(\frac{D_m}{s}\right)\right], \quad (7)$$

$$\frac{dV_B}{dx}=2j\omega k_L\left[I_B\log_e\left(\frac{D}{r}\right)+I_A\log_e\left(\frac{D_m}{s}\right)\right], \quad (8)$$

$$V_A=\frac{2}{j\omega k_C}\left[\frac{dI_A}{dx}\log_e\left(\frac{D}{r}\right)+\frac{dI_B}{dx}\log_e\left(\frac{D_m}{s}\right)\right], \quad (9)$$

$$V_B=\frac{2}{j\omega k_C}\left[\frac{dI_B}{dx}\log_e\left(\frac{D}{r}\right)+\frac{dI_A}{dx}\log_e\left(\frac{D_m}{s}\right)\right]\cdot \quad (10)$$

The solutions are:

$$V_A=C_Ae^{j\omega x/c}+D_Ae^{-j\omega x/c}, \quad (11)$$

$$V_B=C_Be^{j\omega x/c}+D_Be^{-j\omega x/c}, \quad (12)$$

$$I_A=E_Ae^{j\omega x/c}+F_Ae^{-j\omega x/c}, \quad (13)$$

$$I_B=E_Be^{j\omega x/c}+F_Be^{-j\omega x/c}. \quad (14)$$

where C_A, D_A, C_B, D_B, E_A, F_A, E_B, and F_B are constants.

By reintroducing equations (11), (12), (13), and (14) in (7) and (8), or (9) and (10), the constants E_A, F_A, E_B, and F_B may be expressed in terms of C_A, D_A, C_B, and D_B. Accordingly,

$$E_A=\frac{2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D}{r}\right)C_A-2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D_m}{s}\right)C_B}{\left[2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D}{r}\right)\right]^2-\left[2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D_m}{s}\right)\right]^2},$$

$$E_B=\frac{2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D}{r}\right)C_B-2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D_m}{s}\right)C_A}{\left[2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D}{r}\right)\right]^2-\left[2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D_m}{s}\right)\right]^2},$$

$$F_A=-\frac{2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D}{r}\right)D_A-2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D_m}{s}\right)D_B}{\left[2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D}{r}\right)\right]^2-\left[2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D_m}{s}\right)\right]^2},$$

$$F_B=-\frac{2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D}{r}\right)D_B-2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D_m}{s}\right)D_A}{\left[2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D}{r}\right)\right]^2-\left[2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D_m}{s}\right)\right]^2}.$$

By definition

$$2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D}{r}\right)=Z_0,$$

and

$$2\sqrt{\frac{k_L}{k_C}}\log_e\left(\frac{D_m}{s}\right)=Z_m.$$

Z_0 is the surge impedance of each pair of lines in free space when the other pair is far off.

Z_m is the mutual impedance between the two pairs of lines (producing the interaction between the lines).

$$Z_0=120\log_e\left(\frac{D}{r}\right),$$

$$Z_m=120\log_e\sqrt{1+\left(\frac{D}{s}\right)^2}.$$

D, r, and s are lengths which must be expressed in the same units, although any system of units may be used. Also,

$$\frac{j\omega x}{c}=\frac{j2\pi fx}{\lambda f}=j2\pi\left(\frac{x}{\lambda}\right)\cdot$$

Letting

$$\theta=2\pi\left(\frac{x}{\lambda}\right),$$

and rewriting equations (11), (12), (13), and (14) with the proper substitutions:

$$V_A=C_Ae^{j\theta}+D_Ae^{-j\theta}, \quad (15)$$

$$V_B=C_Be^{j\theta}+D_Be^{-j\theta}, \quad (16)$$

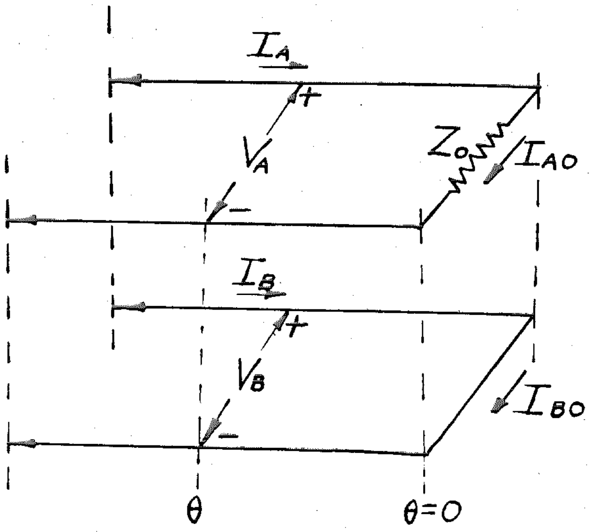

Fig. 4

$$I_A=\left[\frac{Z_0C_A-Z_mC_B}{Z_0^2-Z_m^2}\right]e^{j\theta}-\left[\frac{Z_0D_A-Z_mD_B}{Z_0^2-Z_m^2}\right]e^{-j\theta}, \quad (17)$$

$$I_B=\left[\frac{Z_0C_B-Z_mC_A}{Z_0^2-Z_m^2}\right]e^{j\theta}-\left[\frac{Z_0D_B-Z_mD_A}{Z_0^2-Z_m^2}\right]e^{-j\theta}. \quad (18)$$

Boundary Conditions

A few of the more important boundary conditions merit consideration. In general it is necessary to consider what happens both at the receiving and sending ends of both pairs of lines in order to obtain the complete solution of equations (15), (16), (17), and (18). We shall first consider a case involving only one set of boundary conditions.

Assuming I_{A0} and I_{B0} are the terminating currents in lines A and B respectively, Fig. 4, and applying equations (15), (16), (17), and (18):

At $\theta=0$

$$I_A=\frac{Z_0C_A-Z_mC_B}{Z_0^2-Z_m^2}-\frac{Z_0D_A-Z_mD_B}{Z_0^2-Z_m^2}=I_{A0},$$

$$V_A=Z_0I_A=C_A+D_A,$$

$$V_B=C_B+D_B=0,$$

$$I_B=\frac{Z_0C_B-Z_mC_A}{Z_0^2-Z_m^2}-\frac{Z_0D_B-Z_mD_A}{Z_0^2-Z_m^2}=I_{B0}.$$

Solving these equations for the constants:

$$C_A=I_{A0}Z_0+\frac{I_{B0}Z_m}{2}, \qquad C_B=\frac{I_{A0}Z_m+I_{B0}Z_0}{2},$$

$$D_A=\frac{-I_{B0}Z_m}{2}, \qquad D_B=-\frac{I_{A0}Z_m+I_{B0}Z_0}{2}.$$

Therefore, for any value of θ, from $\theta=0$ until the next boundary condition is reached:

$$V_A=\left[\frac{2I_{A0}Z_0+I_{B0}Z_m}{2}\right]e^{j\theta}-\left[\frac{I_{B0}Z_m}{2}\right]e^{-j\theta},$$

$$V_B=\left[\frac{I_{A0}Z_m+I_{B0}Z_0}{2}\right]e^{j\theta}-\left[\frac{I_{A0}Z_m+I_{B0}Z_0}{2}\right]e^{-j\theta},$$

$$I_A=\left[\frac{I_{A0}(2Z_0^2-Z_m^2)}{2(Z_0^2-Z_m^2)}\right]e^{j\theta}-\left[\frac{I_{A0}Z_m^2}{2(Z_0^2-Z_m^2)}\right]e^{-j\theta},$$

$$I_B=\left[\frac{-I_{A0}Z_0Z_m+I_{B0}(Z_0^2-Z_m^2)}{2(Z_0^2-Z_m^2)}\right]e^{j\theta}+\left[\frac{I_{A0}Z_0Z_m+I_{B0}(Z_0^2-Z_m^2)}{2(Z_0^2-Z_m^2)}\right]e^{-j\theta}.$$

It will be noted that I_A does not contain terms involving I_{B0}. In other words, for a fixed I_{A0}, the current on line A is completely independent of the current on line B. For moderately loose coupling, line A is very nearly a flat line with respect to current distribution.

In a typical case,

$$Z_0=500, \qquad \frac{D}{s}=2,$$

$$Z_m=120\log_e\sqrt{1+\left(\frac{D}{s}\right)^2}=97.$$

ρ_0, the ratio of the backward traveling wave to the forward traveling wave of I_A is given by:

$$\rho_0=-\frac{F_A}{E_A}=\frac{Z_m^2}{2Z_0^2-Z_m^2}=\frac{(97)^2}{2(500)^2-(97)^2},$$

$$\rho_0=0.0191.$$

Q, the standing wave ratio on line A is expressed by:

$$Q=\frac{1+\rho_0}{1-\rho_0}=\frac{1+0.0191}{1-0.0191},$$

$$Q=1.04.$$

Thus, as stated above, line A is essentially flat with respect to I_A. Actually the presence of line B lowers the effective surge impedance of line A from Z_0 to $\dfrac{Z_0^2-Z_m^2}{Z_0}$ and therefore mismatches line A to its termination in Z_0. It must be remembered, however, that the voltage distribution on line A is not flat, and depends on the current on line B.

The Coupled Section Filter

By coupling a quarter wave resonant section into a line transmitting power to a load (Fig. 5), a power transfer at the resonant frequency of the section can be prevented. This is one of the simplest forms of transmission line filters.

$\theta=0$

$$V_B=0=C_B+D_B,$$

$$V_A=C_A+D_A,$$

$$I_A=\frac{Z_0C_A-Z_mC_B}{Z_0^2-Z_m^2}-\frac{Z_0D_A-Z_mD_B}{Z_0^2-Z_m^2},$$

$$V_A=I_AZ.$$

$\theta=90°$

$$I_B=0=\frac{Z_0C_B-Z_mC_A}{Z_0^2-Z_m^2}(j)-\frac{Z_0D_B-Z_mD_A}{Z_0^2-Z_m^2}(-j),$$

$$V_s=V_A=jC_A-jD_A.$$

Solving for the constants:

$$C_A=\frac{-jV_s}{2},\qquad C_B=\frac{-jZ_0V_s}{2Z_m},$$

$$D_A=\frac{jV_s}{2},\qquad D_B=\frac{jZ_0V_s}{2Z_m},$$

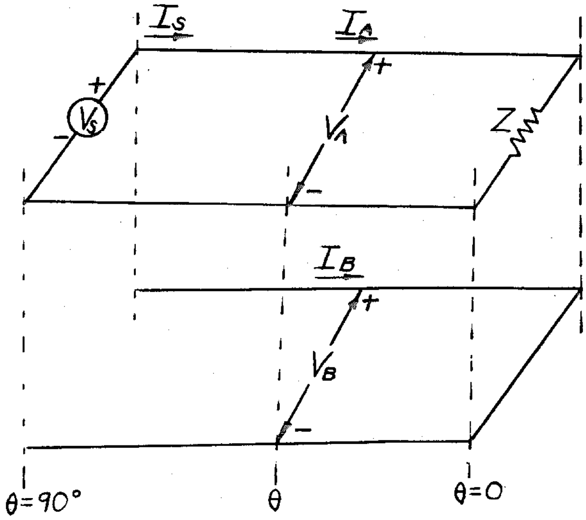

Fig. 5

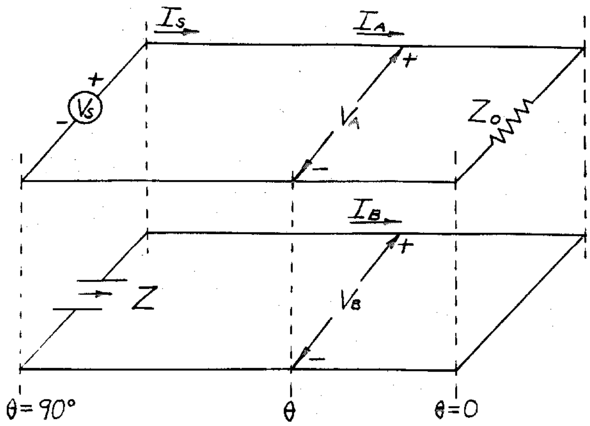

Fig. 6

The currents and voltages on the lines are:

$$V_A=\frac{-jV_s}{2}e^{j\theta}+\frac{jV_s}{2}e^{-j\theta},$$

$$I_A=0,$$

$$V_B=\frac{-jZ_0V_s}{2Z_m}e^{j\theta}+\frac{jZ_0V_s}{2Z_m}e^{-j\theta},$$

$$I_B=\frac{-jV_s}{2Z_m}e^{j\theta}-\frac{jV_s}{2Z_m}e^{-j\theta}.$$

In trigonometric form the currents and voltages are:

$$V_A=V_s\sin\theta,$$

$$I_A=0,$$

$$V_B=V_s\left[\frac{Z_0}{Z_m}\right]\sin\theta,$$

$$I_B=\frac{-jV_s}{Z_m}\cos\theta.$$

The coupled section blocks line A since $V_A/I_A=\infty$ when $\theta=90°$. Further, the smaller the coupling the greater are the resonant voltage and current on line B. It should be remembered that these are ideal lines without dissipation, and for a practical case it is not possible to build up an infinite voltage and current on line B by letting Z_m approach zero.

Mechanical Modulation of the Voltage and Current on a Transmission Line

By coupling a resonant quarter wave section into a line and detuning it by a rotating variable condenser (Fig. 6), the voltage on the line can be modulated. By the use of properly shaped rotor and stator plates, sinusoidal modulation is readily obtainable.

To simplify the equations, the length of B is assumed to be 90°. In practice the length of B is slightly less because of the capacitive loading of the stator plates.

$\theta=0$

$$V_A=C_A+D_A,$$

$$I_A=\frac{Z_0C_A-Z_mC_B}{Z_0^2-Z_m^2}-\frac{Z_0D_A-Z_mD_B}{Z_0^2-Z_m^2},$$

$$V_A=Z_0I_A,$$

$$V_B=0=C_B+D_B.$$

$\theta=90°$

$$V_s=V_A=jC_A-jD_A,$$

$$I_s=I_A=\frac{Z_0C_A-Z_mC_B}{Z_0^2-Z_m^2}(j)-\frac{Z_0D_A-Z_mD_B}{Z_0^2-Z_m^2}(-j),$$

$$V_B=jC_B-jD_B,$$

$$I_B=\frac{Z_0C_B-Z_mC_A}{Z_0^2-Z_m^2}(j)-\frac{Z_0D_B-Z_mD_A}{Z_0^2-Z_m^2}(-j),$$

$$V_B=-ZI_B.$$

Solving for I_s in order to derive the input impedance into line A:

$$I_s=\frac{V_sZ_0^3}{(Z_0^2-Z_m^2)^2+ZZ_m^2Z_0},$$

$$Z_s=\frac{V_s}{I_s}=\frac{(Z_0^2-Z_m^2)^2+ZZ_m^2Z_0}{Z_0^3}.$$

If $Z_m=0$

$$Z_s=Z_0.$$

If $Z_m \ll Z_0$

$$Z_s=Z_0+\left[\frac{Z_m}{Z_0}\right]^2Z.$$

When Z equals zero, the input impedance is equal to Z_0 and optimum power transfer takes place. The load voltage is then equal in magnitude to V_s. When Z equals infinity, the input impedance is infinite, no power transfer takes place, and no voltage can appear across the load. The quarter wave section B reflects Z, the impedance appearing across its open end, into line A. Z is effectively a series condenser of larger capacity in line A.

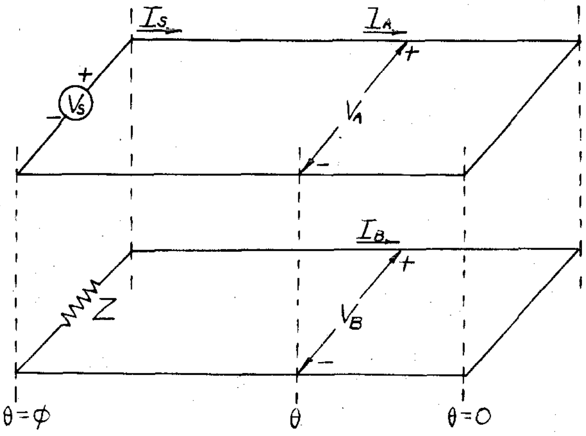

Fig. 7

Coupling Power from a Vacuum Tube

Fig. 7 illustrates one of the forms of coupling often used in coupling power from tank circuits of transmitting power amplifiers.

$\theta=0$

$$V_A=0=C_A+D_A,$$

$$V_B=0=C_B+D_B.$$

$\theta=\phi$

$$V_A=V_s=C_Ae^{j\phi}+D_Ae^{-j\phi},$$

$$I_A=\frac{Z_0C_A-Z_mC_B}{Z_0^2-Z_m^2}e^{j\phi}-\frac{Z_0D_A-Z_mD_B}{Z_0^2-Z_m^2}e^{-j\phi}=I_s,$$

$$V_B=C_Be^{j\phi}+D_Be^{-j\phi},$$

$$I_B=\frac{Z_0C_B-Z_mC_A}{Z_0^2-Z_m^2}e^{j\phi}-\frac{Z_0D_B-Z_mD_A}{Z_0^2-Z_m^2}e^{-j\phi},$$

$$V_B=-ZI_B.$$

Solving for the constants:

$$C_A=\frac{-jV_s}{2\sin\phi}, \qquad D_A=\frac{jV_s}{2\sin\phi},$$

$$C_B=\frac{ZZ_mC_A}{ZZ_0+j\tan\phi(Z_0^2-Z_m^2)}, \qquad D_B=-C_B.$$

Solving for the input impedance into line A:

$$Z_s=\frac{ZZ_0+j\tan\phi(Z_0^2-Z_m^2)}{Z_0-jZ\cot\phi}.$$

If $Z_m=0$

$$Z_s=\frac{Z_0(Z+jZ_0\tan\phi)}{Z_0-jZ\cot\phi}=jZ_0\tan\phi.$$

If $\phi=0$

$$Z_s=\frac{ZZ_0}{Z_0-jZ\cdot\infty}=0.$$

If $\phi=90°$

$$Z_s=\frac{ZZ_0+j\infty(Z_0^2-Z_m^2)}{Z_0}=\infty.$$

If Z is real and equal to R:

$$Z_s=\frac{RZ_0+j\tan\phi(Z_0^2-Z_m^2)}{Z_0-jR\cot\phi}.$$

If Z_s is to be real,

$$\frac{\tan\phi(Z_0^2-Z_m^2)}{RZ_0}=\frac{-R\cot\phi}{Z_0},$$

$$\tan^2\phi(Z_0^2-Z_m^2)=-R^2.$$

The last equation is impossible since Z_m is always less than Z_0. Therefore Z_s can never be real.

This method of coupling is quite satisfactory for coupling power out of a tank circuit since it is only necessary to couple a fairly low resistive component into the tank in order to load up the vacuum tube. However, the method does not seem desirable when coupling from one transmission line to another.

The Ideal Method of Coupling

Fig. 8 illustrates one of the most popular and probably the most effective method of transmission line coupling.

$\theta=0$

$$V_A=0=C_A+D_A,$$

$$V_B=C_B+D_B,$$

$$I_B=\frac{Z_0C_B-Z_mC_A}{Z_0^2-Z_m^2}-\frac{Z_0D_B-Z_mD_A}{Z_0^2-Z_m^2},$$

$$V_B=I_BR.$$

$\theta=\phi$

$$V_A=C_Ae^{j\phi}+D_Ae^{-j\phi}=V_s,$$

$$V_B=C_Be^{j\phi}+D_Be^{-j\phi}=0,$$

$$I_A=\frac{Z_0C_A-Z_mC_B}{Z_0^2-Z_m^2}e^{j\phi}-\frac{Z_0D_A-Z_mD_B}{Z_0^2-Z_m^2}e^{-j\phi}=I_s.$$

The constants are given by:

$$C_A=\frac{-jV_s}{2\sin\phi},\qquad D_A=\frac{jV_s}{2\sin\phi},$$

$$C_B=\frac{-jZ_mRV_se^{-j\phi}}{2\sin\phi[RZ_0\cos\phi+j\sin\phi(Z_0^2-Z_m^2)]},$$

$$D_B=\frac{jZ_mRV_se^{j\phi}}{2\sin\phi[RZ_0\cos\phi+j\sin\phi(Z_0^2-Z_m^2)]}.$$

The input impedance, Z_s, is:

$$Z_s=[Z_0^2-Z_m^2]\sin\phi\times\left[\frac{RZ_0\cos\phi+j\sin\phi(Z_0^2-Z_m^2)}{Z_0\sin\phi\cos\phi(Z_0^2-Z_m^2)+jR(Z_m^2-Z_0^2\cos^2\phi)}\right].$$

When ϕ is an integral number of half wavelengths, $\sin\phi$ is equal to zero and no coupling takes place.

When Z_m equals zero

$$Z_s=Z_0^2\sin\phi\left[\frac{RZ_0\cos\phi+jZ_0^2\sin\phi}{Z_0^3\sin\phi\cos\phi-jZ_0^2R\cos^2\phi}\right],$$

$$Z_s=jZ_0\tan\phi.$$

When Z_s is real for any value of ϕ

$$\frac{\sin\phi(Z_0^2-Z_m^2)}{RZ_0\cos\phi}=\frac{R(Z_m^2-Z_0^2\cos^2\phi)}{Z_0\sin\phi\cos\phi(Z_0^2-Z_m^2)}.$$

Note: This operation cannot be performed at the quarter wave points since it would involve dividing by zero.

$$R(Z_m^2-Z_0^2\cos^2\phi)=\frac{\sin^2\phi(Z_0^2-Z_m^2)^2}{R}.$$

Resubstituting this last equation in the second term of the denominator of the general expression for Z_s,

$$Z_s=R.$$

When ϕ equals 90 degrees, substituting this

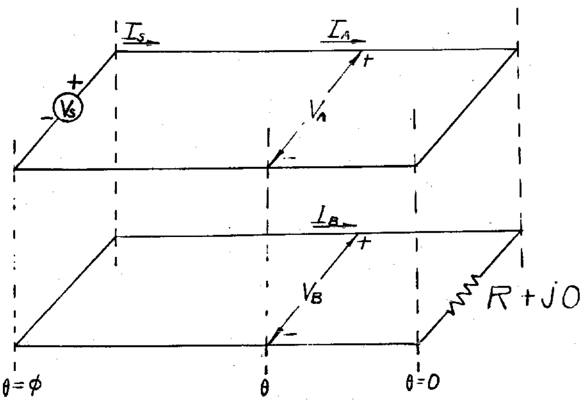

Fig. 8

value of ϕ in the general equation for Z_s,

$$Z_s = \frac{(Z_0^2 - Z_m^2)^2}{R Z_m^2}.$$

It will thus be seen that a quarter wave coupling loop can become an extremely effective coupling device. By varying Z_m, Z_s can be made to assume any value desired from zero to infinity. Further, the reciprocal of R is coupled into line A, a property common to ordinary inductive circuits with lumped constants; and, since R is real, Z_s must also be real. For coupling between transmission lines this method is ideal.

Conclusion

The equations derived in the preceding pages apply to a large number of problems met with in practice. When, however, the spacings between conductors become small in comparison with the diameters of the conductors, they merely represent approximations. The degree of approximation is, roughly equivalent to that obtained when the well known formula 276 log $2D/d$ is used in place of 120 $\cosh^{-1}(D/d)$ to calculate the surge impedance of a two wire transmission line.

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Antennae for Ultra-High Frequencies

Wide-Band Antennae

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*Editor's Note: This paper presents a general study of ultra-high frequency antenna problems and a comparison of the theories developed by different authors. Discussion of their contributions is illustrated by reproductions of figures from their published papers. Grateful acknowledgment is made for permission to reproduce this material.**

Because of the length of Professor Brillouin's paper, it is being published in two installments. Sections 10 to 19, inclusive, will appear in the next issue of this journal.

1. Statement of the Antenna Problem

THE classical discussion of antennae is usually based on some simplified assumptions which do not fit very exactly into a rigorous theory. They were developed in connection with the theory of antennae for long wave-lengths, and it can be proved that they represent a good first procedure for antennae of very small diameter and great length. Such assumptions are, however, hardly justified for ultra-high frequencies where the radiating systems in use must be short and very often are built with large diameters, especially in the case of wide band antennae. Consequently, the theory must be revised and based directly on Maxwell's field equations. Solutions were attempted in the early days of the theory of electromagnetic waves in connection with the discussion of Hertz's experiments. Hertz, as a matter of fact, always worked with oscillators of very small dimensions, giving ultra-short damped waves. Around the year 1900, a number of very important theoretical papers were published [1–6] on the problem of antennae, dealing with spherical, ellipsoidal and cylindrical antennae. M. Brillouin and P. Debye discussed the different types of vibrations for a sphere. M. Brillouin also computed the oscillations of prolate spheroids, while M. Abraham dealt with the problem of very long ellipsoids. These old papers are still very important since they give the basis of the modern theory of antennae. While the problem was practically forgotten for many years because of the expansion of radio towards greater wave lengths, the last few years have witnessed its revival because of great technical improvements enabling one to produce and apply sustained oscillations of ultra-high frequencies, and a number of interesting papers have been published (some of them duplicating old results); nevertheless the ramifications of the problem are very far from being exhausted. The present report is aimed at giving an account of the results achieved, together with a discussion of the weak points of the different theoretical methods in use and an attempt to draw some general practical conclusions.

Assuming the choice of a *certain shape of antenna* (the ellipsoid, for instance), consideration may be given to:

A. Free Antenna Vibrations

In effect one assumes that a certain type of oscillation has been excited in the antenna, which is afterwards left to itself and oscillates freely while radiating energy progressively to a great distance. The problem here is to find the field distribution corresponding to each natural mode of vibration, together with the proper frequency and damping of a particular vibration. If one assumes a time dependence in

$$e^{i\omega t}$$

the solution of the Maxwell equations (with the boundary conditions on the surface of the antenna) will be found possible only for certain values of ω (the so-called "proper" values) and these will be derived as complex quantities

$$\omega = \omega_r + i\omega_i, \quad e^{i\omega t} = e^{-\omega_i t + i\omega_r t}, \quad (1)$$

* See note at end of present installment.

showing that the "proper" vibrations have a frequency ω_r, and a coefficient of damping ω_i.

Electrotechnicians like to define, for each type of oscillator, a "Q factor" which measures the sharpness of resonance. For a usual L, C, R, circuit, the definition is

$$Q=\frac{\omega L}{R}, \quad \omega_i=\frac{R}{2L}, \tag{2}$$

where ω means proper frequency of the circuit and corresponds to ω_r. Hence, from the discussion of *free* vibration, we may compute

$$Q_f=\frac{\omega_r}{2\omega_i}, \tag{3}$$

where the subscript "f" indicates free vibration. This distinction is necessary since antennae have properties which are not exactly similar to those of ordinary electrical tuning circuits.

B. Receiving Antenna

Here the antenna is supposed to be acted upon by the field of an incident plane wave. Such a wave excites vibrations in the antenna, and some energy is dissipated by the Joule effect while a certain amount is radiated again in all directions. These conceptions are useful for studying the resonance properties of the receiving antenna and for defining another Q factor, which will be called Q_r.

C. Transmitter

The antenna is supposed to be cut and connected at the end of a line feeding the antenna. This can be assumed to be equivalent to inserting a certain electromotive force of given frequency across the gap. Resonance curves will be obtained, from which a Q_t factor may be defined.

The first two problems (A, free antenna, and B, receiving antenna) have been discussed in the older papers, while the last one (C, transmitter) has been considered only in recent papers.

Note on Unit System

Throughout this paper, the units used are the M.K.S. system of Giorgi rationalized units. Explanations regarding them can be found in Stratton's book [7] (p. 10–23).

ϵ_0	permittivity of free space $(1/36\pi)10^{-9}$ farad/meter	(4)
μ_0	permeability of free space $4\pi 10^{-7}$ henry/meter	
$\kappa_e=\epsilon/\epsilon_0$	usual dielectric constant of air (isotropic medium)	
$\kappa_m=\mu/\mu_0$	usual permeability	
c	velocity of light $\epsilon_0\mu_0 c^2=1$	

Some authors use the Heaviside-Lorentz rationalized units, which correspond to

$$\epsilon_0=\frac{1}{4\pi c}, \quad \mu_0=\frac{4\pi}{c}.$$

In non-rationalized symmetrical Heaviside-Lorentz units

$$\epsilon_0=\mu_0=\frac{1}{c}.$$

2. *Free Vibration for a Sphere*

This problem is discussed in papers [1, 4, 5, 6] and a good summary may be found in Stratton's book [7] (pp. 554–560). There are two fundamental types of oscillations:

I—E waves, also called electric waves, or transverse magnetic waves, for which there is no radial component of the magnetic field; the electric field has a radial component.

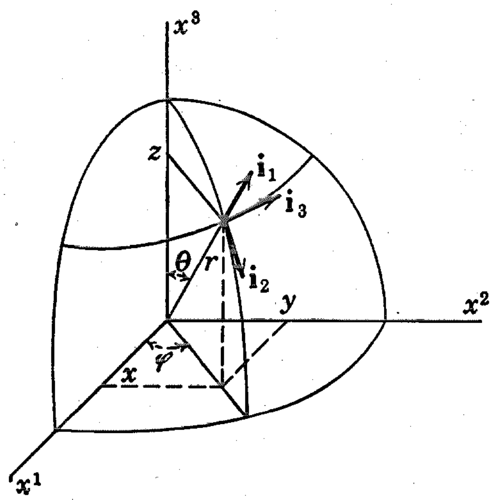


Fig. 1—Spherical Coordinates.

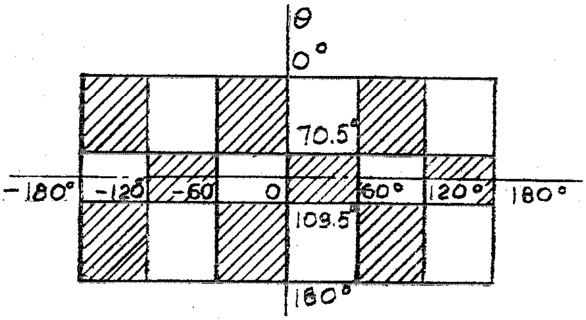


Fig. 2—Nodes of the function $\sin 3\varphi P_5^3(\cos\theta)$ on the developed surface of a sphere. The function has negative values over the shaded areas.

II—H waves, or magnetic waves, or transverse electric, with no radial electric field, but with a radial component of the magnetic field.

For both E and H waves, an infinite series of proper vibrations is obtainable, each characterized by two integers, m, n, and eventually by a subscript e or o to distinguish between even vibrations ($\cos m\varphi$) and odd vibrations ($\sin m\varphi$). Fig. 1 shows the usual definitions of the spherical angles φ, θ; the field distribution is obtained in terms of the well-known tesseral harmonics.

$$\begin{aligned} Y_{emn} &= P_n^m(\cos\theta)\cos m\varphi, \\ Y_{omn} &= P_n^m(\cos\theta)\sin m\varphi, \end{aligned} \tag{5}$$

where P_n^m represents the Legendre functions.

These tesseral harmonics divide the surface of the sphere into a number of zones, with m nodal lines along meridian circles (due to $\cos m\varphi$ or $\sin m\varphi$) and $n-m$ nodal lines along circles parallel to the equator. For instance, Fig. 2 shows the subdivisions on the sphere for $n=5$, $m=3$.

If one is not specially interested in the position of the origin of the φ angles on the sphere, the e or o subscript may be dropped and only the essential n and m suffixes retained. The whole system of proper vibrations, for the sphere, is then proved to consist of E_{mn} and H_{mn} waves.

These waves exhibit a very high damping, even when the metal is a very good conductor, i.e., the amount of energy radiated at a great distance is always very large. On account of high radiation losses, one may completely neglect the Joule losses in the metal and assume a perfectly conducting sphere.

It should be noticed that the r dependence of the field is obtained by means of functions which depend only on n, and not explicitly on m. Hence the boundary condition on the sphere (no tangential electric field on the radius a) involves n alone, and the proper frequencies of the sphere also depend only on n, while the other number m can be taken arbitrarily, provided it does not exceed n:

$$m = 0\cdot 1\cdot 2\cdot 3\cdots, \quad m \leqslant n. \tag{6}$$

Thus all the E_{mn} vibrations, with the same n value, have the same frequency and the same damping, while they correspond to very different field distributions where m meridians and $n-m$ parallels are the nodal lines.

The proper frequencies, damping and Q factors for the most important vibrations of the sphere are best expressed as follows:

$$\begin{aligned} \rho &= ka = \omega\frac{a}{c}, \quad && a \text{ radius of the sphere,} \\ & && \lambda \text{ wave length,} \\ Q_f &= \frac{\omega_r}{2\omega_i} = \frac{\rho_r}{2\rho_i}, \quad && k = \frac{\omega}{c} = \frac{2\pi}{\lambda}. \end{aligned} \tag{7}$$

A general rule is that for magnetic modes of order n, one finds n different ρ values, while electric modes of order n yield $n+1$ different ρ values. The most striking fact in Table I is the

TABLE I

n	Magnetic Modes		Electric Modes	
	$i\rho$	Q_f	$i\rho$	Q_f
1	-1	0	$-0.5 \pm i\,0.86$	0.86
2	$-1.5 \pm i\,0.86$	0.286	$\begin{cases}-1.6 \\ -0.7 \pm i\,1.81\end{cases}$	0 1.3
3	$\begin{cases}-2.26 \\ -1.87 \pm i\,1.75\end{cases}$	0 0.47	$\begin{cases}-2.17 \pm i\,0.87 \\ -0.83 \pm i\,2.77\end{cases}$	0.2 1.67

extraordinary low values of the Q factors, some of which are even zero and correspond to damping without oscillation. This certainly shows that spherical antennae are a typical example of wide band antennae.

Reverting to consideration of E and H waves, some characteristics of their structure may be noted. Electric E vibrations are the only ones exhibiting radial electric fields. As is well known, a radial field component on the surface of a metal

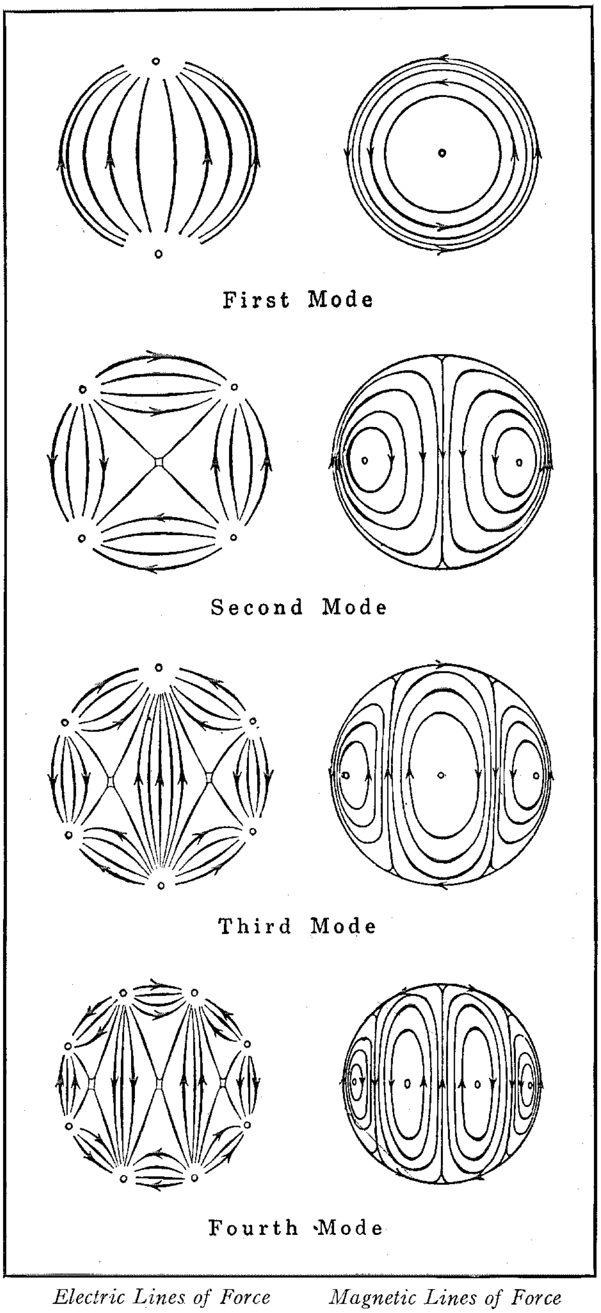


Fig. 3—Lines of force corresponding to the first four modes of electric type of vibrations of a metal sphere.

sphere means an electric charge density on the sphere. Hence electric waves correspond to oscillations whereby alternate sectors on the sphere are positively or negatively charged at a certain moment; and, during oscillation, currents flow on the surface of the sphere from each sector (+ charged) to the next ones (− charged).

Since magnetic H modes do not contain any radial electric field components, they do not excite any superficial electric charge densities on the sphere but only closed currents. The field created by these oscillations is similar to that obtained with a closed circuit system. Very interesting as illustrations of these general remarks are sketches by G. Mie; they show some of the oscillations but not the complete set of the first possible oscillations of a spherical antenna. (See Fig. 3, reproduced from Stratton's book, p. 567.)

The same sketches can be used for representing different types of oscillation. Considering first the *electric oscillations*, let us look at the two upper figures. The diagram on the left shows the distribution of electric lines of force on a sphere surrounding the spherical antenna. If the polar axis is assumed to be vertical and in the plane of the drawing, this is an $m=0$, $n=1$, vibration, since it exhibits no meridian nodal line and $n-m=1$ parallel nodal line (the equator). In such a case, the diagram on the right represents the distribution of magnetic lines of force, but here the polar axis is perpendicular to the plane of the drawing; and the magnetic lines run along the equator and the parallels near the equator.

If we choose a different position of the polar axis, the same sketch may represent an electric mode $m=1$, $n=1$, with one meridian nodal line and $(m-n=0)$ no nodal line on the equator. In the left diagram (electric lines) one must assume the polar axis to be perpendicular to the plane of the figure, while in the diagram on the right (magnetic lines) the polar axis should be taken as vertical in the plane of the drawing.

The second group of diagrams may be best considered as representing the $m=2$, $n=2$, electric mode with two meridian nodal lines and $(n-m=0)$ no parallel nodal line. The polar axis is perpendicular to the plane of the left sketch (electric lines) and vertical in the plane of the figure on the right (magnetic lines). In the same way, the two last groups of diagrams correspond to $m=n=3$ and to $m=n=4$.

In this discussion, a "nodal line" signifies a line parallel to the main magnetic line of force, and perpendicular to the electric lines. As a rule, the magnetic and electric lines of force cross each other at right angle on the sphere. The

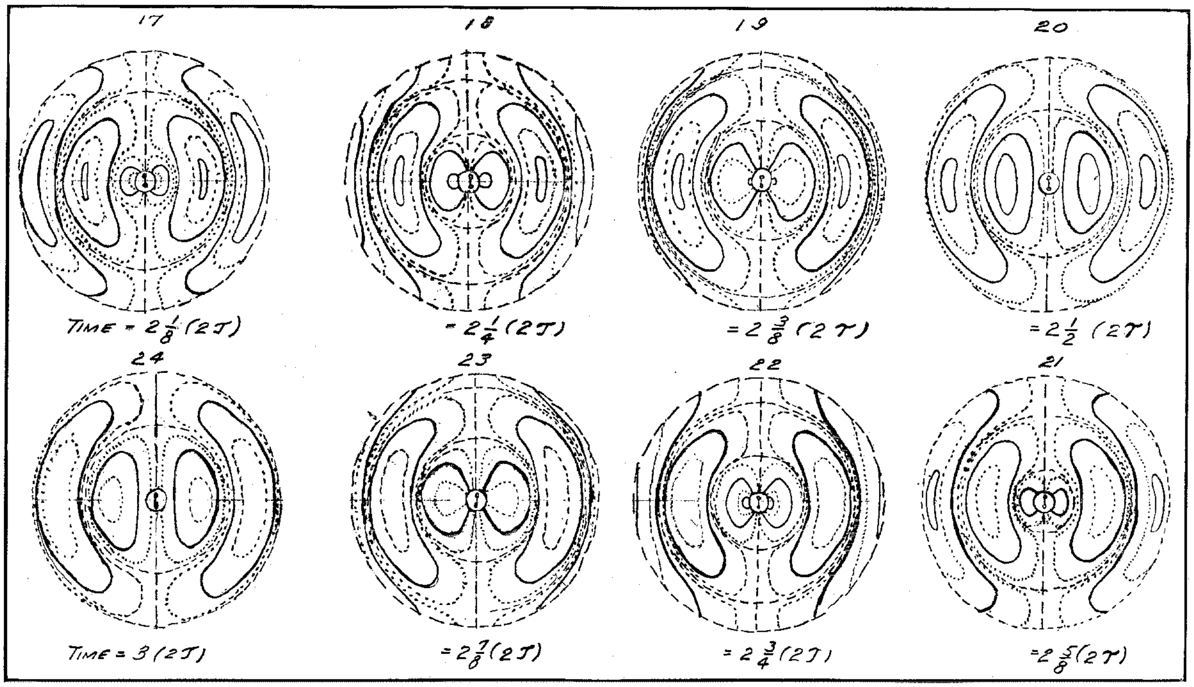


Fig. 4— Lines of force in a spherical wave spreading out of a dipole.

vibrations with $m<n$ are not shown on the sketches (Fig. 3).

Considering now the *magnetic oscillations*, the sketches on the right represent the distribution of the electric lines of force while the diagrams on the left show the magnetic lines of force. If the polar axis is taken as vertical in the plane of the sketches on the right, it must be set as perpendicular to the plane of those on the left. The diagrams correspond to $m=n=1$, $m=n=2$, $m=n=3$, and $m=n=4$.

In all these illustrations, only the projections of the lines of force (either electric or magnetic) on the sphere are represented. It must be noted that these lines are actually bent out of the sphere and converge in opposite direction on another sphere. This can be better understood by examination of Fig. 4, which refers to the similar problem of electromagnetic waves radiated from an infinitely small hertzian oscillator.

3. Comparison with Other Similar Problems—Hydrogen Atom in Wave Mechanics—Waves Rotating Around the Sphere

The two functions Y of equation 5 can be combined to constitute a wave which is no longer a pure standing wave, but one exhibiting mixed behaviour: a standing wave in θ and a propagating wave in φ, as is made evident by completing the functions Y with their $e^{i\omega t}$ factors (omitted in (5)) and writing

$$Z_{\pm m, n} = (Y_{emn} \pm i Y_{omn}) e^{i\omega t} = P_n^m(\cos\theta) e^{i\omega t \pm im\varphi}. \quad (8)$$

This linear combination offers another solution of Maxwell's equations and represents a wave spinning about the Z axis of the sphere with an angular velocity $\mp\omega/m$: it is the way the waves are usually combined in the corresponding problem of wave-mechanics (the hydrogen atom, for instance). The proper frequencies are, in this case, usually expressed in terms of the three "quantum numbers," called n, l, m, corresponding to p, n, m, in our problem; p is an additional integer—the number of the root chosen among the different roots ρ. In the quantum mechanical problem, it is well known that the proper values depend only on n, l (corresponding to our p, n), and not upon m so long as the spherical symmetry is preserved. These integers have received names corresponding to their physical meaning (Table II).

TABLE II

Q.M. Problem	Electromagnetic Problem
n total quantum number	p
l total angular momentum	n
m angular momentum about $0z$	m

Hence, the fact that one finds a number of proper frequencies corresponding to given n and m numbers is a general result, as a third integer p is necessary to fix the type of oscillation. For a sphere of finite conductivity, the distinction between the different p oscillations is quite obvious, as they correspond to the same field distribution in θ and φ, but to different penetrations. The p and n numbers define the field distribution as a function of r and, accordingly, the current and charge distribution inside the metal. However, when one reaches the limit of infinite conductivity, currents and charges are exactly superficial in all cases, and the distinction between the different p solutions becomes less clear. It can be seen only if one looks at the field distribution outside the sphere, where the r dependence of all the fields is again determined by the number p of the root chosen.

4. *Solid Sphere and Spherical Cavity*

In addition to the oscillations discussed in the preceding section P. Debye also finds another set of oscillations [6] (p. 77, Table II), the nature of which must be explained as these additional solutions seem to have confused many writers. Stratton [7] sketches (p. 560) the explanation, which is the following: These additional vibrations are apparently *undamped* for a perfect metal of infinite conductivity; but, as soon as the finite conductivity is taken into account, the oscillations are completely aperiodic and practically die out in an extremely short time. They correspond to charge and current distribution inside the solid sphere, a very unstable state of affairs, and, furthermore, one which can not be created by any experimental procedure. Hence these motions correspond, in the case of a metal sphere, to a mathematical solution of no practical value.

The case is different when the solid sphere is a dielectric imbedded in a medium with different dielectric properties. Mathematically this second problem is another special case of the general problem: a sphere with $k_1^2 = \epsilon_1\mu_1\omega^2 + i\sigma_1\mu_1\omega$ in a medium with $k_2^2 = \epsilon_2\mu_2\omega^2 + i\sigma_2\mu_2\omega$ (ϵ = dielectric constant, μ = permeability, σ = conductivity). When σ_1 and σ_2 are zero, both types of oscillations are of importance; the damped vibrations of the previous sections correspond roughly to the vibrations outside the sphere, while the second group of undamped vibrations are standing waves inside the dielectric sphere.

If the external medium is assumed to be a metal, we encounter oscillations inside a spherical closed cavity, another well-known problem. Very small damping occurs with good conductors and the damping drops to zero (infinite Q factor)

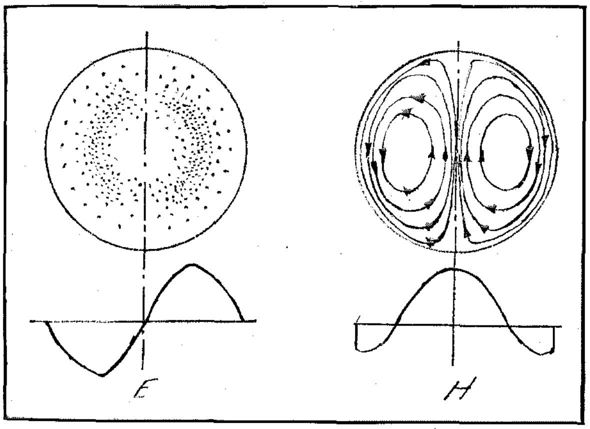


Fig. 5—The lowest magnetic mode of oscillation in a spherical resonator, showing the field in a meridian plane.

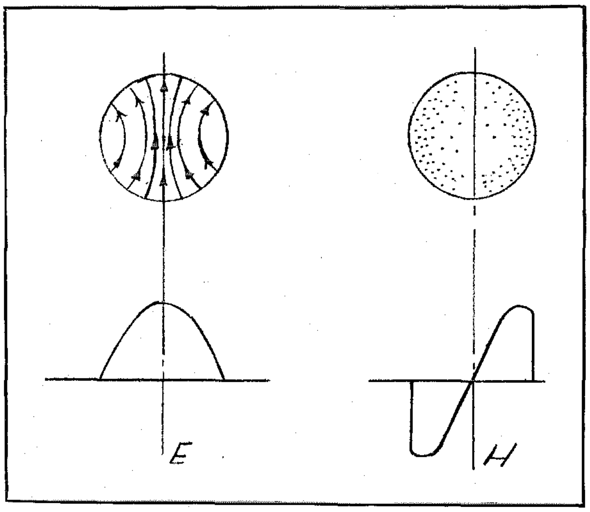


Fig. 6—The lowest electrical mode of oscillation in a spherical resonator, showing the field in a meridian plane.

with a perfect metal. Oscillations inside the spherical tank are those of the second Debye type, as is easily seen from the preceding explanation, and especially by comparison with the problem of the dielectric sphere in a dielectric medium. It is also obvious from the fact that the derived proper values (Debye,[6] Table II) coincide with those of the spherical cavity (Stratton,[7] p. 560, eq. 34).

Much confusion results from the fact that, in the case of a perfect metal of infinite conductivity, two different physical problems merge into one mathematical statement. Considering an antenna of ideal metal with infinite conductivity and a certain mass of metal inside a surface S, the mathematical problem is to find the solution of Maxwell's equations yielding an electric field perpendicular to the surface S. Now, if a cavity with the same shape be assumed, the metal being outside the surface S, let us determine the vibrations inside this tank resonator. The fundamental boundary problem remains the same as before, and the distinction results only from the consideration of the behaviour of the fields at very great distance or inside the surface S. For comparative purposes and some further application, the main results relative to the proper oscillations of a spherical cavity are summarized. The lowest modes only will be considered (see Stratton, pp. 560–563); they yield

Lowest magnetic mode

$$\lambda_{11}=1.4a, \quad \rho_{11}=\omega_{11}\frac{a}{c}=4.5,$$

Lowest electric mode

$$\lambda'_{11}=2.28a, \quad \rho'_{11}=2.75.$$

The field distributions of these two vibrations are illustrated in Figs. 5 and 6 (taken from Stratton's book). The corresponding frequencies are much higher than the lowest frequencies of the external vibrations.

As an example, let us imagine the device indicated in Fig. 7. A *hollow metallic sphere* is provided with *two circular holes* with the poles as centers, and the fundamental *electric vibration* is excited inside the sphere. The internal distribution of the lines of force is very similar to the case of Fig. 6, but a few lines of force escape

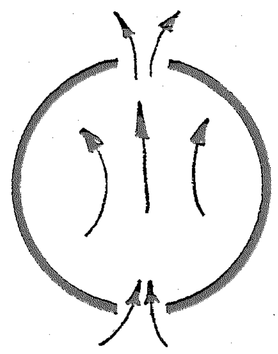


Fig. 7.

through the holes. They will excite a field outside the sphere, consisting mainly of the symmetrical 3rd oscillation, $r=3$—electric mode. A vibration of similar symmetry and similar frequency ($\rho=2.77+i\,0.83$) may be found in Debye's Table II.[6] This electric vibration outside the sphere has very high damping ($Q=1.67$) and radiates very strongly. Its coupling with the internal vibration can be controlled at will by a variation of the size of both polar openings. One can thus build a spherical oscillator with an adjustable Q factor, yielding an antenna system with any desired band width. With such a structure of cylindrical symmetry about the polar axis, the external vibrations to be excited should be ones exhibiting no meridian nodal line but only parallel nodal lines, namely,

$$m=0, \quad n=3, \quad \text{(electric mode)}.$$

This mode is *not* represented in the sketches of Mie (Fig. 3).

The same structure might work on the *magnetic modes;* the lowest magnetic mode ($\rho=4.5$) would be produced inside the spherical cavity (Fig. 5) and radiated outside through the polar openings, thus exciting an outside magnetic vibration $m=0$, $n>3$. Computations for these higher modes are not available but, from the low ρ values found for the magnetic modes (Table I), one may infer that n should be very high, probably from 6 to 10.

Instead of two symmetrical apertures surrounding both poles, one might equally well use only one aperture about one of the poles. In the first case, external vibrations with *odd n* values could be excited if the internal vibration is an *odd* one (for instance $n=1$, lowest mode). In the second case, all vibrations n might be excited outside the sphere.

5. *Prolate Spheroids as Antennae*

This problem was first stated by M. Abraham [3] who specially discussed the case of very thin ellipsoids. A general discussion for all ellipticities was given by M. Brillouin [4] and then again by Leigh Page, N. I. Adams and R. M. Ryder,[8] who not only computed the free damped oscillations (problem A, §1) but also the case of the receiving antenna (problem B, §1). Stratton and Chu [9] have very carefully discussed the problem C of the transmitting antenna (sphere and spheroid).

In an ellipsoid of revolution with a the semi-axis of the ellipsoid in the direction of symmetry and b the semi-axis in a plane perpendicular to the axis of symmetry, one must distinguish between two cases:

$$\begin{matrix}\text{Prolate Spheroid} & a>b, \\ \text{Oblate Spheroid} & a<b,\end{matrix} \quad \text{Sphere } a=b. \quad (9)$$

The first case has been carefully discussed but very little, if anything, has been done on the second problem, which might offer some distinct advantages for certain important types of vibrations. The limit of infinitely thin oblate spheroids ($a \ll b$) would lead to the flat disk. The prolate spheroid, at the limit $a \gg b$ tends to the rectilinear antenna.

In both cases, the axis of symmetry is taken as the line of the poles and the coordinates used are ξ, η, φ, where ξ, η represent elliptic coordinates in a meridian plane, while φ is the angle of longitude. Fig. 8 shows the confocal ellipses (η = const., ξ variable) and hyperbolas (ξ = const., η variable), which constitute the well known system of elliptic coordinates, with the transformation formulae.

$$\begin{aligned} &x=f\xi\eta, \quad y^2=f^2(1-\xi^2)(\eta^2-1), \\ &-1 \leqslant \xi \leqslant 1, \quad 1 \leqslant \eta < \infty, \\ &r^2=x^2+y^2=f^2(\xi^2+\eta^2-1), \\ &2f = \text{focal distance.} \end{aligned} \qquad (10)$$

Oblate spheroids would correspond to a vertical polar line and horizontal equator of Fig. 8, while prolate spheroids are found by taking the axis of symmetry along Ox and the meridian plane vertical.

In both cases the problem is found to yield two separate sets of solutions, corresponding to the electric or the magnetic vibrations of the sphere. These vibrations depend on three integers, the first of which, m, corresponds to the longitude angle φ since all the fields depend upon φ by $\cos m\varphi$ and $\sin m\varphi$ or $e^{\pm im\varphi}$. In the spherical problems, the proper frequencies (and damping factors) do not depend upon m but, in the spheroidal case, the m coefficient comes out explicitly. In addition to it, another n integer, corresponding to the ξ and η variables will be found; and, for the higher types of oscillation, a third integer (like p in the spherical problem) would be needed to distinguish between the different vibrations corresponding to the different roots of the vibration equation. As a matter of fact, only axially symmetric waves ($m=0$) for prolate spheroids exclusively have been considered.

M. Abraham, who first attacked the problem, especially discussed the case of very thin ellipsoids, and found solutions when the ratio b/a is small enough to be neglected. The fields and frequencies can be expressed by expansions* in ϵ_A, ϵ_A^2, $\epsilon_A^3 \cdots$

$$\epsilon_A = \frac{1}{2\Omega}, \quad \Omega = 2\log_e \frac{2a}{b} \approx 2 \log_e \frac{2f}{b}, \qquad (11)$$

* ϵ_A should not be confused with dielectric permittivity ϵ.

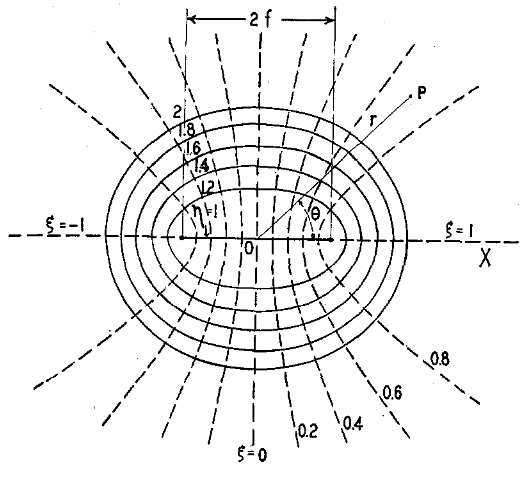


Fig. 8—Spheroidal Coordinates.

since $a \approx f$ for these very long spheroids. He obtained the following results:

vibration type	$n=1$	2	n	
λ wave length of free oscillation	$\frac{\lambda}{4a}=1+5.6\epsilon_A^2$	$\frac{1}{2}[1+3.3\epsilon_A^2]$	$\frac{1}{n}\left[1+\frac{4.8+2\log n}{n}\epsilon_A^2\right]$	(12)
δ logarithmic decrement	$\delta=9.74\epsilon_A$	$6.23\epsilon_A$	$\frac{9.66+4\log n}{n}\epsilon_A$	

Here, δ is the logarithmic decrement of the oscillations. If our solution for the proper frequency is a complex number

$$\omega=\omega_r+i\omega_i, \quad \delta=2\pi\frac{\omega_i}{\omega_r}=\frac{\pi}{Q_f}, \tag{13}$$

according to the definition of the Q_f factor for free oscillations (eq. 3).

M. Brillouin [4] discusses the free oscillations of ellipsoids of all eccentricities from the sphere to the very thin and long ones. He computes a coefficient which he calls θ and which (loc. cit., pp. 331 and 372) is defined as

$$i\omega=-\theta\frac{c}{f}, \quad \theta=-i\omega\frac{f}{c}, \quad 2f=\text{focal distance}. \tag{14}$$

He calls Ω the velocity c of light and $1/s_1$ the ellipsoid eccentricity which, according to Fig. 8, corresponds to $1/\eta_0$ and is related to the length of both axes by

$$\frac{1}{s_1^2}=\frac{1}{\eta_0^2}=1-\left(\frac{b}{a}\right)^2. \tag{15}$$

Later (pp. 360 and 403), discussing the case of very long ellipsoids, he defines a small quantity ϵ which we shall call ϵ_B to avoid confusion with M. Abraham's notations:

$$s_1=1+\epsilon_B. \tag{16}$$

Inasmuch as only very small b/a ratios are involved,

$$\epsilon_B=\frac{1}{2}\left(\frac{b}{a}\right)^2.$$

Then (p. 368, eq. 87) M. Brillouin takes

$$\frac{1}{\eta}=\log\frac{2+\epsilon_B}{\epsilon_B}\approx\log\frac{2}{\epsilon_B}=\log\frac{4a^2}{b^2}=2\log\frac{2a}{b}=\Omega. \tag{17}$$

(in M. Abraham's notation). Comparing (17) with (11),

M. Brillouin's $\eta=2\epsilon_A$ in M. Abraham's notation. (18)

Subsequently (p. 381, eq. 101, 102) M. Brillouin, for the nth mode of oscillation, finds

$$\theta_n=\frac{n\pi i}{2}+\eta\theta_n', \quad \omega_n=-n\pi\frac{c}{2f}+in\theta_n'\frac{c}{f}. \tag{19}$$

Comparison with M. Abraham's formulae necessitates computing the logarithmic decrement (eq. 13),

$$\delta=2\pi\frac{\omega_i}{\omega_r}=-2\pi\frac{\theta_r}{\theta_i}=-4\eta\frac{\theta_n'}{n}=-8\epsilon_A\frac{\theta_n'}{n}. \tag{20}$$

The logarithmic decrements obtained by M. Brillouin are about 1/2 Abraham's for the first two terms; the discrepancy decreases for higher modes of oscillations. Since the computations were made by different methods of series expansions, it seems difficult to find the origin of the disparity. Further, M. Brillouin points out a consistent difference between even and odd vibrations that does not appear in Abraham's formulae. Such a difference is very likely to be correct inasmuch as similar results have been

$n=$	1	2	3	4	5	6	7	8	9	10
$-\theta_n'=$	0.61	0.77	0.707	0.8	1.35	1.21	1.37	1.3	1.39	1.33
$\frac{\delta}{\epsilon_A}=-\frac{8\theta_n'}{n}=$	4.88	3.08	1.888	1.6	2.16	1.61	1.57	1.3	1.24	1.07
M. Abraham's Results										
	9.74	6.23	4.32	3.8	3.22	2.8	2.49	2.25	2.05	1.88

obtained in other cases. A number of numerical tables and dates are found in his book relative to the different functions introduced in the discussions of both electric and magnetic oscillations.

A modern treatment of the problem is found in a very interesting paper by Page and Adams,[8] who first discuss the free vibrations of the prolate spheroid for the symmetrical oscillation ($m=0$) of the modes $n=1$, 3, 5. Two different types of expansions are used; the first applies to eccentricities smaller than 0.8

$$0 \leqslant \frac{1}{\eta_0} < 0.8, \quad 0.6 < \frac{b}{a} \leqslant 1.$$

The second type of series expansions can be used for eccentricities very near unity

$$\eta_0^2 = 1 + t, \quad t \approx 2\epsilon_B \approx \frac{b^2}{a^2}. \tag{21}$$

These authors use a parameter

$$l = \frac{1}{\log \dfrac{\eta_0+1}{\eta_0-1} - 2} \approx \frac{1}{\log \dfrac{4a^2}{b^2} - 2}$$

$$= \frac{1}{2 \log \dfrac{2a}{b} - 2} = \frac{1}{\Omega - 2}. \tag{22}$$

For very long ellipsoids, the ratio $2a/b$ is very large, and Ω is the same large parameter found in the previous papers. The formulae developed could be used for numerical computations of the proper frequencies and damping for the modes 1, 3, 5, but calculations have been completed only for the fundamental vibration ($n=1$). The results are summarized in Tables III and IV.

TABLE III

WAVE-LENGTHS AND LOGARITHMIC DECREMENTS FOR FUNDAMENTAL FREE OSCILLATION OF SPHEROIDS OF ECCENTRICITIES BETWEEN 0 AND 0.8

$1/\eta_0$	$\lambda/4a$	δ
0.0	1.814	3.628
0.1	1.809	3.618
0.2	1.794	3.588
0.3	1.770	3.537
0.4	1.734	3.461
0.5	1.686	3.356
0.6	1.625	3.214
0.7	1.549	3.024
0.8	1.455	2.772

TABLE IV

LONG SPHEROIDS, ECCENTRICITIES NEAR TO UNITY

$l=0$	0.02	0.05	0.08	0.1	0.125
$b/a=0$	$1.02\ 10^{-11}$	$3.34\ 10^{-5}$	$1.42\ 10^{-3}$	$4.96\ 10^{-3}$	$1.35\ 10^{-2}$
$\lambda/4a=1$	1.001	1.004	1.009	1.015	1.023
$\lambda/4a$ M.Abraham $=1$	1.0006	1.0035	1.009	1.014	1.022
$\delta=0$	0.098	0.247	0.396	0.494	0.613
$Q_f=\pi/\delta=\infty$	32	12.75	7.95	6.4	5.15

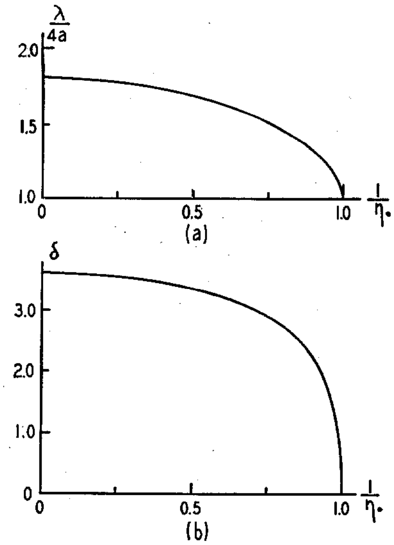


Fig. 9—(a) wave-length λ; (b) logarithmic decrement δ of fundamental free oscillation of spheroid of eccentricity $1/\eta_0$.

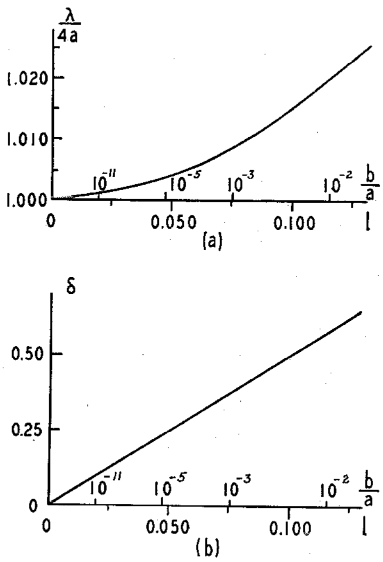


Fig. 10—(a) wave-length λ; (b) logarithmic decrement δ of fundamental free oscillation of spheroid of major axis "a" and minor axis "b."

The summarized results check very well with M. Abraham's formulae. As for the logarithmic decrement, the curve is practically a straight line

$$\delta = 4.94l = \frac{4.94}{\Omega - 2} \qquad (23)$$

M. Abraham obtains

$$\delta = \frac{9.74}{2\Omega} = \frac{4.87}{\Omega}, \qquad (23a)$$

which shows a very small discrepancy for large Ω. These formulae should not be used for values of l larger than those shown in Table III as they soon diverge (commencing with $\Omega = 2$) which means

$$\frac{\eta_0 + 1}{\eta_0 - 1} = e^2 = 7.6, \quad \frac{1}{\eta_0} = 0.8, \quad \frac{b}{a} = 0.6.$$

Thus, quite a large gap exists between $b/a = 0.6$, lower limit of the first method, and $b/a = 0.0135$, upper limit of the second method. Figs. 9 and 10 are taken from Page and Adams, while Fig. 11 is an attempt at drawing the complete curves for $\lambda/4a$ and δ as functions of b/a. The interpolation in the gap (from 0.0135 to 0.6) is rather uncertain, and may be wrong by a few percent around $b/a = 0.3$.

As noted in eq. 13 the logarithmic decrement δ corresponds to a $Q_f = \pi/\delta$. Hence ellipsoids of various eccentricities yield all Q factors from the very small ones obtained for the sphere up to the highest values for very fine wires.

The above discussion of numerical results deals only with the first fundamental symmetrical vibration ($n = 1$, $m = 0$) and it would be very important to complete it for even vibrations ($n = 2$, 4) and higher odd vibrations ($n = 3$, 5). The points to check would be: (1) Is there any consistent difference between even and odd modes (M. Brillouin versus M. Abraham)? (2) What occurs in the case of the many roots $p = 1, 2 \cdots$ found for higher modes in the case of the sphere (Tables I and II)? (3) What about the magnetic modes for which M. Brillouin made rather extensive calculations? (4) Modes of vibrations with $m = 1, 2, 3, \cdots$?

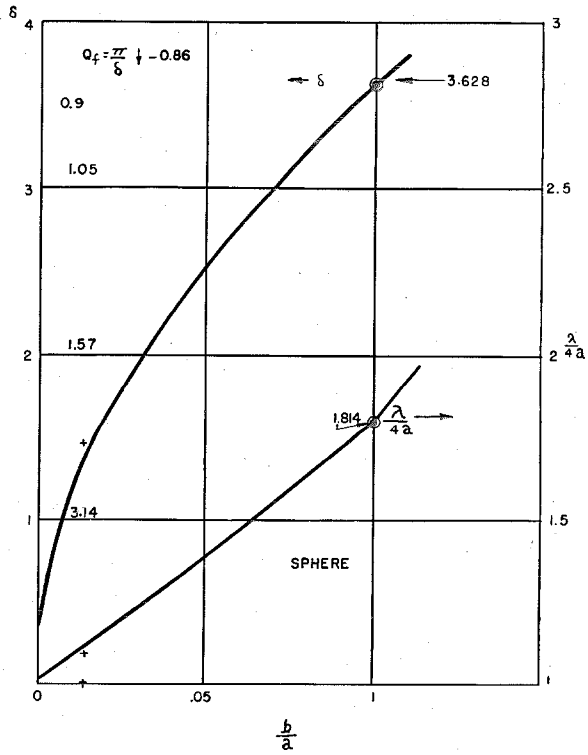


Fig. 11—Prolate Spheroids.

6. Spheres and Spheroids as Receiving Antennae

This is problem B of section 1: an incident plane wave impinges on the antenna, inducing vibrations that in turn radiate energy into space. If the antenna system were composed of a perfect metal, no energy would be lost and the whole energy absorbed per second from the incident wave would be reradiated in all directions. This problem was discussed, for the spherical case, by G. Mie [5] and P. Debye [6] and a good summary of their results is found in Stratton's book [7] (pp. 564–573). Consideration was confined to optical problems (light scattered by spheres or colloidal suspensions); the basic theory has been formulated, but numerical calculations of actual importance in antenna theory remained to be performed.

A theoretical discussion of practical value has been given by Page and Adams, and by Ryder,[8] for the case of long prolate spheroids of eccentricity near unity. They assume an electric field

$E_0e^{i\omega t}$ parallel to the x axis of symmetry of the ellipsoid, and consider the case of long and thin ellipsoids, so that this field can be treated as practically constant over the whole region occupied by the conducting spheroid. Expansions with respect to the parameter l of eq. (22) are used as in the previous section and formulae are given for the current I_0 at the center of the antenna as a function of

$$\epsilon_{PA}=\frac{\omega}{c}f=2\pi\frac{f}{\lambda}. \qquad (24)$$

where f = half focal distance.

The ϵ_{PA} of Page and Adams should not be confused with the ϵ_A of M. Abraham or ϵ_B of M. Brillouin. The resonance frequency occurs when I_0^2 is a maximum, a condition that does not yield a current in exact phase with the field. Actually, the current leads the electromotive force at resonance by an angle φ which increases with increasing l.

The resonant wave-length is slightly shorter than the wave-length of free oscillation, as can be seen by comparing Tables IV and V. The

TABLE V

WAVE-LENGTHS AND CURRENTS AT RESONANCE

l	b/a	$\lambda/4a$	$I_0/\kappa e^{-i\omega t}$	I_{00}/κ	ϕ
0.000	0.00	1.0000	1.045	1.045	0°.0
0.020	1.02 $(10)^{-11}$	1.0004	$1.045-0.028i$	1.045	1°.5
0.050	3.34 $(10)^{-5}$	1.0027	$1.045-0.071i$	1.047	3°.9
0.080	1.42 $(10)^{-3}$	1.007	$1.045-0.113i$	1.050	6°.2
0.100	4.96 $(10)^{-3}$	1.011	$1.045-0.141i$	1.055	7°.7
0.125	1.35 $(10)^{-2}$	1.017	$1.046-0.175i$	1.061	9°.5

resonance condition would occur when

$$b_1(\epsilon_{PA})=1-0.38\epsilon^2_{PA}-0.010649\epsilon^4_{PA}$$
$$+0.000164\epsilon^6_{PA}=0.1475l^2\epsilon^2_{PA}\frac{1-0.0056l}{(1-0.43837l)^2}, \qquad (25)$$

$$k=2\pi caE_0, \qquad (26)$$

if the medium surrounding the antenna were a vacuum. For another dielectrical medium, the factor k would include $\sqrt{\kappa_e/\kappa_m}$ where κ_e is the dielectric constant and κ_m is the permeability. Fig. 12 shows the variation of $\lambda/4a$, I_{00}/k and φ as functions of the parameter l.

Page and Adams also computed the resonance curves for $l=0.02$, 0.05 and 0.1, as shown in

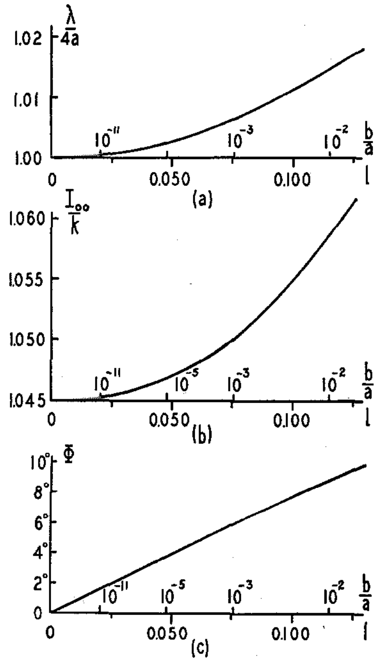


Fig. 12—(a) wave-length λ; (b) current amplitude I_{00}; (c) lead φ of current at resonance.

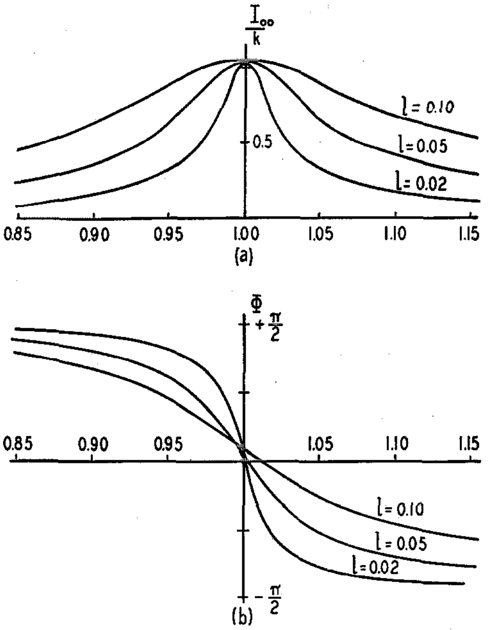


Fig. 13—(a) current amplitude I_{00}; (b) lead φ of current plotted against ratio ϵ/ϵ_r of frequency to frequency at resonance.

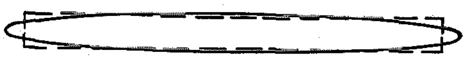


Fig. 14—Prolate spheroid equivalent to cylinder.

Fig. 13 and Tables VI, VII and VIII. For such long spheroids, one may note a small deviation from the classical type of resonance curve, but the Q factors computed from these curves (taking $1/Q=2(\Delta\omega/\omega)$ for $I/I_{\max}=1/\sqrt{2}$) do not differ very much from the Q_f factors obtained from free vibrations.

TABLE VI

CURRENT FOR $l=0.02$, $b/a=1.02\ (10)^{-11}$, $\epsilon_r=1.5701$

ϵ/ϵ_r	$I_0/\kappa e^{-i\omega t}$	I_{00}/κ	ϕ
0.875	$0.011-0.120i$	0.120	84°.8
0.950	$0.081-0.292i$	0.303	74°.6
0.975	$0.275-0.474i$	0.548	59°.9
0.990	$0.726-0.495i$	0.879	34°.3
1.000	$1.045-0.028i$	1.045	1°.5
1.010	$0.757+0.453i$	0.882	−30°.9
1.025	$0.313+0.464i$	0.560	−56°.0
1.050	$0.107+0.302i$	0.320	−70°.5
1.125	$0.022+0.137i$	0.139	−80°.7

TABLE VII

CURRENT FOR $l=0.05$, $b/a=3.34\ (10)^{-5}$, $\epsilon_r=1.5666$

ϵ/ϵ_r	$I_0/\kappa e^{-i\omega t}$	I_{00}/κ	ϕ
0.850	$0.042-0.238i$	0.242	80°.0
0.900	$0.104-0.347i$	0.363	73°.3
0.950	$0.352-0.529i$	0.635	56°.4
0.975	$0.707-0.525i$	0.880	36°.6
1.000	$1.045-0.071i$	1.047	3°.9
1.025	$0.783+0.418i$	0.888	−28°.1
1.050	$0.450+0.482i$	0.660	−46°.9
1.100	$0.182+0.360i$	0.403	−63°.2
1.150	$0.100+0.270i$	0.288	−69°.7

TABLE VIII

CURRENT FOR $l=0.10$, $b/a=4.96\ (10)^{-3}$, $\epsilon_r=1.5539$

ϵ/ϵ_r	$I_0/\kappa e^{-i\omega t}$	I_{00}/κ	ϕ
0.850	$0.144-0.428i$	0.451	71°.5
0.900	$0.306-0.546i$	0.625	60°.8
0.950	$0.669-0.575i$	0.882	40°.7
1.000	$1.045-0.141i$	1.055	7°.7
1.050	$0.822+0.360i$	0.897	−23°.6
1.100	$0.504+0.450i$	0.676	−41°.8
1.150	$0.326+0.410i$	0.524	−51°.5

Table IX shows that the curves of Fig. 11 could be used, at least as a first approximation and for small values of b/a, to give an idea of the properties of spheroidal antennae.

TABLE IX

$l=$	0.02	0.05	0.1
Q from resonance curve	32	13.1	6.1
Q_f from free oscillations	32	12.75	6.4

These results are not very easily checked with experimental data inasmuch as actual experiments never were performed on ellipsoids, but on cylindrical wires. It has been noticed that, for all eccentricities, the diameter of a prolate spheroid falls to half of its maximum value ($2b$) at a distance $x=0.866a$ from the center. Hence the prolate spheroid which best fits a right circular cylinder is one which is slightly wider at the center and somewhat longer, as illustrated in Fig. 14, but this is merely a rough first approximation. Page and Adams also computed the current distribution along the antenna; at resonance, it is given practically by

$$\frac{I_\xi}{I_0}=\cos\frac{\pi}{2}\xi \tag{27}$$

at a distance ξa from the center. The expression contains imaginary (out of phase) terms when the frequency differs from the resonance frequency. Using this current distribution, and knowing the electromotive force due to the external field, one is in a position to calculate the amount of energy absorbed per second (and reradiated at great distance). Writing $\frac{1}{2}R_pI_0^2$, a *radiation resistance* R_p is obtained (in ohms)

$$R_p=87.67\left(\frac{4a}{\lambda}\right)^2\left(\frac{\kappa_m}{\kappa_e}\right)^{1/2}\frac{a_1^4}{s_1}\times\left[1+\frac{\epsilon^2_{PA}}{150}b_1\left(1+\frac{2}{3}l\right)\right] \tag{28}$$

where a_1, s_1, b_1 are functions of ϵ_{PA} defined by their series expansion, and l is the parameter (eq. 22).

Tables X, XI, and Fig. 15 summarize the results of computations based on this formula.

TABLE X

RADIATION RESISTANCE AT RESONANCE

l	$\sqrt{\frac{\kappa_m}{\kappa_e}}R_p$ (ohms)
0.000	73.1
0.020	73.0
0.050	72.8
0.080	72.3
0.100	71.8
0.125	71.1

TABLE XI

RADIATION RESISTANCE $(\kappa_m/\kappa_e)^{1/2}R_p$, IN OHMS FOR FREQUENCIES NEAR RESONANCE

l \ ϵ/ϵ_r	0.850	0.900	0.950	1.000	1.050	1.100	1.150
0.050	55.3	61.0	66.9	72.8	78.7	84.6	90.6
0.100	54.5	60.2	66.0	71.8	77.7	83.6	89.4

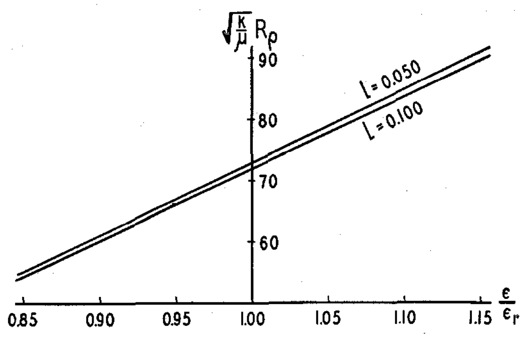


Fig. 15—Radiation resistance R_p in ohms plotted against ratio ϵ/ϵ_r of frequency to frequency at resonance.

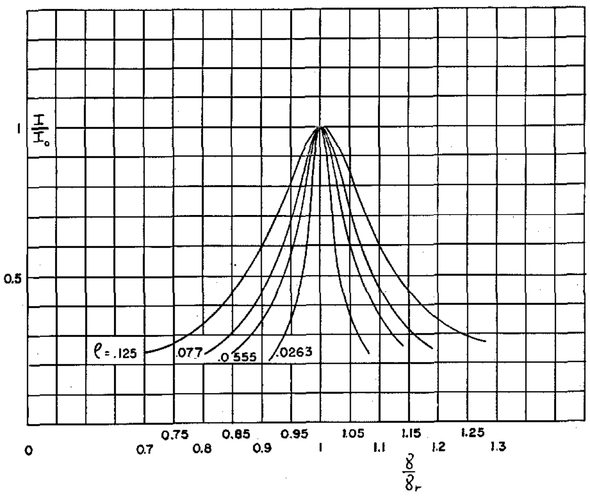


Fig. 16—Resonance curve for prolate spheroids of different ellipticities.

Page and Adams also give formulae for the field radiated at great distance. It should be emphasized here that the properties of the antenna, for forced vibrations, are very different from those of a classical LCR circuit. This is best illustrated by Table XI (or Fig. 15), which shows how much the radiation resistance varies with frequency, even for small departure from resonance. Ryder [8] developed a theory of forced vibrations induced in a long ellipsoid by the field of an incident wave polarized along the axis of symmetry. He used M. Abraham's method of approximation and obtained results which agree very well with those of M. Abraham but not so well with Page and Adams. The difference lies essentially in the two formulae (23) and (23a) for the logarithmic decrements or the corresponding Q factors.

Table XII summarizes the results for the resonance on the first fundamental vibration.

Computations have been performed for the following values of Ω and l:

$\Omega=\log\frac{\eta_0+1}{\eta_0-1}=$	10	15	20	40
$l=\frac{1}{\Omega-2}$	$=0.125$	0.077	0.0555	0.0263
$\frac{b}{a}$	$=1.35\ 10^{-2}$	$1.11\ 10^{-3}$	$9.08\ 10^{-5}$	$4.12\ 10^{-9}$
$\frac{\Delta\nu}{\nu_r}$	$=0.155$	0.103	0.078	0.039

The resonance curves drawn from the numerical data are shown in Fig. 16. (R_p is the radiation resistance, X the imaginary part of the impedance (reactance), φ the phase angle and ν/ν_r the ratio of actual frequency to resonance frequency.) For the first harmonic, near resonance, the band width * $2(\Delta\nu/\nu_r)$ can be easily obtained, and

$$2\frac{\Delta\nu}{\nu_r}=\frac{1.55}{\Omega}=\frac{1}{Q}. \tag{29}$$

Accordingly, one may compute the corresponding Q factors, which agree completely with M.

* The band width $2(\Delta\nu/\nu_r)$ is the frequency interval inside of which the current is greater than $(1/\sqrt{2})I_{0r}$.

TABLE XII

PROPERTIES OF THE FIRST HARMONIC NEAR RESONANCE

			$\log \frac{\eta_0+1}{\eta_0-1}=10$			$\log \frac{\eta_0+1}{\eta_0-1}=15$			$\log \frac{\eta_0+1}{\eta_0-1}=20$			$\log \frac{\eta_0+1}{\eta_0-1}=40$		
θ_1	ν/ν_r	R_p ohms	φ degrees	X ohms	I_0/I_{0r}	φ degrees	X ohms	I_0/I_{0r}	φ degrees	X ohms	I_0/I_{0r}	φ degrees	X ohms	I_0/I_{0r}
−1.2	0.75	51	72.6	−160	0.28									
−1.0	0.80	53	69.4	−140	0.34									
−0.8	0.84	56	64.7	−119	0.42	72.5	−180	0.29						
−0.6	0.88	60	57.7	− 95	0.53	67.2	−142	0.38	72.5	−190	0.30			
−0.4	0.919	63.7	46.4	− 67.0	0.684	57.6	−101	0.532	64.6	−134	0.426	76.6	−268	0.230
−0.2	0.959	68.04	27.7	− 35.66	0.884	38.2	− 53.5	0.784	46.4	− 71.3	0.688	64.5	−142.7	0.429
−0.1	0.980	70.47	14.7	− 18.44	0.966	21.4	− 27.65	0.930	27.6	− 36.87	0.885	46.3	− 73.74	0.690
−0.05	0.990	71.75				11.1	− 14.07	0.981	14.6	− 18.76	0.967	27.6	− 37.51	0.885
−0.025	0.995	72.41										14.6	− 18.92	0.967
0	1	73.08	0	0	1.000	0	0	1.000	0	0	1.000	0	0	1.000
0.025	1.005	73.76										−14.6	19.26	0.968
0.05	1.010	74.46				−11.1	14.57	0.980	−14.6	19.43	0.967	−27.6	38.86	0.885
0.1	1.020	75.89	−14.7	19.79	0.967	−21.4	29.68	0.930	−27.5	39.57	0.885	−46.2	79.14	0.690
0.2	1.041	78.93	−27.5	41.08	0.885	−38.0	61.61	0.787	−46.2	82.16	0.692	−64.4	164.3	0.432
0.4	1.081	85.8	−46.0	88.9	0.689	−57.3	133	0.538	−64.3	178	0.432	−76.4	356	0.233
0.6	1.12	94	−57.2	146	0.53	−66.7	219	0.39	−72.1	292	0.30			
0.8	1.16	104	−64.1	210	0.43	−72.1	320	0.30						
1.0	1.20	116	−68.7	300	0.35									
1.2	1.25	130	−72.0	400	0.29									

Abraham's results (eq. 23a):

$$\text{Ryder} \qquad Q=\frac{\Omega}{1.55},$$

$$\text{M. Abraham} \quad Q=\frac{\pi}{\delta}=\frac{\pi}{4.87}\Omega=\frac{\Omega}{1.55}.$$

Further, Q factors computed from Page and Adams for free or forced oscillations also yield consistent results; however, the two groups of results differ slightly (Fig. 17), due to the discrepancy between formulae (23) and (23a).

Here again, for antennae, one must stress the large variation of the resistance term R_p as a function of frequency. It is very surprising that the resonance curves are still almost symmetrical and that a Q factor can be defined which summarizes the whole behaviour near resonance. This point will be discussed hereinafter (section 9).

7. *Spheroids as Transmitting Antennae*

Thus far, only the properties of free vibrations or receiving antennae have been discussed; Ryder also takes into consideration the forced vibrations in a transmitting antenna. (Problem C, §1.) As an example, he considers a vertical half spheroid (vibrating almost as a quarter-wave antenna) excited at the base above a perfectly conducting ground. The antenna and its image are excited together and may be regarded approximately as a single, centrally driven prolate spheroid. Calling $Ve^{-i\omega t}$ the driving e.m.f. between antenna and ground, one must assume an electromotive force $2\,Ve^{-i\omega t}$ in the gap between both half spheroids. This will excite the fundamental vibrations ($n=1$) together with higher vibrations of the same parity ($n=3,\ 5\cdots$). Ryder considers the contribution of the first two modes ($n=1,\ 3$) and finds the current at the

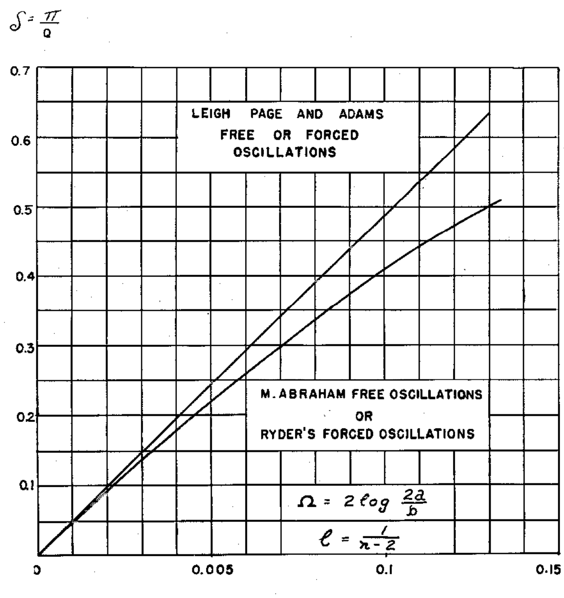


Fig. 17—Logarithmic decrement for thin ellipsoids according to different authors.

center of the spheroid, at *resonance frequency* for $n=1$,

$$\sqrt{\frac{\kappa_m}{\kappa_e}}I(0)=0.02738\,V-\frac{i}{\Omega}0.00545\,V. \tag{30}$$

The first term, which corresponds to the resonance on the fundamental, is the most important; the second term, entirely out of phase, is the contribution of the mode $n=3$.

The definition of "resonance" leads to some difficulties due to the superposition of the different modes: the maximum central current is obtained for a frequency which differs slightly from the frequency yielding zero phase angle, or the one for which there is maximum energy radiation at great distance. All these discrepancies are very small for the long and thin ellipsoids discussed by Ryder, but these differences would become larger and larger for broader spheroids and for spheres.

Returning to the centrally excited half-wave antenna, one may compute its effective radiation resistance R_{eff}, resulting from both terms ($n=1$, $n=3$) of eq. (30):

$$R_1I_1^2+R_3|I_3|^2=R_{\text{eff}}|I|^2,$$

$$R_{\text{eff}}=73.08\sqrt{\frac{\kappa_m}{\kappa_e}}\left[1-\frac{0.04}{\Omega^2}\right]\text{ ohms.} \tag{31}$$

Thus the radiation resistance of the antenna is just slightly lowered by the presence of the higher mode ($n=3$). The radiation field at great distance is practically the same as if the third harmonic were absent. The superposition of the fundamental and third harmonic yields current distribution which differs materially from the sinusoidal distribution along the antenna but the third harmonic, being out of phase, does not contribute much to the energy radiation at great distance.

Many other interesting numerical results can be found in Ryder's paper, as for instance the radiation resistance at resonance for the pth mode:

This is the radiation resistance corresponding to the pth mode at resonance, as if it were the only one excited. As a rule, other modes are excited at the same time (as shown in eq. 31 for the fundamental vibration) and require corrective terms. The radiation resistance of each harmonic, in the neighborhood of resonance, varies with the frequency, as seen in Table XII for the first resonance.

The results obtained by Ryder apply only to very thin ellipsoids of eccentricity near unity. They have been completed and extended by Stratton and Chu [9] who discussed the resonance properties of ellipsoids of all eccentricities ranging from the very long ones up to the sphere. They assumed an electromotive force applied across the equator, and computed the radiation impedance to be measured at this driving point. The method is essentially the same as in Ryder's paper (the reader should note that ξ and η are interchanged), and the boundary condition corresponds to ellipsoids of infinite conductivity. Hence the tangential component of the electric field is zero everywhere except in the region near the equator where the driving voltage V is applied. The authors use the spheroidal functions investigated by Stratton [10] and are thus enabled to discuss all eccentricities completely. The current I_0 across the equator is obtained and the ratio

$$Y_i=\frac{I_0}{V}=\frac{1}{Z_i} \tag{33}$$

represents the input admittance Y_i, which is the reciprocal of the input impedance Z_i. The input admittance Y_i is the sum of an infinite number of terms, each of which corresponds to one of the normal modes of vibration of the ellipsoid. The result is somewhat similar to the one obtained with an infinite number of circuits connected in parallel with the voltage V impressed on the common terminals

$p=$	1	2	3	4	5	6	7	8	9	10
$R_p=$	73.08	93.37	105.4	114	120.7	126.1	130.8	134.8	138.3	141.4

(32)

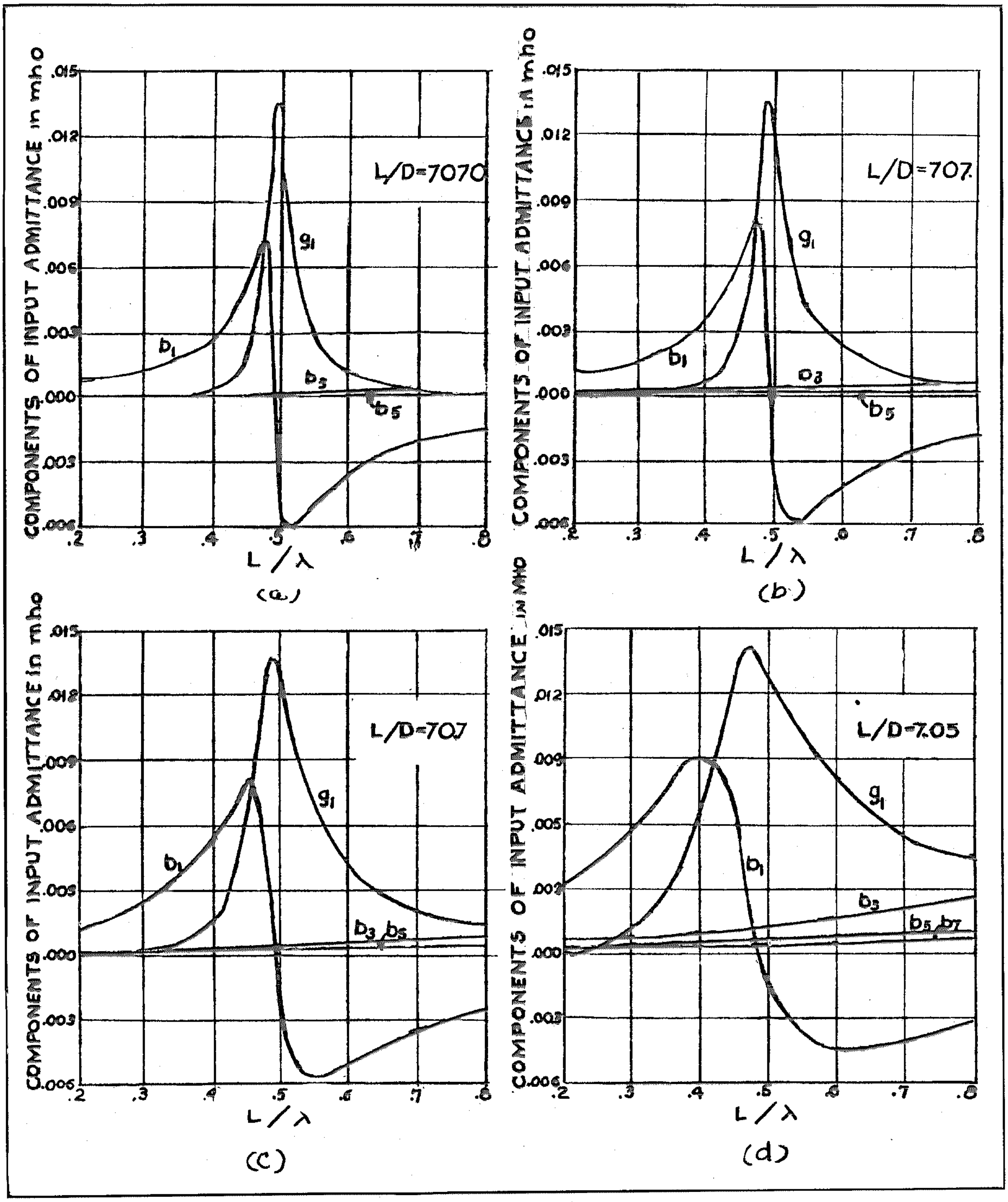


Fig. 18—Components of input admittance as functions of L/λ.

$$Y_i = \sum_l Y_{l+1},$$

$$Y_{l+1} = -\frac{ik_2 f}{60}\frac{(\xi_0^2-1)Re_l^4(\xi_0)[Se_l'(0)]^4}{\frac{d}{d\xi_0}[(\xi_0^2-1)^2 Re_l^4(\xi_0)]N_l} mhos, \quad (34)$$

k_2, ϵ_2, μ_2 refer to the vacuum, outside the conductor; $k_2=\omega\sqrt{\epsilon_2\mu_2}$. *Re* and *Se* are the functions defined by Stratton, and $1/\xi_0$ (instead of $1/\eta_0$ as in Page and Adams; Ryder) represents the eccentricity of the ellipsoid while N_l is a normalizing factor. The total input admittance Y_i is obtained by taking the sum of Y_{l+1} over even values of l. The real part of the admittance component is the conductance g_{l+1} while the imaginary part is the susceptance b_{l+1}.

$$Y_{l+1}=g_{l+1}+ib_{l+1}. \quad (35)$$

The results are shown in Fig. 18 where the components of input admittance are plotted versus L/λ for a series of values of the ratio L/D. L is the length of the major axis, D the length of the minor axis of the metal spheroid, and λ is the wave length measured in free space (L/D corresponds to a/b in the preceding section).

radiation resistance is zero at $L/\lambda=0$ and reaches the value 72 ohms when L/λ is slightly less than one-half. This is in accord with the result obtained by computing the radiation from retarded potentials, assuming a sinusoidal current distribution. The fact that this occurs at a point slightly below $L/\lambda=0.5$ shows the effect of a finite thickness of antenna and of current distribution that deviates from the sinusoidal. The radiation resistance attains a maximum somewhat below $L/\lambda=1$ and passes through subsequent minima near $L/\lambda=3/2,\ 5/2\ \cdots$ (this checks the results of Ryder, eq. 32) at the resonance points of the higher modes $n=3,\ 5\ \cdots$. The reactance curves also are included on

$$\frac{L}{D}=\frac{\xi_0}{\sqrt{\xi_0^2-1}}=\frac{1}{\sqrt{1-\dfrac{1}{\xi_0^2}}} \qquad \text{Stratton and Chu} \qquad \text{eccentricity } \frac{1}{\xi_0}$$

$$\frac{b}{a}=\sqrt{1-\frac{1}{\eta_0^2}} \qquad \text{eq. (15)} \qquad \text{eccentricity } \frac{1}{\eta_0} \quad (36)$$

The curves for very long ellipsoids ($L/D=7070$ or 707) are very similar to the usual resonance curves, and these first two cases correspond to the ones discussed by Ryder. When the ratio L/D decreases (broad ellipsoids), the shape of the curves changes gradually and their dissymmetry increases. The first mode of vibration exhibits a resonance near $L/\lambda=0.5$, while the higher modes ($n=3,\ 5,\ 7$) would show similar properties around

$$\frac{L}{\lambda}=\frac{3}{2},\frac{5}{2},\frac{7}{2}\cdots.$$

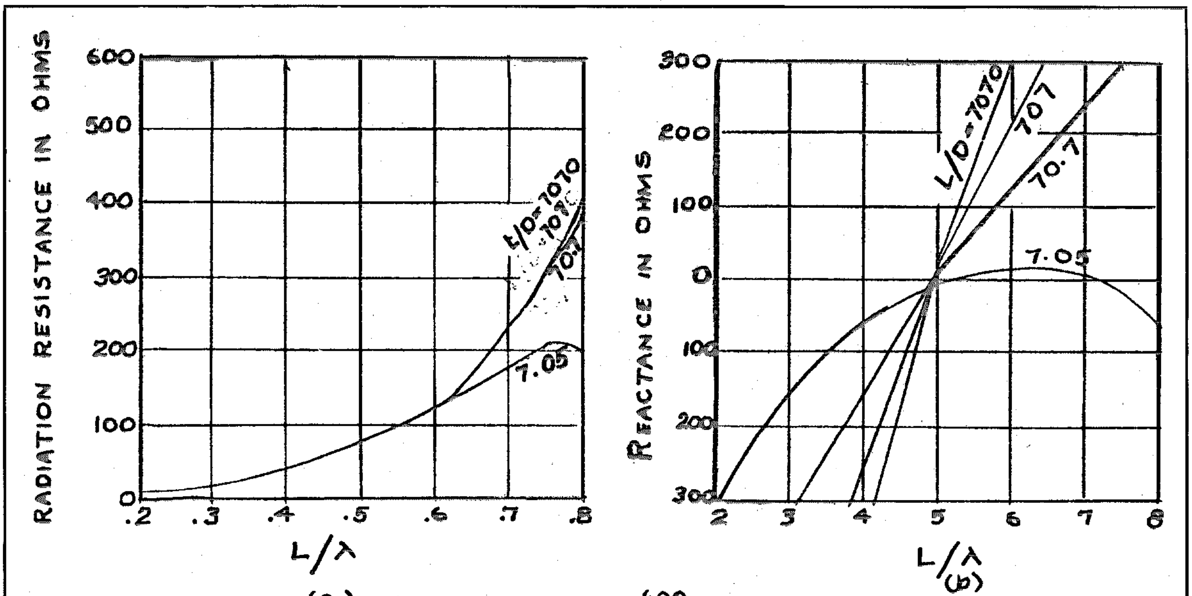
It is obvious from the shape of the curves that the definition of a Q factor would be impossible for broad ellipsoids, and particularly for the sphere. In these cases the b and g curves appear completely distorted; the g curve is very flat and the b curve is always positive (no negative part for frequencies above resonance).

For thin and long ellipsoids, the successive resonances are well separated from each other. The curves plotted in Fig. 18 are not far wrong if the contribution of higher modes be ignored. On this basis, the real and imaginary parts and the absolute magnitude of the input impedance have been plotted in Fig. 19. The real part or graph "b" of Fig. 19. They pass through zero near resonance, become positive, then negative, and start again from negative to positive near the next resonance point. This pattern is especially marked for the ellipsoid $L/D=7.05$. This ellipsoid has a very small reactance over a large range of L/λ (0.4 to 0.8), a property which may prove of great practical value since, at the same time, the radiation resistance does not vary too much (from 72 to 210 ohms).

The last graph "C" shows the total input impedance. Reverting to the curve for $L/D=7.05$, it passes through a minimum for $L/\lambda=0.45$, which means $\lambda/4a=\lambda/2L=1.1$ and $b/a=D/L=0.14$. This point is represented by a cross in Fig. 11 and lies very near the curve for proper frequencies. From Figure 19-C one can compute the approximate width of the band of resonance.

$$\frac{b}{a}=0.14, \quad 2\frac{\Delta\omega}{\omega}=\frac{0.57-0.36}{0.45}=\frac{1}{2.15}=\frac{1}{Q}.$$

The Q factor of about 2.15 compares with 2.25 obtained from Fig. 11. Thus the definition of the Q factor holds rather well for broad ellipsoids. (These two points are indicated by crosses in Fig. 11.)



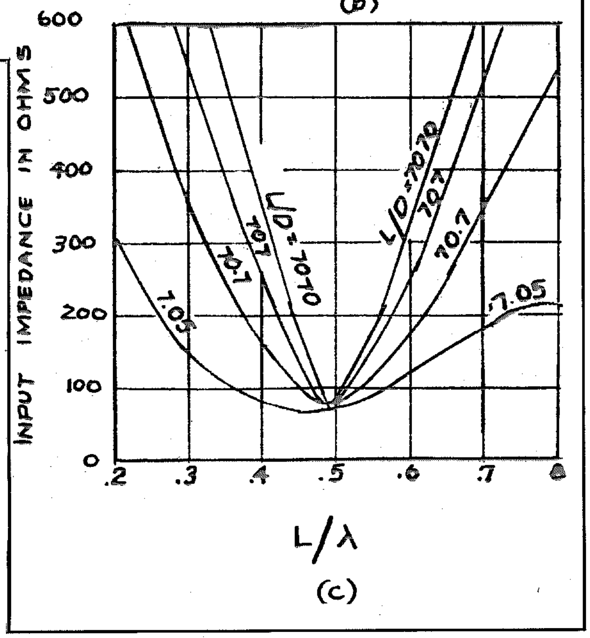


Fig. 19—*Input impedance and its real and imaginary parts as functions of* L/λ.

8. Spheres as Transmitting Antennae

The forced oscillations of a sphere are discussed by Stratton and Chu in Part II of their paper.[7] This example shows an exaggeration of some of the characteristics already noticed for the case of ellipsoids: flat resonance curves, superposition of the successive modes of vibration, and distortion of the admittance curves.

Here again the input admittance Y_i (eq. 33) is obtained as the sum of an infinite number of terms, each corresponding to a certain mode of vibration as in (eq. 34). The corresponding formula (Stratton and Chu, eq. 20, p. 238) reads

$$Y_i=\frac{I_0}{V}=\sum_n Y_n,$$

$$Y_n=\pi\frac{2n+1}{n(n+1)}\frac{[P_n'(0)]^2}{Z_n}=g_n+ib_n, \qquad (37)$$

where

$$P_n'(\cos\theta)=-\frac{d}{d\theta}P_n(\cos\theta)$$

is a Legendre polynomial, which for $\cos\theta=0$ is

$$P_n'(0)=\frac{i^{n-1}2^{1-n}n!}{\left(\frac{n-1}{2}!\right)^2}. \qquad (38)$$

Only odd modes of vibration ($n=1, 3, 5, 7\cdots$) appear in these formulae inasmuch as even modes can not be excited by an electromotive force applied across the equator. Hence $(P_n'(0))^2$ is a real quantity. Z_n represents a complex function depending on $2\pi a/\lambda$; a is the radius of the sphere and λ the wave length in free space. For a perfectly conducting sphere the formula for Z_n

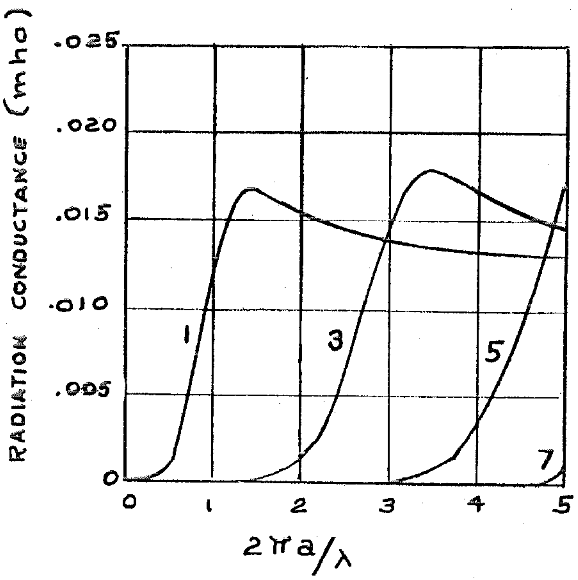


Fig. 20—Radiation conductance of first few modes as functions of $2\pi a/\lambda$.

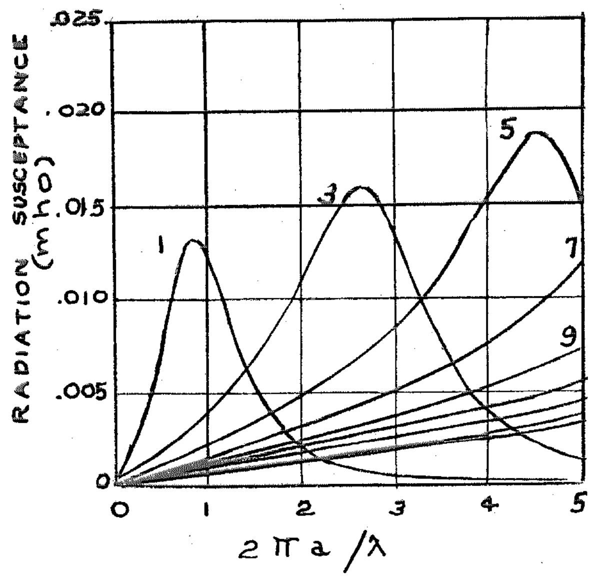


Fig. 21—Radiation susceptance of first few modes as functions of $2\pi a/\lambda$.

reduces in the following two extreme cases to

$$\frac{a}{\lambda}\to\infty,\quad Z_n=\sqrt{\frac{\mu_2}{\epsilon_2}},$$

$$\frac{a}{\lambda}\to 0,\quad Z_n=i\sqrt{\frac{\mu_2}{\epsilon_2}}\left[-\frac{n\lambda}{2\pi a}+\frac{2\pi a}{\lambda(2n-1)}\cdots\right],\quad (39)$$

$\sigma_1=\infty$ conductivity of the metallic sphere,

$\epsilon_2,\ \mu_2$ dielectric constant and permeability of the free space in rationalized units.

The real and imaginary parts of Y_n are, respectively, conductance g_n and susceptance b_n. If one assumes the conductivity σ_1 of the sphere to be infinite, the admittance Y_n reduces to a pure conductance as $2\pi a/\lambda\to\infty$ and contains only susceptance when $2\pi a/\lambda\ll n$. For intermediate values of $2\pi a/\lambda$ the admittance is complex.

In Figs. 20 and 21 the real and imaginary parts of the admittance Y_n are plotted as functions of $2\pi a/\lambda$. The conductivity of the sphere is assumed infinite; hence there is no loss in the metal and the ordinates represent radiation conductance and radiation susceptance. The conductance associated with each mode of vibration rises to a peak at resonance, and then drops off gradually to a constant value. The total conductance rises step-wise with increasing a/λ, as shown on Fig. 23. Thus, as the applied frequency is increased, the harmonic terms add their contributions successively, each new term giving rise to a corresponding increase in the radiation. Conversely, for a fixed wave length λ, an increase in the radius a of the sphere increases its effectiveness as a radiator.

From Fig. 21 it appears that the radiation susceptance of each successive harmonic rises to a peak near resonance and then falls rapidly to zero as the wave length decreases. The *susceptance is always positive*, showing that the *admittance is capacitive.* It is evident from the figure that the sum of the susceptances fails to approach a limit. The ratio Y_n/Y_{n-2} in fact approaches unity as $n\to\infty$. This is a consequence of the assumption that the impressed voltage is applied across a surface of discontinuity at the equator $\theta=\pi/2$, a singularity implying an infinite value of field intensity. Actually the voltage will be applied across a strip or gap of small, but finite, width with a corresponding decrease in the magnitudes of the higher harmonics. The general form of the individual conductance and susceptance curves will not be materially altered, but their sum then converges as it obviously must on physical grounds. The situation in the case of the sphere is similar to the one for ellipsoids, where the same mathematical difficulty is encountered. For instance, if one assumes

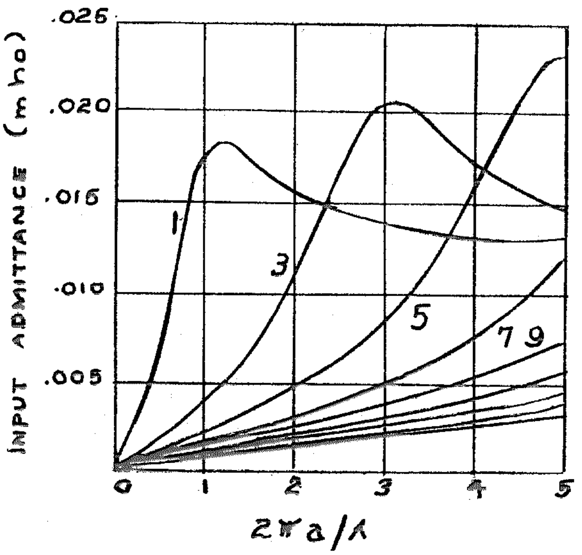


Fig. 22—Input admittance of first few modes as functions of $2\pi a/\lambda$.

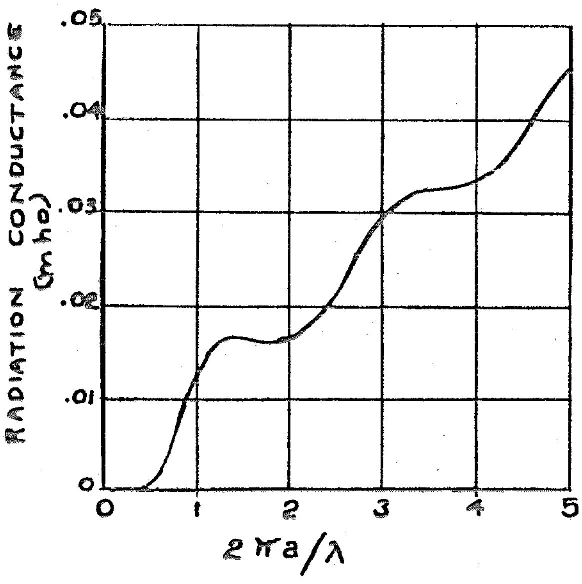


Fig. 23—Total radiation conductance as function of $2\pi a/\lambda$.

the voltage to be applied across a gap or a strip of about $a/10$ across the equator, the ninth harmonic ($n=9$) will be only weakly excited and all higher harmonics will be very weak and can be practically omitted.

Fig. 22 illustrates the amplitudes or absolute values of the first few admittances, each of them taken separately. A more interesting graph is shown in Fig. 23, where the total radiation conductance

$$g = \sum g_n \qquad (40a)$$

is plotted as a function of $2\pi a/\lambda$. This curve is shown again in Fig. 24, together with the curve giving the total susceptance

$$b = \sum b_n, \quad n = 1, 3, 5, 7 \qquad (40b)$$

computed for the first modes of vibration (harmonics 1, 3, 5, 7) according to the above assumption of applying voltage across the equator along a gap or strip wide enough to prevent the excitation of higher modes. From these two curves, the absolute value

$$|Y| = \sqrt{g^2 + b^2} \qquad (40c)$$

of the input admittance is obtained.* It should be noted that all three curve rise step-wise, and show some interesting features. For $1 < (2\pi a/\lambda) < 2.2$ and $3 < (2\pi a/\lambda) < 4$, there is a wide band of almost constant g, b, $|Y|$.

These two wide bands might prove very useful for practical purposes (wide band transmission antennae). Since the susceptance of each mode (or harmonic term) is always positive, the input admittance under no condition becomes a real quantity. The total *admittance is always capacitive.*

It is apparent that the resonance phenomena associated with a radiating sphere differ fundamentally from those encountered in conventional circuit theory. This point will be discussed in the next section where consideration is given to modification of the theory of the usual L.C.R. circuits in order to make it fit the resonance curves observed for long ellipsoids.

From the results obtained for long ellipsoids, one may plot curves rather similar to the ones of Fig. 24 for the sphere. This is illustrated in Fig. 25, which should be considered only as a *qualitative* description of the facts as it was drawn without using any numerical data. For a

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* Stratton and Chu do not give any numerical tables, but only the curves of Figs. 20, 21, 22, 23. The curves of Fig. 24 have been plotted by the author from these data, and cannot be very accurate. Errors of a few per cent should be allowed for.

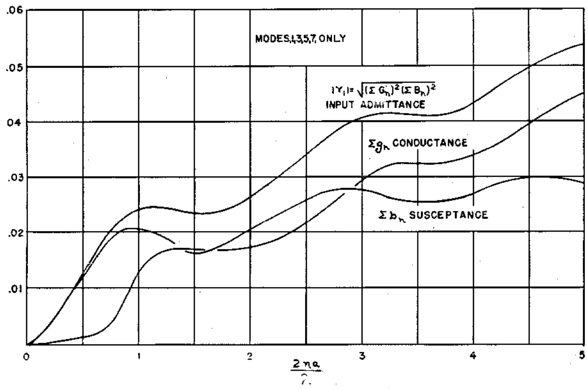

Fig. 24—Total input conductance g, susceptance b, admittance |Y| of a sphere, taking into account modes 1, 3, 5, 7 only.

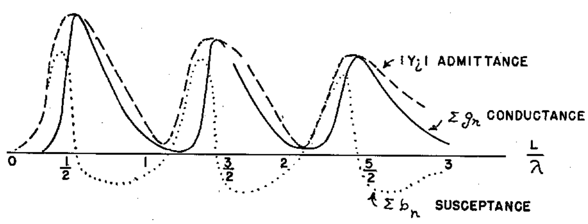

Fig. 25—Qualitative behaviour of b, g, |Y| for a thin ellipsoid.

long ellipsoid, the successive resonances on the individual harmonics are separated rather widely. For each harmonic resonance, the curves g, b, $|Y|$ should repeat approximately the same behaviour as on the fundamental resonance. The only well established numerical result seems to be the regular increase of the radiation resistance (§7, eq. 32) which results in a corresponding decrease of the conductance, as shown in Fig. 25.

Fig. 25 represents an attempt to show the progressive distortion of the curves when starting with a much elongated ellipsoid (which is equivalent to a number of usual tuning circuits all connected in parallel) and ending with broad ellipsoids and sphere (Fig. 24).

9. *Antenna Comparison with Properties of Usual Resonant Circuits*

A comparison with usual L.C.R. circuits will enable one to get a better understanding of the special properties of antennae. Summarizing briefly the fundamental properties of the L.C.R. circuit:

resistance R — constant

reactance $J = L\omega - \dfrac{1}{C\omega}$ — zero at resonance ω_0

impedance $Z = R + iJ$ — $E = ZI$

absolute value of the input impedance $|Z|$ — $|Z|^2 = R^2 + J^2$ (41)

admittance $Y = g + ib = \dfrac{1}{Z} = \dfrac{R - iJ}{R^2 + J^2}$.

The frequency dependence of these quantities is shown in Fig. 26. Assuming a small variation $\delta\omega$ of frequency near resonance, a corresponding small change δZ in impedance occurs.

$$\omega = \omega_0 + \delta\omega, \qquad Z = Z_0 + \delta Z = R + \delta Z,$$

$$\frac{\delta Z}{Z_0} = \frac{i}{R}\left(L\delta\omega + \frac{\delta\omega}{C\omega_0^2}\right) = 2iQ\frac{\delta\omega}{\omega_0}, \qquad (42)$$

$$\frac{\delta|Z|^2}{Z_0^2} = 4Q^2\left(\frac{\delta\omega}{\omega_0}\right)^2, \quad Q = \frac{L\omega_0}{R} = \frac{1}{RC\omega_0}.$$

If the resonance is flat, the formulae for δZ or $\delta|Z|^2$ can be used only in the neighborhood of resonance. In the case of sharp resonance, the same formulae may apply even for a large change δZ or $\delta|Z|^2$. As an example: a decrease of current by a factor $1/\sqrt{2}$ means an increase of $|Z|^2$ by a factor 2 or

$$I = \frac{1}{\sqrt{2}}I_0, \quad |Z|^2 = 2|Z_0|^2, \quad \delta Z = iZ_0, \quad \frac{\delta\omega}{\omega_0} = \frac{1}{2Q}, \qquad (43)$$

as shown in the fourth drawing of Fig. 26. The "band width" $2\delta\omega/\omega_0$ or band of frequencies yielding $|I| \geqslant (1/\sqrt{2})I_0$ is equal to $1/Q$. This classical type of circuit with lumped properties is especially simple since it is assumed that the current and charge distribution along the whole circuit remains constant and independent of the frequency applied. Similarly, for usual circuits, change in current distribution is confined to minute variation occasioned by the skin effect factor.

The situation is different in the case of antennae. Here one must deal with continuous distribution of currents and charges along the aerial, and this distribution depends greatly on frequency. A certain vibration (mode $n = 9$ or ninth harmonic, for instance) is characterized by $n+1$ nodes in the current distribution including the two nodes at the ends of the antenna, but the position of the nodes may vary considerably,

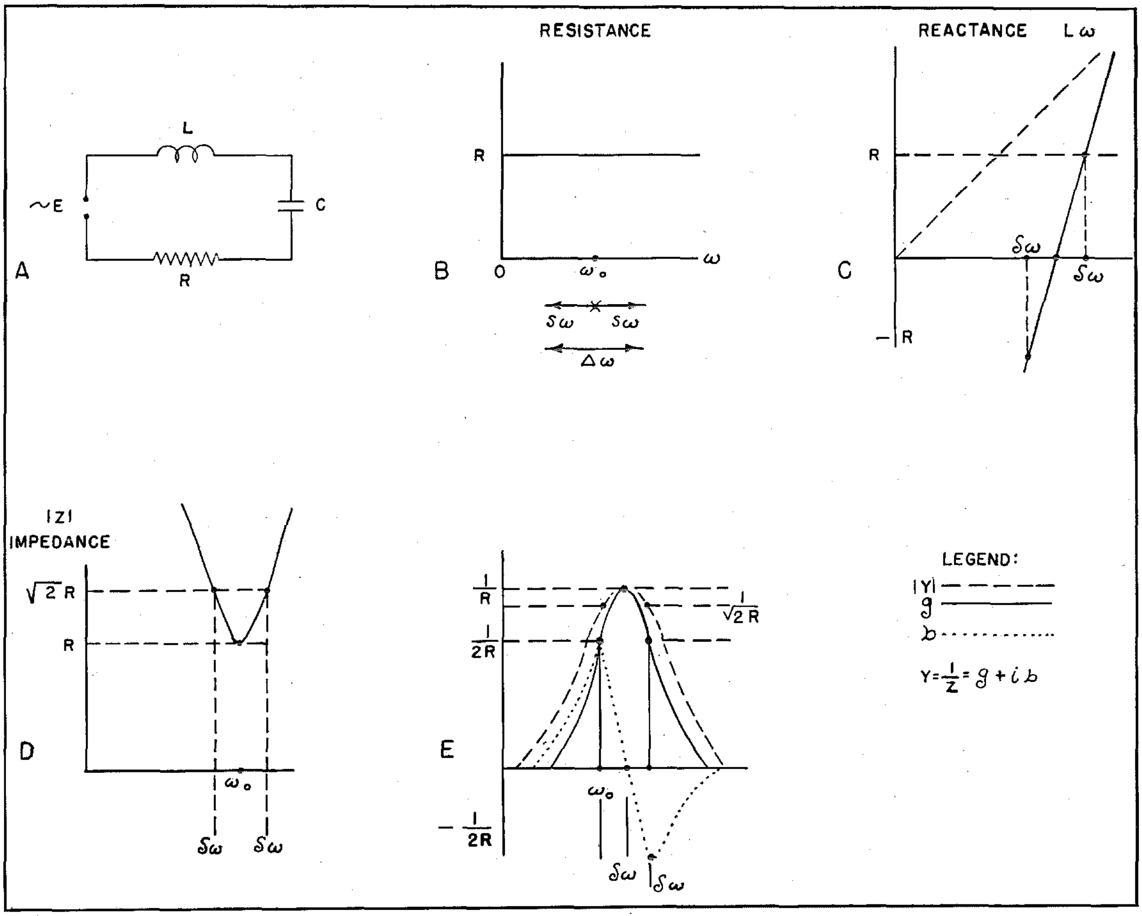

Fig. 26—Comparison of an antenna with a conventional circuit.

as does the current distribution between the nodes. This is shown in Fig. 27, at least qualitatively, and emphasized by Ryder as of great significance.

Change in current distribution results in a corresponding change in the L, C, R coefficients for the equivalent circuit. The variation of the resistance term is clearly shown in Fig. 15 (from L. Page and Adams) and Fig. 19a (from Stratton and Chu).

In attempting to explain the special properties of antennae it will be assumed that, in the neighborhood of resonance, their L, C, and R coefficients vary linearly with frequency (Fig. 28). For the resistance term, this results clearly from the curves; and further, it is known that resistance increases with ω. Consequently, in a small interval of frequencies,

$$\begin{aligned} &\omega=\omega_0+\Delta\omega, \qquad L_0\omega_0=\frac{1}{C_0\omega_0},\\ &R=R_0\left(1+\rho\frac{\Delta\omega}{\omega_0}\right), \quad \rho>0,\\ &L=L_0\left(1+l\frac{\Delta\omega}{\omega_0}\right),\\ &C=C_0\left(1+\gamma\frac{\Delta\omega}{\omega_0}\right), \quad Z=R+i\left(L\omega-\frac{1}{C\omega}\right), \end{aligned} \tag{44}$$

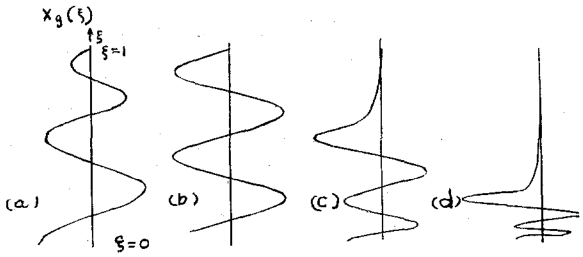

Fig. 27—Qualitative distribution of current in the ninth harmonic: (a) low frequency, (b) resonant frequency, (c) and (d) above resonance.

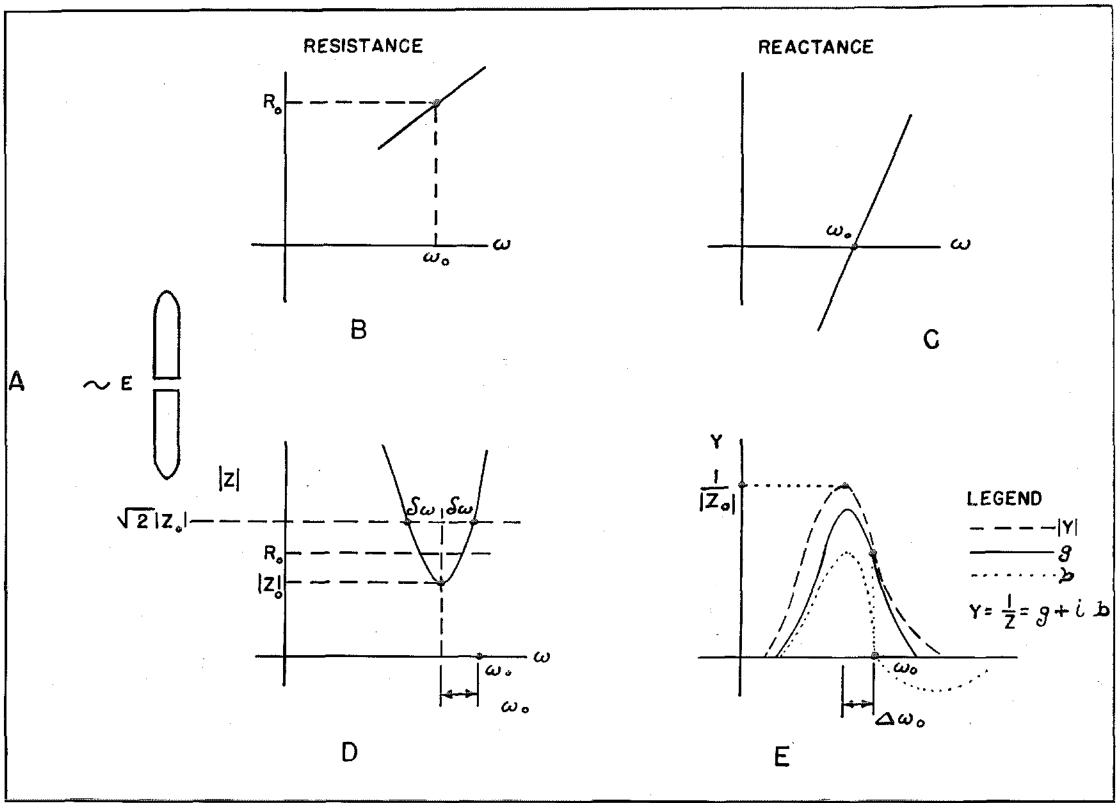

Fig. 28—Properties of a circuit with variable L.C.R.

ω_0 being the frequency which yields zero reactance.

$$L\omega - \frac{1}{C\omega} = L_0\omega_0\left[1+(1+l)\frac{\Delta\omega}{\omega_0}\right] - \frac{1}{C_0\omega_0}\left[1-(1+\gamma)\frac{\Delta\omega}{\omega_0}\right] = L_0\omega_0(2+l+\gamma)\frac{\Delta\omega}{\omega_0},$$

$$\frac{Z}{R_0} = 1+\frac{\Delta\omega}{\omega_0}\left[\rho+i\frac{L_0\omega_0}{R_0}(2+l+\gamma)\right], \quad (45)$$

$$\frac{|Z|^2}{R_0^2} = 1+2\rho\frac{\Delta\omega}{\omega_0} + \left(\frac{\Delta\omega}{\omega_0}\right)^2\left[\left(\frac{L_0\omega_0}{R_0}\right)^2(2+l+\gamma)^2+\rho^2\right],$$

$|Z|^2$ is not minimum for ω_0 but for a slightly lower frequency, corresponding to

$$\frac{\Delta\omega_0}{\omega_0} = -\frac{\rho}{\rho^2+\left(\frac{L_0\omega_0}{R_0}\right)^2(2+l+\gamma)^2}, \quad (46)$$

a result in qualitative agreement with the theoretical computation and with the curves of Fig. 13 or 19.

On this "apparent" resonance frequency $\omega_0+\Delta\omega_0$ the impedance Z_0 is not real but contains an imaginary term (reactance)

$$\frac{Z_0}{R_0} = 1-\rho\frac{\rho+i\frac{L_0\omega_0}{R_0}(2+l+\gamma)}{\rho^2+\left(\frac{L_0\omega_0}{R_0}\right)^2(2+l+\gamma)^2},$$

$$\frac{|Z_0|^2}{R_0^2} = \frac{\left(\frac{L_0\omega_0}{R_0}\right)^2(2+l+\gamma)^2}{\rho^2+\left(\frac{L_0\omega_0}{R_0}\right)^2(2+l+\gamma)^2} < 1. \quad (47)$$

Hence $|Z_0|^2$ on the point of apparent resonance $\omega_0+\Delta\omega_0$, is smaller than the R_0^2 measured on ω_0 (zero reactance point).

The variation of $|Z|^2$ near this apparent resonance can be computed by taking

$$\Delta\omega = \Delta\omega_0 + \delta\omega$$

where $\delta\omega$ is the distance (in frequency) from apparent resonance $\omega_0+\Delta\omega_0$ and calling $\delta|Z|^2$ the change of $|Z|^2$

$$|Z|^2 = |Z_0|^2 + \delta|Z|^2;$$

the result is

$$\frac{\delta|Z|^2}{R_0^2}=\left(\frac{\delta\omega}{\omega_0}\right)^2\left[\rho^2+\left(\frac{L_0\omega_0}{R_0}\right)^2(2+l+\gamma)^2\right]. \quad (48)$$

In comparison with formulae corresponding to a usual circuit, an "apparent" Q_a factor may be defined as

$$\frac{\delta|Z|^2}{|Z_0|^2}=4Q_a^2\left(\frac{\delta\omega}{\omega_0}\right)^2 \quad (49)$$

where

$$Q_a^2=\frac{R_0^2}{4|Z_0|^2}\left[\rho^2+\left(\frac{L_0\omega_0}{R_0}\right)^2(2+l+\gamma)^2\right]$$

$$=\frac{1}{4}\left[\frac{\rho^2+\left(\frac{L_0\omega_0}{R_0}\right)^2(2+l+\gamma)^2}{\frac{L_0\omega_0}{R_0}(2+l+\gamma)}\right]^2. \quad (50)$$

Thus our circuit with variable LCR exhibits a resonance curve very similar to the one found for a usual circuit except that the "apparent resonance" yielding maximum current occurs at a frequency $\omega_0+\Delta\omega_0$ lower than the ω_0 corresponding to zero reactance. Further, the sharpness of resonance can be measured by a Q_a factor

$$Q_a=\frac{1}{2}\frac{\rho^2+\left(\frac{L_0\omega_0}{R_0}\right)^2(2+l+\gamma)^2}{\frac{L_0\omega_0}{R_0}(2+l+\gamma)}$$

$$=\frac{L_0\omega_0}{R_0}\left(1+\frac{l+\gamma}{2}\right)+\frac{\rho^2R_0}{2L_0\omega_0(2+l+\gamma)}, \quad (51)$$

corresponding qualitatively to the results obtained for ellipsoids of eccentricity near unity. Moreover, the required correction of the Q factor should be small if the Q_0 factor of the equivalent circuit ($L_0\omega_0/R_0$) is large and if the ρ, l, γ terms are not too large. This shows why the Q_a factor obtained from the resonance curve for forced oscillations may practically coincide with $Q_f=\pi/\delta$ computed from the logarithmic decrement δ of free oscillations.

All these explanations are purely qualitative, but they give a fairly good picture of the behaviour of elongated ellipsoids. Broad ellipsoids or spheres can not be treated in the same way as they behave very differently and show no similarity with usual tuning circuits. Variations of L, C, R, are great for such antennae and the Q factors are exceedingly small. This marked exaggeration of all these anomalous features would make any attempt at a classical explanation very dangerous.

(To be concluded)

NOTE: Figs. 1, 2, 3, 5 and 6 are reproduced from J. A. Stratton's "Electromagnetic Theory" (McGraw-Hill); Figs. 8, 9, 10, 12, 13, 14 and 15, from "The Electrical Oscillations of a Prolate Spheroid," Paper I, by L. Page and N. I. Adams, Jr. (*Physical Review*, Vol. 53, May 15, 1938); and Figs. 18, 19, 20, 21, 22 and 23 from "Steady-State Solutions of Electromagnetic Field Problems" by J. A. Stratton and L. J. Chu (*Journal of Applied Physics*, Vol. 12, No. 3, March 1941).

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E. M. Deloraine

Elected a Director of I. T. & T.

E. M. Deloraine, General Director of the Laboratories Division of Federal Telephone and Radio Corporation, manufacturing associate of International Telephone and Telegraph Corporation, was elected a Director of I. T. & T. at a meeting of the Board of Directors on February 2, 1944.

Mr. Deloraine was born in Paris, France. In 1917, he joined the Signal Corps of the French Army and was assigned to research work at the Eiffel Tower under General Ferrie. Resuming his studies after the Armistice at l'Ecole de Physique et Chimie, Paris University, he was graduated in 1920 as a physicist with the highest honors. After an additional year of work at the Eiffel Tower, he joined the London engineering staff of International Western Electric Company, reporting to Sir Frank Gill. His initial duties were concerned with broadcasting at experimental station 2WP, and, up to 1925, he was responsible for part of the developments in Great Britain pertaining to the establishment of the first commercial trans-Atlantic telephone circuit.

In 1925, I. T. & T. purchased the International Western Electric Company, changing its name to International Standard Electric Corporation. In 1927 he became active in the direction of the I. S. E. Communication Laboratories and also started I. S. E.'s Paris Laboratories. He was made European Technical Director of I. S. E. in 1933.

During this period Mr. Deloraine was in direct charge of developments leading to the establishment of the first Madrid-Buenos Aires radio telephone circuit. Under his direction in 1931, single sideband shortwave radio telephony was demonstrated between Buenos Aires and Madrid, and between Madrid and Paris, establishing the well-known improvements in transmission efficiency and economies made possible by the application of the single sideband method to shortwave working.

In 1929 he demonstrated long distance telephone communication to ships at sea, conducting for the first time telephone conversations with the S.S. Berengaria in mid-ocean.

In the following years Mr. Deloraine made most important contributions in the utilization of ultra-high frequencies. It was under his direction that telephone and printer communications were established across the English Channel in 1931 and 1933 on approximately 1700 megacycles, using very sharp beams. In 1936 and 1937 his efforts made possible the first multichannel ultra-shortwave telephone link, first operated across the English Channel and later between Scotland and Ireland. Subsequently, his contributions to ultra-high frequency technique made possible the construction of the Eiffel Tower television transmitter, providing the highest power used in television transmission.

His role in the achievement of high-power broadcasting also was important. He was responsible for the Prague Station with 120-kw carrier power, inaugurated in 1932, followed two years later by the Budapest Station with the same carrier power and unique for its anti-fading mast antenna over 1,000 feet high, the highest antenna ever constructed.

In 1939 he made a proposal to the French Post and Telegraph Administration for a high frequency broadcasting center of twelve stations, each of 150-kw carrier power. His project was adopted and an order for four stations was placed with Le Materiel Telephonique, an I. T. & T. associate. The order for the other eight stations, following the same specifications, was allotted to two other concerns.

Mr. Deloraine was highly successful in directing experiments in connection with automatic radio compasses. This technique was demonstrated in the U. S. A. for aircraft for the first time in 1937.

He came to the United States in 1941 and took charge of the organization of the laboratories unit for the Federal Telephone and Radio Corporation, and under his direction the F. T. & R. Laboratories have made outstanding technical advances. His administrative responsibilities have not lessened his enthusiasm for specific personal contributions; he is continuing his inventive activities and is filing many patent applications despite his executive duties as head of a sizable organization.

Mr. Deloraine was made Chevalier of the Legion of Honor in 1938 for exceptional services to the French Post and Telegraph Administration. In 1939 he was elected Vice President of the French Institute of Radio Engineers. He was a member, from 1927 on, of the International Consultative Committee of Long Distance Telephony. He is also a member of the French and Belgium Societies of Electricians, and of the French Astronomical Society, as well as a Fellow of the (American) Institute of Radio Engineers.

The above cited contributions will not be unfamiliar to readers of "Electrical Communication" since they have been recorded at some length in this journal.

Haraden Pratt

Awarded Medal of Honor

Haraden Pratt, Vice President, Chief Engineer and Director, Mackay Radio and Telegraph Company, Vice President and Director, Federal Telephone and Radio Corporation, Fellow, Director, Secretary and Past President, Institute of Radio Engineers—was presented with the Medal of Honor of the I.R.E. at the Institute's dinner held in New York City on January 28, 1944. The award, made annually, is for distinguished service in the field of radiocommunication.

The specific achievements for which Mr. Pratt was selected for the 1944 Medal of Honor were stated in the following citation which accompanied the formal presentation:

"In recognition of his engineering contributions to the development of radio, of his work in the extension of communication facilities to distant lands and of his constructive leadership in Institute affairs."

Mr. Pratt's radio career started in 1906 in the amateur wireless telegraph field in San Francisco. From 1910 to 1914 he was Secretary and President of the Bay Counties Wireless Telegraph Association, one of the few radio clubs in existence in those days. At the same time he was a commercial wireless telegraph ship and shore station operator and installer for the United Wireless Telegraph Company and the Marconi Wireless Telegraph Company of America in San Francisco.

After receiving his degree at the University of California, he was engaged as engineer in the construction and operation of the Marconi 300-kw trans-Pacific radio stations at Bolinas and Marshall, California. From 1915 to 1920 he was Expert Radio Aide, Bureau of Steam Engineering, U. S. Navy Department. He was placed in charge of Radio Laboratory and engineering work at the Mare Island Navy Yard, California; later, he was sent to Washington, D. C. to take charge of construction and maintenance of all high power Navy radio stations, including those of private companies operated by the Navy Department during the last war.

From 1920 to 1923 he was Engineer of the Federal Telegraph Company at Palo Alto, California, in charge of factory operations and construction of Federal's Pacific Coast radio telegraph system, now part of Mackay Radio and Telegraph Company's radio network.

From 1925 to 1927 he constructed and supervised operation of a shortwave, point-to-point radio telegraph system for Western Air Express. From 1927 to 1928 he was in charge of development of radio aids for Air Navigation, Bureau of Standards, Department of Commerce, Washington, D. C.

He became Chief Engineer of Mackay Radio in 1928 and, soon thereafter, was made Vice President. He served as Company Representative at meetings of the International Radio Consultative Committee in Bucharest in 1937 and at the International Radio and Telegraph Conferences in Cairo in 1938. He also served as U. S. Government Technical Advisor at the International Radio Conference at Washington, D. C. in 1927 and on the Consultative Committee on Radio at Copenhagen in 1931. From 1939 to 1942 he was a Director of the American Standards Association.

Mr. Pratt is the Delegate of the Institute of Radio Engineers to the Radio Technical Planning Board and is Chairman of the Panel on Radio Communications of the Radio Technical Planning Board.

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COMPANHIA RADIO INTERNACIONAL DO BRASIL: *Radio Telephone and Telegraph Services*.....Rio de Janeiro
Additional Stations: São Salvador (Bahia), Belem, Curityba, Fortaleza, Natal, Porto Alegre, Recife.

COMPANHIA TELEFONICA PARANAENSE, S. A.: *Telephone Operating System* ....Curityba

COMPANHIA TELEPHONICA RIO GRANDENSE: *Telephone Operating System* ....Porto Alegre

CHILE

*COMPAÑÍA STANDARD ELECTRIC, S.A.C.: *Manufacturer of Communication and Other Electrical Equipment*
Santiago

COMPAÑÍA DE TELÉFONOS DE CHILE: *Telephone Operating System* ....Santiago

COMPAÑÍA INTERNACIONAL DE RADIO, S. A. (CHILE): *Radio Telephone and Telegraph Services*.....Santiago

CUBA

CUBAN TELEPHONE COMPANY: *Telephone Operating System* ....Havana

RADIO CORPORATION OF CUBA: *Radio Telegraph Service*
Havana

MEXICO

MEXICAN TELEPHONE AND TELEGRAPH COMPANY: *Telephone Operating System*................Mexico City

PERU

COMPAÑÍA PERUANA DE TELÉFONOS LIMITADA: *Telephone Operating System*......................Lima

PUERTO RICO

PORTO RICO TELEPHONE COMPANY: *Telephone Operating System*..............................San Juan

RADIO CORPORATION OF PORTO RICO: *Radio Telephone Service and Radio Broadcasting*............San Juan

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Associate Companies in the British Empire

*STANDARD TELEPHONES AND CABLES, LIMITED: *Manufacturer of Communication and Other Electrical Equipment* ....London, England
Branch Offices: Birmingham, Leeds, England; Glasgow, Scotland; Cairo, Egypt; Calcutta, India; Pretoria, South Africa.

*CREED AND COMPANY, LIMITED: *Manufacturer of Teleprinters and Other Communication Equipment*
Croydon, England

INTERNATIONAL MARINE RADIO COMPANY, LIMITED: *Supplies, Operates and Maintains Marine Communication and Navigational Equipment*..Liverpool, England

*KOLSTER-BRANDES LIMITED: *Manufacturer of Radio Equipment* ....Sidcup, England

*STANDARD TELEPHONES AND CABLES PTY. LIMITED: *Manufacturer of Communication and Other Electrical Equipment* ....Sydney, Australia
Branch Offices: Melbourne, Australia; Wellington, N. Z.

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